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14. Abstract/Notes

This paper presents some aspects of control for nonlinear systems of the type $\dot{z} = Az + Nz + Bu$, where A is linear, N is nonlinear, B is linear, z is the state and u is the control. It is assumed that the dynamics of the linear part (i.e., $\dot{z} = Az$) can be described in terms of a strongly continous semigroup on an appropriate Banach space (the state space), so that this formulation includes distributed parameter systems and delay systems, as well as lumped parameter systems. Theoretical results in controllability are obtained by using fixed point theorems. It is considered the space X of functions from the interval [0,T] to Z (the possible trajectories of the system). In this setting, some mappings $F:X \to X$ are constructed and the solution of the problem of nonlinear controllability is obtained via the fixed points of such F's.

15. Remarks

This paper has been presented at the IV Encontro Regional de Matemática Aplicada e Computacional (IV ERMAC) held in Belo Horizonte - MG, from 26 to 28 of March, 1987).

SOME ASPECTS OF CONTROL FOR SEMILINEAR DISTRIBUTED PARAMETER SYSTEMS

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Here we consider semilinear systems of the type

$$\dot{z} = Az + Nz + Bu, \quad z(0) = z_0 , \tag{1}$$

where A is a linear operator on a Banach space Z (the state space), N a nonlinear operator, B a linear operator from an input space U to Z and u the control.

The problem of controllability is to find a control $u \in \mathcal{U}$ (where $\mathcal{U}$ is a space of functions from [0,T] to the input space U and assumed to be a reflexive Banach space) which drives the system from the initial state z_0 at t = 0 to a given desired $z_d \in Z$ at t = T.

As an example of system (1), consider the following diffusion process on $Z = L^2(0,1)$ described by the partial differential equation.

$$\frac{\partial z}{\partial t}(x,t) = \frac{\partial^2 z}{\partial x^2}(x,t) + Nz(,t) + u(x,t) \tag{2}$$

with boundary and initial conditions

$$z_x(0,t) = z_x(1,t) = 0, \qquad\qquad z(x,0) = z_0(x),$$

where $b(.) \in L^2(0,1)$ and N is a nonlinear operator on $L^2(0,1)$ such as

$$Nz = z^4,\ zz_x,\ z^2 z_x, \text{ etc.}$$

The above system can be expressed in the form (1) by setting $U = L^2(0,1)$, B the identity on U and A the linear operator on $L^2(0,1)$

$$Az = \frac{\partial^2 z}{\partial x^2} = z_{xx}$$

with domain

$$D(A) = \{z \in L^2(0,1) : \frac{\partial^2 z}{\partial x^2} \in L^2(0,1), \frac{\partial z}{\partial x} = 0 \text{ at } x = 0,1\}$$

It can be shown (see p. 46 of [1]) that A generates a strongly continuous semigroup $S(t)$ on Z given by

$$S(t)z = \sum_{n=1}^{\infty} e^{-n^2\pi^2 t}\phi_n < z, \phi_n > , \quad \text{where } \phi_n(x) = \sqrt{2} \sin n\pi x.$$

Fixed point theorems have been used since the beginning of the century to show local existence theorems for differential equations. Recently, several papers [2 - 11] have been published using fixed point theorems to provide existence of solutions of the problem of nonlinear controllability.

The nonlinear control problem (1) written in the mild form is.

$$z(t) = S(t)z_0 + L(t)Nz(.) + L(t) Bu. \tag{3}$$

For $t = T$ this equation becomes

$$z(T) - S(T)z_0 - L(T) Nz(.) = L(T)Bu = Gu.$$

Now define $u^* \in \mathcal{U}$ by

$$u^* = G^\dagger[z_d - S(T)z_0 - L(T)Nz^*(.)]. \tag{4}$$

So, the control u^* depends on the knowledge of the actual trajectory $z^*(.)$ of the system. Substituting $z(.)$ and u in (3) by $z^*(.)$ and u^* given in (4), one gets

$$z^*(.) = S(.)z_0 + L(.)Nz^*(.) + L(.)BG^\dagger[z_d - S(T)z_0 - L(T)Nz^*(.)],$$

which is an expression of the actual trajectory depending on itself. Clearly, $z^*(.)$ is a fixed point of the mapping $\phi : X \to X$ defined by

$$\phi z(.) = S(.)z_0 + L(.)Nz(.) + L(.)BG^{\dagger}[z_d - S(T)z_0 - L(T)Nz(.)].$$

That is $z^*(.) = \phi z^*(.)$.

The aim is to impose conditions on the various operators so that the map ϕ has a fixed point on some suitable space of trajectories X. If for the moment this is assumed, then

$$(\phi z^*(.))(T) = S(T)z_0 + L(T)Nz^*(.) + GG^{\dagger}[z_d - S(T)z_0 - L(T)Nz^*(.)].$$

But $GG^{\dagger}$ is the orthogonal projection of Z onto Range (G), hence if

$$[z_d - S(T)z_0 - L(T)Nz^*(.)] \in \text{Range (G)},$$

then

$$(\phi z^*)\ (T) = z_d$$

and so $z^*\ (T) = z_d$.

Several fixed point theorems for ϕ have been developed in [2 - 11]. In most of these papers no attempt was made to optimize the conditions on the various operators and often crude estimations have been made. The crucial assumption here is that Range(G) is closed in Z, since this means that $G^{\dagger}$ is bounded (or continuous). The way one can achieve this can be illustrated by the following example.

Consider the nonlinear diffusion equation (2). If $U = L^2(0,T;L^2\ (0,1))$, it is easy to show that the linear part is exactly controllable to $H_0(0,1)$ which is not closed in $L^2(0,1)$ although, of course, it is dense. One way of ensuring that the Range (G) is closed in Z is to choose $Z = H_0^1(0,1)$. The restriction of the semigroup S(t) to H_0^1 is also a semigroup and for any small $\delta > 0$

$$S(.) \in L^1[0,T;L(H^{-1+\delta}(0,1),\ H_0^1(0,1))].$$

This allows for a large class of nonlinearities Nz such as z^4, z_x^2 etc. [11].

Range (G) can also be made closed by enlarging the control space; for example let $\mathcal{U}$ be the space of functions from $[0,T]$ to $U = L^2(0,1)$ defined by: $u \in \mathcal{U}$ if

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t)\phi_n(x)$$

with the norm

$$\|u(.,.)\|^2 = \sum_{n=1}^{\infty} \int_0^T (u_n^2(t)/2n^2\pi^2)dt.$$

Then, the linear system can be exactly controlled to $Z = L^2(0,1)$. The semigroup $S(t)$ also satisfies

$$S(.) \in L^1[0,T; (H^{-2+\delta}(0,1), L^2(0,1))],$$

so nonlinearities Nz, which are local contractions from $L^2(0,1)$ to $H^{-2+\delta}(0,1)$, can be allowed.

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