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14. Abstract/Notes

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NONLINEAR THEORY OF THE FREE-ELECTRON LASER

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ABSTRACT

A theory of Raman free-electron laser using a circularly polarized electromagnetic pump is investigated. Coupled wave equations that describe both linear and nonlinear evolution of stimulated Raman scattering are derived. The dispersion relation and the growth rate for the parametric instability are obtained. Nonlinear processes that may lead to saturation of the free-electron laser are discussed.

The generation of high-power coherent radiation using stimulated emission of backscattered radiation from intense relativistic electron beams has received considerable interest lately. This new radiation device called Free-Electron Laser (FEL) has a great advantage of being readily tunable over a wide frequency range, from submillimeter to optical regions, by varying the pump frequency or the electron beam energy. Many applications[1] can be found for FEL, for example, plasma heating, diagnostics, laser pellet fusion, isotope separation and radar.

The operative mechanism of FEL is based on the double Doppler effect. Consider a pump wave (ω_0, $\vec{k}_0$) propagating with a wavevector antiparallel to the velocity ($\vec{v}_b$) of a relativistic electron beam in the laboratory frame. In the beam frame, in which the quantities are primed, the frequency and wavenumber of the pump are first Doppler-shifted to

$$\omega_0' = \gamma_b(\omega_0 + v_b k_0), \quad k_0' = \gamma_b(k_0 + v_b\omega_0/c^2), \qquad (1), (2)$$

where $\gamma_b = (1 - v_b^2/c^2)^{-1/2}$. For a given backscattered wave, with $\vec{k}_s'$ antiparallel to $\vec{k}_0'$, the frequency and wavenumber in the laboratory frame are double Doppler-shifted to

$$\omega_s = \gamma_b(\omega_s' + v_b k_s'). \qquad (3)$$

Since typically in the beam frame $\omega_0' \cong \omega_s'$ and $k_s' \cong k_0'$, moreover in the laboratory frame $\omega_0 \cong ck_0$ for pump frequency much larger than the characteristic frequencies of the system, combining (1)-(3) then gives

$$\omega_s \cong 4\gamma_b^2\omega_0 \qquad (4)$$

in the limit $v_b/c \to 1$. For a relativistic electron beam the factor γ_b^2 can be much greater than unity, thus the scattered frequency can be much larger than the pump frequency. According to the Manley-Rowe relation, the ratio of the scattered wave energy W_s to the incident wave energy W_0 can be as large as $W_s/W_0 = \omega_s/\omega_0$. As a result, intense radiation with frequency upconversion and power gain is achieved.

FEL can operate in two distinct regimes[2], namely, the Compton regime in which the pump wavelength is smaller than or comparable to the Debye wavelength of the electrons and the interaction is dominated by wave-particle resonance, and the Raman regime in which the pump wavelength is much greater than the Debye wavelength and the interaction is dominated by wave-wave coupling.

Most research on FEL has dealt with a zero-frequency pump wave[3], namely, a magnetostatic ondulator since the electromagnetic source with power density high enough to excite the stimulated backscattering is not readily available. It has been suggested[4] that the radiation generated by an ondulator FEL can be reflected to act as a finite-frequency pump to interact with the same relativistic electron beam. In addition, rapidly developing high-power gyrotrons and relativistic magnetrons[5] can also provide suitable radio-frequency sources to operate an electromagnetic-pump FEL.

In this paper a theory of Raman FEL using a circularly polarized electromagnetic pump is investigated. The physical mechanism of stimulated Raman scattering can be described as follows. The introduction of a large-amplitude electromagnetic pump wave excites a transverse oscillation of the beam electrons. This transverse electron velocity coupled to electron density oscillations (space-charge waves) produces a current, which then generates a backscattered electromagnetic wave. The beating between the pump and scattered electromagnetic waves produces a density modulation and a grouping of the electrons into bunches along the beam axis. The growth of the space-charge waves gives increasing coherence to the scattering process, resulting in a growing scattered wave which in turn increases the density modulation still further. Hence there is a feedback mechanism in this process which may give rise to a parametric instability and exponential growth of both the space-charge wave and the scattered radiation. The instability is terminated when nonlinear saturation mechanisms such as pump depletion or particle trapping set in.

Consider the interaction of an intense electromagnetic pump wave ($\vec{k}_0 = -k_0\hat{z}$) with a cold counterstreaming relativistic electron beam ($\vec{v}_b = v_b\hat{z}$, $n_e = n_0$). Plane wave solutions are treated ($\nabla = \hat{z}\ \partial/\partial z$), thus the transverse canonical momentum of electrons is constant, i.e., $\vec{p}_{e\perp} \equiv m\gamma\vec{v}_{e\perp} = e\vec{A}_\perp/c$. Assume immobile ions to serve as the neutralizing background, and neglect self-fields associated with the electron beam.

The wave equation describing the transverse component of the vector potential ($\vec{A} = \vec{A}_\perp$, $A_z = 0$) is

$$\left(\frac{\partial^2}{\partial t^2} - c^2\ \nabla^2\right)\vec{A} = 4\pi c\,\vec{J}\,. \tag{5}$$

Separating the electron density n_e into homogeneous and inhomogeneous parts, i.e., $n_e = n_0 + n(z,t)$, (5) becomes

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \frac{\omega_p^2}{\gamma}\right)\vec{A} = -\frac{\omega_p^2}{n_0\gamma}\,n\vec{A}\,, \tag{6}$$

where $\gamma = (1 + \vec{p}_e \cdot \vec{p}_e)^{1/2}$ and $\omega_p^2 = 4\pi n_0 e^2/m$.

In order to cast the governing equations into the form of coupled wave equations, one needs a corresponding wave equation for the perturbed electron density n, which can be obtained from the following equations:

$$\left(\frac{\partial}{\partial t} + \vec{v}_e \cdot \nabla\right)\vec{p}_e = e\,\nabla\phi - \frac{e}{c}\,\vec{v}_e \times (\nabla \times \vec{A}), \tag{7}$$

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\vec{v}_e) = 0, \tag{8}$$

$$\nabla^2\phi = -4\pi\rho. \tag{9}$$

For the sake of simplicity all nonlinearities in (7)-(9), with the exception of the ponderomotive term in the RHS of (7), are neglected. This implies that nonlinear steepening terms that may lead to wavebreaking of space-charge waves are ignored ($n/n_0 \ll 1$); in addition all waves are assumed to have finite, but nonrelativistic ($v_{0\perp}^2/c^2$, $v^2/c^2 \ll 1$) amplitudes, so that

$$p_{ez} = p_{oz} + p_z \,, \qquad p_z = m\gamma_0^3\, v_z \,, \tag{10}$$

where $\gamma_0 = (1 - \vec{v}_0\cdot\vec{v}_0/c^2)^{-1/2} \approx (1 + p_{0z}^2/m^2c^2)^{1/2}$. Under these assumptions (7)-(9) become

$$(\frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla)\vec{p} = e\nabla\phi - \frac{e}{c}\,\vec{v}_e \times (\nabla\times\vec{A}), \tag{11}$$

$$(\frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla)n + \frac{n_0}{m\gamma_0^3}\,\nabla\cdot\vec{p} = 0, \tag{12}$$

$$\nabla^2\phi = 4\pi e n\,. \tag{13}$$

Substituting (12) and (13) into the equation resulting from taking the divergence of (11), then yields a wave equation for the space-charge wave

$$\left[(\frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla)^2 + \frac{\omega_p^2}{\gamma_0^3}\right] n = \frac{n_0 e^2}{2m^2c^2\gamma_0^4}\,\nabla^2(\vec{A}\cdot\vec{A}), \tag{14}$$

where $\gamma = \gamma_0$ is assumed in the RHS. (6) and (14) form a set of coupled wave equations that describes the stimulated Raman scattering and demonstrates clearly the feedback mechanism for the excitation of parametric instability.

The pump is taken to be a circularly polarized electromagnetic wave

$$\vec{A}_0 = A_0 \, [\hat{x} \cos(k_0 z - \omega_0 t) + \hat{y} \sin(k_0 z - \omega_0 t)] \, . \tag{15}$$

An integration of the zero-order equation of motion

$$\frac{d\vec{p}_0}{dt} = - \frac{e}{mc\gamma_0} \vec{p}_0 \times (\nabla \times \vec{A}_0) \tag{16}$$

gives

$$\vec{p}_0 = p_{0\perp} [\hat{x} \cos(k_0 z - \omega_0 t) + \hat{y} \sin(k_0 z - \omega_0 t)] + p_{0z} \hat{z} \, , \tag{17}$$

which shows that $\gamma_0 = (1 + p_{0\perp}^2 + p_{0z}^2)^{1/2}$ is constant.

To the zero-order, the ponderomotive coupling terms in the RHS of (6) and (14) vanish (n, $\nabla^2(\vec{A}_0 \cdot \vec{A}_0) \to 0$). Hence the two wave equations are decoupled, yielding the dispersion relations[6] for electromagnetic waves and space-charge waves, respectively,

$$\omega_0^2 - c^2 k_0^2 - \omega_p^2/\gamma_0 = 0 \, , \tag{18}$$

$$(\omega_\ell - k_\ell v_b)^2 - \omega_p^2/\gamma_0^3 = 0 \, . \tag{19}$$

A corresponding dispersion relation for the scattered electromagnetic wave can be obtained from (18) by substituting (ω_0, k_0) for (ω_s, k_s). Note that the slow branch of space-charge waves (19) is a negative-energy mode and plays a crucial role in the operation of FEL.

In linear theory the amplitude of the pump electromagnetic wave A_0 is assumed to be constant with dispersion relation given by (18), the backscattered electromagnetic wave is coupled to the space-charge wave, thus (6) and (14) become

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \frac{\omega_p^2}{\gamma_0}\right)\vec{A}_s = -\frac{\omega_p^2}{n_0\gamma_0}\,n\vec{A}_0\,, \tag{20}$$

$$\left[\left(\frac{\partial}{\partial t} + \vec{v}_0\cdot\nabla\right)^2 + \frac{\omega_p^2}{\gamma_0^3}\right] n = \frac{n_0 e^2}{2m^2c^2\gamma_0^4}\,\nabla^2(\vec{A}_0^{\,*}\cdot\vec{A}_s)\,. \tag{21}$$

Fourier - analyzing (20) and (21) gives the coupled-wave dispersion relation for the Raman backward scattering

$$\left(\omega^2 - c^2k^2 - \frac{\omega_p^2}{\gamma_0}\right)\left\{[(\omega-\omega_0) - (k+k_0)v_b]^2 - \frac{\omega_p^2}{\gamma_0^3}\right\} = \frac{e^2\omega_p^2(k+k_0)^2}{2m^2c^2\gamma_0^5}\,|A_0|^2, \tag{22}$$

where the following matching conditions are imposed

$$\omega_0 = \omega_s - \omega_\ell\,, \qquad k_0 = k_\ell - k_s\,. \tag{23}$$

Note that in the beam frame $\omega_0' = \omega_s' + \omega_\ell'$ (i.e., $\omega_s'/\omega_0' < 1$) since the energy of both scattered electromagnetic wave and positive-energy space-charge wave is provided by the pump wave; however, in the laboratory frame $\omega_s/\omega_0 > 1$ as evidenced in (23) since now the scattered wave energy is provided by both the pump wave and the negative-energy space-charge wave. In addition (23) shows that $\vec{k}_0$ is parallel to $\vec{k}_\ell$, but antiparallel to $\vec{k}_s$; thus the electron density perturbation propagates in the direction opposite to the beam direction.

The dispersion relation (22) shows that the backscattered electromagnetic wave is coupled to the space-charge wave through the pump electromagnetic wave. Since the slow mode of space-charge waves is a negative-energy wave, its interaction with the fast mode of electromagnetic waves may lead to instability, which develops in the regions near the intersection $(\bar{\omega}_0, \bar{k}_0)$ of the dispersion curves of the two interacting modes. An approximate solution of (22) can be obtained by expanding the dispersion relation about $(\bar{\omega}_0, \bar{k}_0)$, assuming weak interaction,

$$\omega = \left(c^2 k^2 + \frac{\omega_p^2}{\gamma_0}\right)^{1/2} + i\,\frac{e(k_0+k)A_0}{2\sqrt{2}\,\gamma_0^{7/4}\,mc(c^2k^2/\omega_p^2+\gamma_0^{-1})^{1/4}} \quad . \tag{24}$$

The real part of ω gives the double Doppler-shifted frequency (4) of the backscattered wave, whereas the imaginary part of ω gives the growth rate describing the exponential growth of unstable waves.

The efficiency of an electromagnetic-pump FEL is defined as the ratio of the scattered electromagnetic energy to the sum of the pump electromagnetic energy and the relativistic electron beam energy, which is determined by the nonlinear saturation mechanisms. Saturation of the backscattered electromagnetic wave may be due to either particle trapping or pump depletion, depending whether the pump field is strong or weak, respectively[7].

Particle trapping occurs when the density fluctuation of the space-charge wave becomes greater than or comparable to the equilibrium electron density ($n/n_0 \geq 1$). When this takes place, the electron dynamics becomes nonlinear and electrons are trapped in the potential well formed by the combined space-charge and ponderomotive potentials (vide the RHS of (7)). In this regime the assumptions used to derive (14) break down; a fully nonlinear and kinetic treatment then becomes necessary.

Pump depletion occurs when the amplitude of the pump wave is depleted to a level below the threshold required to sustain the parametric instability. Saturation by pump depletion can be studied by adding the pump evolution equation, determined from (6),

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \frac{\omega_p^2}{\gamma_0}\right)\vec{A}_0 = -\frac{\omega_p^2}{n_0\gamma_0} n^* \vec{A}_S , \qquad (25)$$

to (20) and (21). The nonlinear evolution of the pump, scattered and space-charge waves, as well as the saturation time for the instability, can be obtained either numerically or analytically in terms of Jacobi's elliptic functions by reducing (25), (20) and (21) to a set of coupled-mode equations[4,8-10].

The three-wave Raman backward scattering discussed in this paper is valid only for relatively low pump intensity. When the pump intensity is large, the space-charge wave loses its linear property and the wave coupling may excite the Raman oscillating two-stream instability (or modified Raman scattering). If the pump intensity is further increased, forward scattered electromagnetic waves ($\omega_S = \omega_0 - \omega_\ell$, $k_S = k_0 + k_\ell$) can also be induced, leading to the Raman forward scattering (or Raman modulational instability). Generation of these four-wave parametric processes[2,7] may modify the characteristics of a FEL.

To conclude, it worths mentioning that recent experiments on ondulator FEL performed at MIT[11], Columbia University[12] and NRL[13] have produced encouraging results. Further FEL experiments using electromagnetic pump should be carried out in order to determine the best scheme for generating stimulated backscattered radiation.

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