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14. Abstract/Notes

This paper presents some results associated with semilinear distributed parameter systems on a state space Z, with some parameter $\alpha \in \mathbb{R}^p$ *to be identified. Recently some techniques have been developed [2, 3, 4, 5, 6, 7 and 8] to solve the joint problem of state and parameter estimation which is to construct the state* $z(t)$, $t \in [0,t]$ *and identify the parameters* $\alpha \in \mathbb{R}^p$ *when the output observation* $y(t)$ *on* $[0,t]$ *is given. These techniques consider a new formulation with on enlarged state* $\bar{z}(t) = (z(t), \alpha)$ *on a new state space* $\bar{Z} = Z \times \mathbb{R}^p$ *and make some observability assumptions (such as overall observability introduced in [4]) on the linearized model. However, conditions for this assumptions to hold have never been proved before. Here we develop some necessary conditions for overall observability.*

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ON PARAMETER IDENTIFICATION AND STATE ESTIMATION FOR DISTRIBUTED PARAMETER SYSTEMS

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ABSTRACT

This paper presents some results associated with semilinear distributed parameter systems on a state space Z, with some parameters $\alpha \in \mathbb{R}^p$ to be identified. Recently some techniques have been developed [2,3,4,5,6, 7, and 8] to solve the joint problem of state and parameter estimation which is to construct the state z(t), t ∈ [0,T] and identify the parameters $\alpha \in \mathbb{R}^p$ when the output observation y(t) on [0,T] is given. These techniques consider a new formulation with an enlarged state $\bar{z}(t) = (z(t),\alpha)$ on a new state space $\bar{Z} = Z \times \mathbb{R}^p$ and make some observability assumptions (such as overall observability introduced in [4])on the linearized model. However, conditions for this assumptions to hold have never been presented before. Here we develop some necessary conditions for overall observability.

1 - INTRODUCTION

Consider the joint problem of state estimation and parameter identification of distributed parameter systems of the type

$$\begin{aligned} \dot{z}(t) &= Az(t) + A_1\alpha + N(z(t),\alpha), \\ z(0) &= z_0 , \\ y(t) &= Cz(t), \end{aligned} \tag{1}$$

where A is a linear operator which generates a strongly continuous semigroup on a Banach space Z(the state space), $A_1 : \mathbb{R}^p \to Z$, N a nonlinear operator from $Z \times \mathbb{R}^p$ to Z, and C a linear operator from the state space Z to $\mathcal{Y}$, a space of output functions from [0,T] to Y (the output space).

System (1) may be derived, for example, from the linearization of a system described by the nonlinear evolution equations

$$\begin{aligned} \dot{z}(t) &= f(z,u,\alpha,t) , \qquad z(0) = z_0 \\ y(t) &= h(z,u,\alpha,t) . \end{aligned} \tag{2}$$

The joint problem of state and parameter estimation is to construct the state z(t), t ∈ [0,T] and identify the parameter $\alpha = (\alpha_1,\alpha_2,\dots,\alpha_p) \in \mathbb{R}^p$ for system (1) when the output observation $y(\cdot) \in \mathcal{Y}$ is given.

We shall see in Section 3 that this problem can be transformed in to state estimation of a system with an enlarged state space $Z \times \mathbb{R}^p$ which has the form

$$\begin{aligned} \dot{\bar{z}}(t) &= \bar{A}\,\bar{z}(t) + \bar{N}\,\bar{z}(t), \qquad \bar{z}(0) = \bar{z}_0 \\ y(t) &= \bar{C}\,\bar{z}(t). \end{aligned} \tag{3}$$

where $\bar{A}$ generates a strongly continuous semigroup $\bar{S}(t)$ on $\bar{Z} = Z \times \mathbb{R}^p$. Some techniques have been developed in [2,3,4, 5,6,7 and 8] to solve the problem of state estimation of a semilinear system of the type (3). Usually they assume that the linear system

$$\begin{aligned} \dot{\bar{z}}(t) &= \bar{A}\,\bar{z}(t) , \qquad \bar{z}(0) = \bar{z}_0 \\ y(t) &= \bar{C}\,\bar{z}(t) \end{aligned} \tag{4}$$

is observable. In Section 4 we present some conditions for observability of (4).

Note that system (1) without the nonlinearity becomes

$$\begin{aligned} \dot{z}(t) &= Az(t) + A_1\alpha, \qquad z(0) = z_0 \\ y(t) &= Cz(t). \end{aligned} \tag{5}$$

System (5) has the mild solution

$$\begin{aligned} z(t) &= S(t)z_0 + \int_0^t S(\tau)A_1\alpha\, d\tau, \qquad z(0) = z_0 \\ y(t) &= Cz(t). \end{aligned} \tag{6}$$

In the next section we given a simple example of a system described by partial differential equations which can be expressed in the form (1) once linearized.

2 - EXAMPLE

Consider the following diffusion process

$$z_t = z_{xx} + \alpha z \ , \tag{7}$$

with boundary conditions

$$z(0,t) = z(\ell,t) = 0 \ , \tag{8}$$

initial condition

$$z(x,0) = z_0(x) \ , \tag{9}$$

and output equation

$$y(t) = \int_0^\ell c(x)\, z(x,t)\, dx \ , \tag{10}$$

where $\alpha \in \mathbb{R}^1$ is the parameter to be identified and $c(\cdot) \in L^2(0,\ell)$.

Take $\bar{z}$ and $\bar{\alpha}$ as an initial guess for the state z and the parameter α, respectively. Assume, for simplicity, that

a) $\bar{\alpha} = 0$;

b) $\bar{z}(x,t) = \bar{z}(x)$ (i.e., $\bar{z}$ is independent of t);

c) $\bar{z}(0) = \bar{z}(\ell) = 0$ (i.e., $\bar{z}$ satisfies the boundary conditions).

Setting $z = \bar{z} + z'$ and $\alpha = \bar{\alpha} + \alpha'$ and using assumption (a) and (b) we obtain

$$z'_t = z'_{xx} + \bar{z}\alpha' + \bar{z}_{xx} + \alpha' z' \tag{11}$$

which is the linearization of the dynamics about $(\bar{z},\bar{\alpha})$. By assumption (c) z' also satisfies the boundary conditions, that is

$$z'(0,t) = z'(\ell,t) = 0 \ . \tag{12}$$

Now define

$$\bar{y} = \langle c(.),\ \bar{z}(\cdot)\rangle_{L^2(0,\ell)} = \int_0^\ell c(x)\bar{z}(x)\, dx$$

$$y' = y - \bar{y} \ . \tag{13}$$

Clearly, since $\bar{y}$ is know, y' is obtained with knowledge of the observation y. Also, by (10), since $z = \bar{z} + z'$,

$$y' = \langle c(\cdot),\ z'(\cdot)\rangle_{L^2(0,\ell)} \ .$$

Now, if we set $Z = L^2(0,\ell)$, A the linear operator on Z defined by

$$Az' = z'_{xx} = \frac{\partial^2 z'}{\partial x^2} \ ,$$

with

$$D(A) = \{z' \in L^2(0,\ell) : \frac{\partial^2 z'}{\partial x^2} \in L^2(0,\ell),\ \frac{\partial z}{\partial x} = 0 \text{ at } x = 0 \text{ and } \ell\} \ ,$$

$p = 1$ (i.e., one parameter $\alpha \in \mathbb{R}^1$), $A_1 : \mathbb{R}^1 \to Z$ given by

$$A_1\, \alpha' = \bar{z}\, \alpha' \ ,$$

the nonlinearity $N(z',\alpha') = \alpha' z' + \bar{z}_{xx}$ and $C : Z \to \mathbb{R}$

$$Cz' = \langle c(\cdot),\ z'(\cdot)\rangle_{L^2(0,\ell)} \ .$$

then, we can write (11) - (12) as

$$\dot{z}' = Az' + A_1\alpha' + N(z',\alpha'),\quad z'(0) = z_0 \tag{14}$$

and the output observation (13) as

$$y' = Cz' \ . \tag{15}$$

Dropping the ' notation, equations (14) - (15) become (1).

3 - THE JOINT STATE AND PARAMETER ESTIMATION

In this section we show that the joint problem of state estimation and parameter identification, as formulated in Section 1 (for system (1) and state space Z) is equivalent to just state estimation of a system of the type (3) with an enlarged state space $\bar{Z} = Z \times \mathbb{R}^p$.

Since $\alpha \in \mathbb{R}^p$, $\dot{\alpha} = \frac{d\alpha}{dt} = 0$ and system (1) can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} A & A_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \alpha \end{bmatrix} + \begin{bmatrix} N(z(t),\alpha) \\ 0 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \alpha \end{bmatrix} \tag{16}$$

with initial condition $(z(0),\alpha(0)) = (z_0,\alpha)$. We call (16) the overall system for (1).

Now, if we define $\bar{z}_0 = (z_0,\alpha)$ and $\bar{z}(t) = (z(t),\alpha)$ for each $t \in [0,t]$, then we can write the overall system (16) in the form (3) with

$$\bar{A} = \begin{bmatrix} A & A_1 \\ 0 & 0 \end{bmatrix},\quad \bar{N} = \begin{bmatrix} N \\ 0 \end{bmatrix} \text{ and } \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}.$$

So, the joint problem of state estimation and parameter identification of (1) is transformed into state estimation of (3).

Note that $\bar{A}$ generates the strongly continuous semigroup $\bar{S}(t)$ on $\bar{Z}$ given by

$$\bar{S}(t) = \begin{bmatrix} S(t) & L_1(t) \\ 0 & I \end{bmatrix},$$

where

$$L_1(t)\alpha = \int_0^t S(\tau)\, A_1\alpha\, d\tau$$

and I is the identity on $\mathbb{R}^p$.

4 - OVERALL OBSERVABILITY

Observe that system (4), the linear part of (3), has the mild solution

$$\begin{aligned} &\bar{z}(t) = \bar{S}(t)\bar{z}_0 \quad , \quad \bar{z}(0) = \bar{z}_0 \, , \\ &y(t) = \bar{C}\, \bar{z}(t). \end{aligned} \tag{17}$$

Basically equations (6) and (17) are the same. However, when we refer to the overall system (17) we mean a system with state space $\bar{Z} = Z \times \mathbb{R}^p$, whereas we regard (6) as a system with state space Z and a perturbation.

Using the concepts of initial observability (see[1]) and continuous initial observability (see[1]) we shall define initial and continuous initial overall observability:

Definition: We say that (6) is initially (continuous initially) overall observable [9] if system (17) is initially (continuous initially) observable.

Clearly, if we have initial overall observability of (6), then $\bar{C}\bar{S}(\cdot) : \bar{Z} \to Y$ is a one-to-one mapping and this clearly implies that $CS(\cdot) : Z \to Y$ is also a one-to-one mapping and therefore we have initial observability of (6). It is also immediate to verify that continuous initial overall observability of (6) implies continuous initial observability of (6).

Suppose now that A_1 is not injective, i.e., kernel $(A_1) \neq \{0\}$. Thus, we can find $\alpha_0 \in \mathbb{R}^p$, $\alpha_0 \neq 0$, such that $A_1\alpha_0 = 0$ and for $\bar{z}_0 = (0,\alpha_0) \neq 0$ we have

$$\bar{C}\bar{S}(\cdot)\bar{z}_0 = C\int_0^{\cdot} S(\cdot - \tau)\, A_1\alpha_0\, d\tau = 0$$

which implies that kernel $(CS(\cdot)) \neq 0$, that is, the overall system cannot be neither initially nor continuous initially observable.

Summarizing these two necessary conditions for overall observability we have

Lemma: If system (6) is initially (continuous initially) overall observable, then

i) system (6) is initially (continuous initially observable, and

ii) A_1 is a one-to-one mapping.

Actually if (ii) does not hold we have more parameters than really needed in the model, since we can redefine α in a new space of parameters $\bar{Z}_1 = \mathbb{R}^p/\text{kernel}\,(A_1)$ of dimension $p' = [p - \dim(\text{kernel}\,(A))] < p$. In the sequel we shall assume, without loss of generality, that A_1 satisfies (ii) in the above lemma, i.e.,

$$\text{Kernel}\,(A_1) = \{0\}.$$

Consider now the following theorem which establishes another necessary condition for overall observability:

Theorem: Let the operator $\Gamma : \bar{Z} \to Z \times Y$ be given by

$$\Gamma = \begin{bmatrix} A & A_1 \\ C & 0 \end{bmatrix}. \tag{18}$$

If the system is initially overall observable

$$\text{kernel}\,(\Gamma) = \{0\}. \tag{19}$$

Proof: Assume that (6) is initially overall observable but (19) does not hold. Hence, there is $(z_0,\alpha) \in D(A) \times \mathbb{R}^p$, $(z_0,\alpha) \neq 0$, such that

$$Az_0 + A_1\alpha = 0, \tag{20}$$

$$Cz_0 = 0. \tag{21}$$

Acting $CS(\cdot)$ on both sides of (20) and integrating on $[0,t]$, $t \geq 0$, we get

$$\int_0^t CS(\tau)Az_0 d\tau + \int_0^t CS(\tau)A_1\alpha d\tau = 0, \quad \text{for all } t \geq 0.$$

Thus, since $z_0 \in D(A)$ we can write

$$CS(t)z_0 - Cz_0 + \int_0^t CS(\tau)A_1\alpha d\tau = 0, \quad \text{for all } t \geq 0 . \tag{22}$$

and hence, substituting (21) in (22) we obtain

$$\bar{C}\bar{S}(t) \begin{pmatrix} z_0 \\ \alpha \end{pmatrix} = 0, \qquad \forall\, t \geq 0$$

and therefore, since D(A) is dense in Z

$$\text{kernel}\,(\bar{C}\bar{S}(\cdot)) \neq \{0\}$$

which is a contradiction to the assumption of overall observable. So (19) holds.

Q.E.D.

5 - CONCLUSIONS

In order to solve the problem of estimation z(t) and identifying α for the nonlinear system (1) it is often required overall observability of the linear system (6). This is the same as observability of (17).

The results of Section 4 give only necessary conditions for overall observability of (6) or equivalently observability of (17). We should note here that, in general, these conditions are not sufficient. However, for the particular case of finite dimension state space Z, it is not difficult to show that the converse of the theorem in Section 4 holds.

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