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In this work we establish the necessary grounds in order to use the concepts of vector theory with wider purposes. The main purpose is to settle a reference theory to deal with the physical, chemical and biological entities that may allow the study of the ecological equilibrium of human life.

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A THEORY FOR CLASSIFIED QUANTITIES

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ABSTRACT

In this work we establish the necessary grounds in order to use the concepts of vector theory with wider purposes. The main purpose is to settle a reference theory to deal with the physical, chemical and biological entities that may allow the study of the ecological equilibrium of human life.

INTRODUCTION

In this work we establish the necessary grounds in order to use the concepts of vector theory with wider purposes. The main purpose is to settle a reference theory to deal with the physical, chemical and biological entities that may allow the study of the ecological equilibrium of human life.

The history of Mathematics does not go backward in time long enough to reach the days and places, when and where the earliest mathematical concepts appeared. The oldest documents (egyptian papyrus, mesopotamian tablets) revealed that considerable progress was already made by the time they were written (Boyer, 1968). Reasoning, however, seems to indicate that among the first concepts was that which is called a classified quantity expressed as:

$$\underline{v} = v\,\hat{e}\,, \tag{1}$$

where v represents a quantitative notion and $\hat{e}$ stands for a qualitative conception of the represented entity. For instance, the expression: some water has a quantitative idea, "some", and a qualitative description, "water".

The Human thinking has more tendency into the analysis than into the synthesis. If man has ever thought about a classified quantity, he soon inferred from it the abstract notion of a quantity (and later a number) and the abstract notion of forms (and later the geometric figures). Therefore, as long as history goes, the mathematical study was already split into the science of numbers (arithmetics) and the science of forms (geometry). Of course, none of these sciences could be associated in a one to one relationship with natural entities. Their incompleteness was felt all along the History of Mathematics (see Boyer, 1968).

Occasionally the two primitive sciences were put together like it was done, for instance, by Newton in his Caleuli. However, only in the nineteenth century appeared the first mathematical entity similar to that defined in (1). It was called vector, characterizing a magnitude (quantitative part) and a direction (qualitative part). Still in the nineteenth century appeared another mathematical notion related to (1): the set. Defined as a collection of objects viewed as a single entity, the set concept is very close to that of a classified quantity. Metric spaces established the link between the two primitive sciences (see Gerla and Volpe, 1985).

The first ideas of this work appeared when the author tried to generalize the concept of abstract vector spaces. In order to acomplish this purpose, one must release the constraints imposed by the operational rules (e.g. the parallelogram rule for vectors). In this case a union of vectors would replace the addition of vectors and so on. The recognition that vectors belong to a larger class of mathematical entities, that of classified quantities, is not new and can be found in the traditional texts on vector analysis (e.g. Craig, 1943; Brand, 1947). However, no formal algebra was suggested towards a unified theory.

To some extent this work follows the ideas of Fréchet (1906). While this author was worried in associating a numerical value to each element of a set and hence studying the resulting set-theoretical relations that are independent of the nature of the set elements, we, here, associate a magnitude and quality to each set and try to look for the theoretical grounds applicable to a collection of classified quantities. Thus, physical entities like 8 kg, chemical entities like 10 atoms of oxygen, and biological entities like 5 wolves etc., are all classified quantities.

In this work we attempt to include all natural system (e.g. physical, chemical, biological) into a unique category (see

Eilenberg and MacLane, 1945). Moreover we show that this category is equivalent to that of vector spaces.

2 - FORMAL THEORY

Once it is established that objects are classified quantities, the next step in the characterization of a category (Eilenbert and MacLane, 1945) is to settle the mappings that the objects can bear.

The necessary mappings must preserve the natural processes of comparing objects (definitions I, II and III), modifying objects (definitions IV, V and VI) as well as those for measuring objects (definitions VII, VIII and IX).

At first, one must remember that magnitude and form are subjective notions. Therefore, if a set of classified quantities is formed on grounds on their shape to sight, no further details should be attributed to it or expected from it. The notions of element, collection and system are also considered primitives. On these grounds, we establish the basic definitions as:

Definition I: A relation is any sort of correspondence established between two considered classified quantities.

This definition refers to the procedure of collecting classified quantities pairwise. The usual mathematical relations of equality, inequality etc., as well as that of "contained in" used in set theory, are all comprised into this definition. The two considered qualities establish the appropriate reference to measure likeness or contrast. Thus, if the classified quantities express vector quantities, the relations of Vector Theory can be used. In nature, where the classified quantities express the components of sets, the Set Theory relations are more appropriate.

Definition II: Two given qualities, $\hat{e}_1$ and $\hat{e}_2$, are said to be independent of each other, if no additional quality $\hat{e}$ can be attributed simultaneously to the two given qualities.

On the same way as the concept of equality is established on the basis of absence of contrast (Boyer, 1968), we here establish the perfect contrast (independence of qualities) as the absence of likeness.

Two vectors are considered independent if they are orthogonal. In nature different classes (mammals, birds, etc.) are assumed to be independent as far as the needs of biological studies are concerned.

Definition III: Two classified quantities, $\underline{v}_1$ and $\underline{v}_2$, are said to be ordered within a system if the interchange of their relative positions modifies the system.

From the vector theory applied to physics we know that two opposite forces acting on a material point produce a motion in the direction of the largest force. The reversal of their magnitudes will certainly modify this system.

In nature, the interchange of the physical position of a fish with that of a bird (to place the fish in a tree and the bird within the water) modifies the forest system.

Definition IV: A transformation of a collection of classified quantities is any action which modifies either the quantity or the quality or both of one or more elements of the collection.

The binary operations on vectors, functions of vectors are the corresponding of transformations found in Vector Theory.

Physical processes like fusion, chemical reaction, animal growth are examples of transformations, which may be found in nature.

Corollary 1 - If the collection constitutes a system, we characterize the <u>transformation of a system</u>.

Corollary 2 - A transformation of an ordered collection of classified quantities is called an <u>ordered transformation</u>.

<u>Definition V</u>: A <u>uniform</u> under a given transformation is a classified quantity to which the transformation does not alter the quality.

In vector theory the eigenvectors correspond to the uniform defined here.

A wolf is an example of a <u>uniform</u> under the action of <u>age</u>, which is found in nature. Here again this statement refers to simple observations of form. A dragon-fly, of course, does not preserve its form in the course of its life.

<u>Definition VI</u>: A <u>binary operation</u> on a set of classified quantities is a transformation which atributes to two elements of the set another classified quantity of the same set.

Equivalent to this definition one finds that of a binary operation in vector theory (e.g. the addition of two vectors; the result of a product of a vector by a scalar for linear vector spaces).

If we consider the whole population of animals in a region, the union of two flocks of these animals constitutes a natural example of a binary operation.

Definition VII: An inner product is a transformation of two classified quantities into a number, whose magnitude symbolizes the extent of likeness of the two classified quantities, subject to the following constraints:

a) the product is symmetric relative to the two classified quantities;

b) it vanishes if the classified quantities are independent.

The entity characterized here is equivalent to the inner product defined in vector theory.

Considering the equivalence between sets and classified quantities, representing natural collections, one possible inner product in this case is:

$$p = \text{mag}\ (\underline{v}_1 \cap E\ \hat{e})\ \text{mag}\ (\underline{v}_2 \cap E\ \hat{e}) \quad , \tag{2}$$

where $\underline{v}_1$ and $\underline{v}_2$ represent the two classified quantities, $E\ \hat{e}$ is the universe of the common quality $\hat{e}$ to $\underline{v}_1$ and $\underline{v}_2$. Here, mag ( ) is the number which expresses the quantity of the classified quantity in parenthesis.

If more than one quality is common to the two classified quantities, the sum of the individual inner products will symbolize the extent of likeness of the considered entities.

Definition VIII: A metric is a transformation of two classified quantities into a number, whose magnitude symbolizes the extent of contrast of the two classified quantities, subject to the following constraints:

a) it is symmetric relative to the classified quantities;

b) it vanishes if the two classified quantities are identical.

In vector theory the metric $d = |\underline{v}_1 - \underline{v}_2|$, where $\underline{v}_1$ and $\underline{v}_2$ are two vectors, is a typical example of the entity characterized in this definition.

In nature, since a classified quantity is equivalent to a set of elements, a metric for it can be expressed using the set theory notation as:

$$d = \text{mag}\,[(\underline{v}_1 \cup \underline{v}_2) - (\underline{v}_1 \cap \underline{v}_2)] \quad . \tag{3}$$

Definition IX: A space of classified quantities is a set of these entities together with a logical relation to establish likeness or contrast and a measure transformation (inner product or metric) to (evaluate) likeness or contrast.

Abstract vector spaces with inner product are examples of the entity characterized here for which likeness is the bases for comparisons.

The space of solids defined in Gerla and Volpe (1985) is a typical example of a space built on the logical grounds of contrast.

Natural spaces can be based on both likeness and contrast, being thus equivalent to metric inner product vector spaces.

3 - DISCUSSION AND CONCLUSION

A set of definitions were proposed in this work to characterize classified quantities with a collection of mappings into a category (Eilenberg and MacLane, 1945). The correspondence between classified quantities and vectors was established throughout. Therefore, it is a straightforward matter to characterize the functor that establishes the equivalence of the category of classified quantities with that of vectors. This may allow the study of ecological equilibrium based on isomorphic transformations of vetor spaces.

The theory is left with its general character so that one could look at it as a generalization of the vector theory, or else as an equivalent theory to this one using the concept of Eilenberg and MacLane (1945).

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