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This paper deals with the theoretical aspects of using spectral analysis in middle atmosphere modelling. We start from the hydrodynamic equations on which a wavelike solution is imposed. This procedure allows us to identify: a) a dispersion relation; b) excitation and damping mechanism; c) mutual coupling between each pair of modes. Next we discuss the problem of choosing appropriate boundary conditions. Finally we comment on the accuracy aspects of this type of modelling.

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THE SPECTRAL ANALYSIS APPROACH TO MIDDLE ATMOSPHERE MODELLING

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ABSTRACT

This paper deals with the theoretical aspects of using spectral analysis in middle atmosphere modelling. We start from the hydrodynamic equations on which a wavelike solution is imposed. This procedure allows us to identify: a) a dispersion relation; b) excitation and damping mechanism; c) mutual coupling between each pair of modes. Next we discuss the problem of choosing appropriate boundary conditions. Finally we comment on the accuracy aspects of this type of modelling.

INTRODUCTION

To model the atmosphere is to find an analytical, empirical or numerical slution to the hydrodynamic system of equations written as:

$$\partial \mathbf{h}/\partial t = \delta \mathbf{h} - \nabla \cdot \underline{\phi} \tag{1}$$

where **h** is the vector of the hydrodynamics parameters: mass, momentum and energy, $\delta \mathbf{h}$ is the local rate of change of these parameters and $\underline{\phi}$ is the matrix of the corresponding fluxes.

In this work we are particlarly interested in the spectral analysis approach to atmospheric modelling. In this approach the driving term $\delta \mathbf{h}$, resulting from the balance between excitation and damping mechanisms, is represented by the sum of a finite number of wavelike components and the derived hydrodynamic parameters are also assumed to have a similar wavelike decomposition. Frequency range response of the middle atmosphere, even when a single component mode excites it (Salby, 1984), as well as the existence of a periodic response in the absence of excitation (Salby, 1981) considerably increase the complexity of this type of approach.

The forcing mechanisms may be steady, such as the regular solar radiation (which drives tides and quasi-stationary planetary waves), or random, such as particle precipitation (which drives gravity waves). The first set of mechanisms have a worldwide influence whereas the second is usually localized. Large amplitude responses, from the middle atmosphere, to one of these mechanisms may drive other oscillations through wave breaking (Lindzen, 1981).

To compromise with the actual situation existing in the middle atmosphere, we opted in this work to solve Equation 1 using the method of separation of variables. This approach has the advantage that it allows us to use relations involving the actual space-time response of the middle atmosphere to forcing mechanisms.

THE HYDRODYNAMIC EQUATIONS

The hydrodynamic equations are detailed using the following relations:

$$h_1 = \rho, \; h_2 = \rho \mathbf{u}, \; h_3 = \rho \, C_p \, T \; ; \tag{2}$$

$$\delta h_1 = 0, \; \delta h_2 = -\nabla p + \rho \mathbf{g} - 2\Omega \times h_2 \, ,$$
$$\delta h_3 = (Q - L) - \mathbf{u} \cdot \delta h_2 + \mathbf{u} \cdot \nabla(\gamma p) \; ; \tag{3}$$

$$\phi_1 = h_1 \mathbf{u}, \; \phi_2 = h_2 \mathbf{u} + \underline{\phi}\mathrm{vis}, \; \phi_3 = (h_3 + \gamma p)\mathbf{u} + \phi_{heat} \tag{4}$$

$$\underline{\phi} = -\eta[\nabla \mathbf{u} + (\nabla \mathbf{u})^t - (2/3)\underline{I} \; \nabla \cdot \mathbf{u}] \tag{5}$$

$$\phi_{heat} = -\lambda \nabla T \tag{6}$$

where ρ is the air density, T its temperature, p its pressure, C_p is the specific heat at constant pressure, γ is the ratio between specific heats, η is the viscosity coefficient and λ is the thermal conductivity. The vector symbols represent the wind velocity by $\mathbf{u}$, the acceleration due to gravity by $\mathbf{g}$ and the earth's angular velocity by Ω. The matrix symbol $\mathbf{I}$ stands for the identity matrix. Thermal excitation is the only energy source considered through the local energy balance (Q-L), where Q is the rate of energy deposition and L the rate of energy loss.

Our next step is to consider that the source terms are space-time periodic functions. The derived atmospheric parameters: ρ, $\mathbf{u}$ and T, are assumed to be composed of a constant added to a space-time periodic function. The pressure, p, relates to these parameters through the equation of state:

$$p = \rho \, RT/M$$

where R is the universal gas constant and M is the mean molecular mass of the air in a.m.u.

Henceforth the symbols ρ, **u** and T will be used for the average density, wind and temperature. The amplitude of the space time component of these parameters will be denoted with the same symbols tildered. Assuming the amplitude of this component can be as large as 30% of that of the constant term, first and second powers of the space-time term need to be considered.

THE SPACE-TIME EQUATIONS WITH NO FORCING MECHANISM

In this section the space-time dependence of the parameters is made explicit by replacing the density, wind and temperature in Equations 2-6 respectively by: $\rho + \tilde{\rho}\ a(t)\ \hat{a}(s)$, $\mathbf{u} + \tilde{\mathbf{u}}\ a(t)\ \hat{a}(s)$, $T + \tilde{T}a(t)\ \hat{a}(s)$; where a(t) is a function dependent only on time and a(s) is a function of space coordinates exclusively. A series of powers $[a(t)a(s)]^{i}$, where i varies from 0 to 3, is obtained. Equating the coefficients of equal powers, on both sides of Equation 1, four sets of equations result. These equations can all be put in the standard form:

$$\mathbf{D}_t(t) + \mathbf{D}_s(s) = \mathbf{C}\ , \tag{8}$$

where **D** stands for a differential expression involving only time derivatives (subscript t) or space derivatives (subscript s). Here **C** represents a constant vector which may eventually be the null vector.

Before we present the equations referring to the to the power i = 1 of $[a(t)a(s)]^{i}$ in the form of Equation 8, some additional simplifying assumptions will be made. The first one is the validity of the hydrostatic equilibrium for the zeroth order equation (i = 0). The other assumptions concern the use of constant values for η, λ, γ, C_P, and M moreover a simplified expression for the viscosity term (valid for incompressible fluids) will be used. The linear equations then become:

$$\left(\frac{\rho}{\rho}\right) \frac{a'(t)}{a(t)} + \left(\mathbf{u} + \frac{\rho}{\rho}\,\mathbf{u}\right) \cdot \frac{\nabla\hat{a}(s)}{\hat{a}(s)} = \left(\frac{\nabla\rho}{\rho}\right) \cdot \mathbf{u} \tag{9}$$

$$\left(\mathbf{u} + \frac{\rho}{\rho}\,\mathbf{u}\right) \frac{a'(t)}{a(t)} - \frac{n}{\rho}\,\mathbf{u}\,\frac{\nabla^2\hat{a}(s)}{\hat{a}(s)} + \left(\mathbf{u}\,\mathbf{u} + \mathbf{u}\,\mathbf{u} + \frac{\rho}{\rho}\,\mathbf{u}\,\mathbf{u}\right) \cdot \frac{\nabla\hat{a}(s)}{\hat{a}(s)} +$$

$$+ RTM^{-1}\left(\frac{\rho}{\rho} + \frac{T}{T}\right) \frac{\nabla\hat{a}(s)}{\hat{a}(s)} = \frac{\rho}{\rho}\,\mathbf{g} - 2\Omega \times \mathbf{u} -$$

$$- 2\,\frac{\rho}{\rho}\,\Omega \times \mathbf{u} - (\mathbf{u}\,\mathbf{u} + \mathbf{u}\,\mathbf{u}) \cdot \frac{\vec{\nabla}\rho}{\rho} - RTM^{-1}\left(\frac{T}{T}\frac{\nabla\rho}{\rho}\quad \frac{\rho}{\rho}\,\frac{\nabla T}{T}\right), \tag{10}$$

$$C_p\, T\left(\frac{\rho}{\rho} + \frac{T}{T}\right) \frac{a'(t)}{a(t)} - \lambda \frac{T}{T} \frac{\nabla \hat{a}(s)}{\hat{a}(s)} + T(C_p + \gamma R M^{-1})\, \mathbf{u} \cdot \frac{\nabla \hat{a}(s)}{\hat{a}(s)} +$$

$$+ TC_p\left(\frac{\rho}{\rho} + \frac{T}{T}\right) \mathbf{u} \cdot \frac{\hat{a}(s)}{\hat{a}(s)} = - TC_p\, \mathbf{u} \cdot \left(\frac{\nabla\rho}{\rho} + \frac{\nabla T}{T}\right), \qquad (11)$$

where the primes indicates the first order derivative relative to the specified variable.

In what follows we consider a short range of altitudes such that $(\nabla\rho)/\rho$, $(\nabla T)/T$, $\mathbf{u}$, ρ, T, $\mathbf{u}$ can all be taken as constants. As a result the time differential expression must be a constant and the space differencial expression should be a constant as well. The constants are written as:

$$\frac{a'(t)}{a(t)} = \omega \qquad (12)$$

$$\frac{\nabla a(s)}{a(s)} = \mathbf{k} \qquad (13)$$

The difference between our approach and the traditional linear approach is that here the angular frequency, ω, is the fundamental angular frequency of the actual observed time variation (resulting from the superposition of several waves). The constant, $\mathbf{k}$, also refers to the actual propagation constant existing in the atmosphere. Both and $\mathbf{k}$ can be purely imaginary numbers, corresponding to sinusoidal functions, or could include a real part, corresponding to damping or growth.

OSCILLATORY RELATIONS

Equations 9-11 can be written in a matrix form as:

$$\mathbf{D}(\omega, \mathbf{k})\mathbf{v} = \mathbf{0} \qquad (14)$$

where $\mathbf{D}(\omega, \mathbf{k})$ is a matrix which depends also on the zeroth order parameters and $\mathbf{v}$ is the vector of the amplitude of the periodic components of the atmospheric parameters.

Considering the unperturbed atmosphere as horizontally stratified the gradients of the zeroth order parameters will be nonzero only along the radial component. It then follows that the elements of the matrix $\mathbf{D}(\omega, \mathbf{k})$ are expressed as:

$$d_{11} = \omega + \mathbf{u} \cdot \mathbf{k}\ , \quad d_{12} = k_r + \frac{\partial\rho}{\partial\rho r}\ , \quad d_{13} = k_\theta\ ,$$

$$d_{14} = k_\psi\ , \ d_{15} = 0\ ; \quad d_{21} = (\omega + \mathbf{u} \cdot \mathbf{k})\ u_r + g + RTM^{-1}\left|k_r + \frac{\partial T}{T\partial r}\right|\ ,$$

$$d_{22} = \left(\omega + \mathbf{u} \cdot \mathbf{k}\right) - k^2\,\eta\,\rho^{-1} + k_r\ u_r + 2u_r\ \frac{\partial\rho}{\rho\partial r}\ ,$$

$$d_{23} = k_\theta\ u_r + u_r\ \frac{\partial\rho}{\rho\partial r}\ , \quad d_{24} = k_\psi u_r + u_r\ \frac{\partial\rho}{\rho\partial r}\ ,$$

$$d_{25} = RTM^{-1}\left(k_r + \frac{\partial\rho}{\rho\partial r}\right);\ d_{31} = (\omega + \mathbf{u} \cdot \mathbf{k})\ u_\theta + k_\theta\ RTM^{-1}\ -$$

$$-\ 2\Omega\ \sin\ \theta\ u_\theta\ , \quad d_{32} = k_r\ u_\theta + u_\theta\ \frac{\partial\rho}{\rho\partial r}\ ,$$

$$d_{33} = (\omega + \mathbf{u} \cdot \mathbf{k}) - k^2\ \eta\ \rho^{-1} + k_\theta\ u_\theta - 2\Omega\ \sin\ \theta\ ,$$

$$d_{34} = k_\psi\ u_\theta\ , \quad d_{35} = k_\theta\ RTM^{-1}\ ;\ d_{41} = (\omega + \mathbf{u} \cdot \mathbf{k})\ u_\psi\ +$$

$$+\ k_\psi\ RTM^{-1} + 2\Omega\ \sin\ \theta\ u_\psi\ ; \quad d_{42} = k_r\ u_\psi + u_\psi\ \frac{\partial\rho}{\rho\partial r}\ ;$$

$$d_{43} = k_\theta\ u_\psi\ , \quad d_{44} = (\omega + \mathbf{u} \cdot \mathbf{k}) - k^2\ \eta\ \rho^{-1} + k_\psi\ u_\psi\ +$$

$$+\ 2\Omega\ \sin\ \theta\ , \quad d_{45} = k_\psi\ RTM^{-1}\ ; \quad d_{51} = (\omega + \mathbf{u} \cdot \mathbf{k})\ TC_p\ ,$$

$$d_{52} = T(C_p + \gamma\ RM^{-1})\ k_r + TC_p\left(\frac{\partial\rho}{\partial\rho r} + \frac{\partial T}{T\partial r}\right)\ ,$$

$$d_{53} = T(C_p + \gamma\ RM^{-1})\ k_\theta\ , \quad d_{54} = T(C_p + \gamma\ RM^{-1})\ k_\psi\ ,$$

$$d_{55} = (\omega + \mathbf{u} \cdot \mathbf{k})\ TC_p - \lambda\ k^2$$

and the vector **v** is given by:

$$\mathbf{v} = (\rho/\rho \quad u_r \quad u_\Theta \quad u_\Psi \quad T/T)^t \ , \tag{16}$$

where r is the radial coordinate, of a spherical coordinate system with origin at the center of the Earth; Θ stands for latitude and Ψ for longitude.

The dispersion relation is obtained by the equation:

$$\text{Det } \mathbf{D} = 0 \ , \tag{17}$$

and will not be developed further in this work. To compute the amplitude of the oscillations we include on the right hand side of Equation 14 the vector of excitation and damping mechanism.

It is possible to observe the strong influence of a constant wind on the matrix **D**. Constant wind can be understood also as a planetary oscillation (of semiannual periodicity) when tidal periodicities are being considered. More generally, any long periodicity oscillation can produce the "constant wind" for a short periodicity oscillation.

In the present approach the most important mechanisms for transmission of energy and momentum (see Lindzen, 1968) were taken into account for a realistic atmosphere. It can be considered as an extension of the theory of gravity waves by Yeh and Liu (1974), and can be applied to tides and planetary waves as well.

The driving mchanisms for oscllations are the ones which produces momentun and/or energy variations. In particular, the regular solar radiation, absorbed as heat energy by ozone (around 45 km altitude) and by water vapour (around 8 km altitudes), constitute an important driving mechanism for tides and planetary waves (see Forbes and Garrett, 1978). Planetary waves can also be generated in the troposphere (see Salby, 1984). As for the gravity waves some sources are identified and others are questioned (Hines, 1972). Concerning these waves only the ones that satisfy Equations 12 and 13 can be treated with the present approach.

From the theoretical point of view we assume a heating mechanism of the form qf(t)f(s). The loss mechanism is the Newtonian cooling of the form Ta(t)a(s). The driving mechanism vector then becomes:

$$\mathbf{w} = \left(0 \quad 0 \quad 0 \quad 0 \quad \frac{q\ f(t) \quad f(s)}{\rho T\ a(t)\ \hat{a}(s)} \ - \ \frac{\alpha T}{T} \right)^t \tag{18}$$

and again we expect, at least in an approximate sense, that:

$$f(t)/a(t) = \beta \, , \tag{19}$$

$$f(s)/a(s) = \kappa \, , \tag{20}$$

where β and are constants. When a small frequency spread response occurs it is more likely that Equations 19 and 20 be satisfied in an average sense.

Mutual coupling between modes affect even the linear equations through the terms: $(\mathbf{uu} + \mathbf{uu}).[\nabla a(s)]/a(s)$ and $(\mathbf{uu} + \mathbf{uu}).(\nabla \rho)/\rho$, although the coupling can be better appreciated in the higher order equations ($i = 2, 3$). The conclusion here is identical to that of Spizzichino (1969), namely the appearance of sums and differences of the frequencies of the two considered modes as well as sums and differences of the respective propagation constants.

DISCUSSION AND CONCLUSIONS

Once established the basic oscillatory relations and the driving mechanisms the problem of spectral analysis modelling can be completed if appropriate boundary conditions are taken. For middle atmosphere modelling it seems convenient to choose the altitudes of 20 km and 70 km as the zeros of local heating (see Forbes and Garrett, 1976). At these two altitudes, the knowledge of the vector $\mathbf{v}$ is necessary to completely account for momentum and energy exchange between the middle atmosphere and its upper and lower boundaries.

The present approach reduces to the traditional forms, to deal with planetary waves (Salby, 1984), tidal waves (Forbes, 1982a,b) and gravity waves (Yeh and Liu, 1974), when we consider $a(t) = \exp j\omega t$ and $a(s) = \exp -j\mathbf{k}.\mathbf{M}$. It is more appropriate, in the study of middle atmosphere waves, because we can deal with the a(t) and a(s) actually observed, including the efects of nonlinearities, mode couplings and interactions which affect the parameters of the medium. Although this aspect has not been fully explored in this work, because of its limited length, we believe that some insight can be gained when applying it to frequency spread responses of the middle atmosphere to oscillatory forcing mechanisms.

The relations obtained with higher order powers of the product [a(t)a(s)] are rich to the extent that, besides the appearance of sums and differences of the basic frequencies and harmonics of each considered fundamental frequency, they include interesting restrictions, like for instance $\mathbf{u}.\mathbf{k} = 0$, resulting from the continuity equation. These relations have not been worked out here, but we can point out that in the momentum equation allowance can be made for a varying η produced by the oscillatory perturbations (see Lindzen, 1981).

The most severe criticism to spectral analysis modelling of the atmosphere has always been that it is not able to reproduce transient situations. This restriction has not been eliminated in the present work but is somewhat attenuated since Equations 12, 13, 19 and 20 may be applied to these transients, at least in a approximate sense.

To conclude, the spectral analysis approach was undertaken in this work from a different point of view, namely using its direct connection to the method of separation of variables. By doing so we could determine a number of relationships more appropriate to the analysis of experimental data, since only first order numerical derivatives are necessary to determine the constant parameters ω and $\mathbf{k}$. Several experimental and theoretcal tests will be necessary to establish the extent to which the proposed method can contribute to improve our understandng of the intricate interactions which govern the behavior of middle atmosphere dynamics.

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