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14. Abstract/Notes

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AN APPROACH FOR ORDER REDUCTION AND PLACEMENT OF ACTUATORS/SENSORS FOR THE CONTROL OF A CLASS OF DISTRIBUTED PARAMETERS SYSTEMS

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ABSTRACT

This work considers some aspects of the dynamics and control of a class of distributed parameter system, namely: Vibrating Structures. Some techniques for order reduction are discussed and compared taking into account their applicability to the control of those structures. To investigate the influence of the number, position and type of actuators and sensors on the reduced model, two structures are considered: a free-free beam and a completely free rectangular plate. Mode dominance distributions are presented for both structures corresponding to different configurations of actuators/sensors. The results show that the dominance measure originally proposed for the selection of critical modes can be also used to specify convenient actuators/sensors arrangements.

1 - INTRODUCTION

The control of distributed parameter systems has given rise to a lot of research in recent years due to its importance under theoretical and practical points of view. Flexible structures are a class of distributed parameter systems whose importance has steadily increased. As examples of such structures one can mention long bridges, large ships, flexible airplanes, rocktes and satellites. The dynamics of these systems is usually described by a hybrid set consisting of ordinary and partial differential equations. Closed solutions do exist for few cases only, and approximate solutions are commonly considered. These solutions present two problems in most of the cases: i) the discretized models possess a large number of degrees of freedom; ii) the higher modes are less accurate than the lower ones. These drawbacks clearly justify the necessity of using an order reduction technique.

Order reduction methods are usually composed by two steps: selection of critical modes [1]-[5] and reduced model evaluation [6]-[8]. The application of standard order reduction methods to the control of vibrating structures is not straightforward because the critical modes are known in advance to be the lower ones (the higher are less accurate and harder to be excited). Another distinguishing feature of this problem is the possibility of using actuators and sensors (a/s) whose position, type and number can be changed.

The objectives of this work [9] are: i) to analyse and compare different order reduction techniques considering their applicability to flexible structures models; ii) to present an approach for determining a/s configurations to ensure that the lower modes become the critical ones. In order to obtain numerical results, two structures, namely a free-free beam and a completely free rectangular plate, have been investigated and modal importance distributions are presented.

2 - STATEMENT OF THE PROBLEM

Consider a linear time invariant system represented by equations in standard state variable form:

$$\dot{x} = Ax + Bu \,, \tag{1}$$

$$y = Cx \,, \tag{2}$$

where $x \in R^p$ and $y \in R^q$, and A,B and C are constant matrices of appropriate dimensions. In this work, capital letters denote matrices and lower case letters represent vectors. Suppose that an appropriate mode selection criterion has been used and m modes are to be retained. After applying the Jordan transformation $x = Vz$ to (1) and (2), the partitioned system can be written:

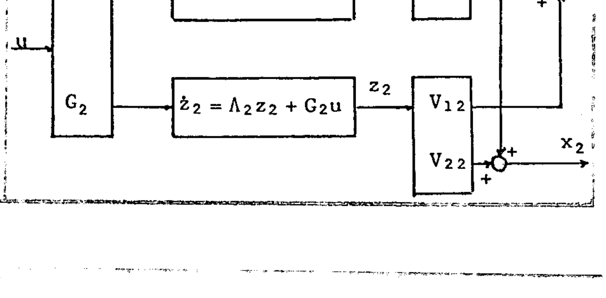
$$\dot{z}_1 = \Lambda_1 z_1 + G_1 u \,, \tag{3}$$

$$\dot{z}_2 = \Lambda_2 z_2 + G_2 u \,, \tag{4}$$

$$y = H_1 z_1 + H_2 z_2 \,, \tag{5}$$

where $\Lambda_1 = \mathrm{diag}(\lambda_1, \ldots \lambda_m)$, $\Lambda_2 = \mathrm{diag}(\lambda_{m+1}, \ldots \lambda_n)$, $G = [G_1^T \; G_2^T]^T = U B$, $H = [H_1 \; H_2] = CV$ and $U = V^{-1}$. Note that Λ_1 constains the critical eigenvalues of Λ and Λ_2 constains the noncritical ones.

The signal flow representation for the original system having the critical and noncritical modes is presented below:



The problem of deriving a reduced order model consists in the evaluation of a system given by:

$$\dot{\tilde{x}}_1 = \tilde{A}\tilde{x}_1 + \tilde{B}u \ , \qquad (6)$$

$$\tilde{x}_2 = \tilde{F}(\tilde{x}_1,u) \ , \qquad (7)$$

where only the dynamics of the critical modes has been considered. The matrices $\tilde{A}$, $\tilde{B}$ and the relation $\tilde{F}$ can be provided by different methods ([6]-[8]) according to different criteria (steady state agreement, minimal error, etc).

3 - DYNAMICAL MODEL OF VIBRATING STRUCTURES

The dynamics of a vibrating structure can be described by a partial differential equation given by:

$$L[w(r,t)] + \frac{\partial}{\partial t} D[w(r,t)] + M(r) \frac{\partial^2 w(r,t)}{\partial t^2} = $$

$$= F(r,t) \ . \qquad (8)$$

In the above L and D are differential operators containing information concerning stiffness and damping properties respectively, M is the mass distribution, F is the distributed force and w is the elastic displacement. To completely define the boundary value problem, there exist boundary conditions $B_i[w(r,t)] = 0$.

Solving the eigenvalue problem related to L and M taking into consideration the boundary conditions B_i, one obtains a set of eigenfrequencies $\omega_i (i = 1,2,\ldots)$ and corresponding eigenfuntions $\phi_i(r)$ $(i = 1,2,\ldots)$. Using the expansion theorem, the elastic displacement can be written as:

$$w(r,t) = \Sigma_i \ \phi_i(r) \ q_i(t) \ , \qquad (9)$$

where q_i is a generalized coordinate representing the vibration amplitude of the i-th mode. Assuming modal damping and using orthogonality properties of the eigenfunctions, (8) can be transformed in a set of ordinary differential equations:

$$\ddot{q}_i + 2 \ \zeta_i \omega_i \ \dot{q}_i + \omega_i^2 \ q_i = n_i(t), \qquad (10)$$

where $n_i(t)$ are the modal inputs given by:

$$n_i(t) = \int_E \phi_i(r) \ F(r,t) \ d \ E(r) \ . \qquad (11)$$

If the first n vibration modes are taken into consideration in the original model, from (10) one can identify the plant matrix A in (1) in the form of equation (12):

$$A = \begin{bmatrix} 0 & I \\ -\Omega & -\Gamma \end{bmatrix} , \qquad (12)$$

with I = identity matrix; $\Omega = \text{diag.} \ (\omega_i^2; \ i = 1, \ldots n)$; $\Gamma = \text{diag.} \ (2\zeta_i\omega_i; \ i = 1,\ldots n)$.

Assuming that there are p concentrated force actuators, the distributed force term in (8) turns to be written in the form:

$$F(r,t) = \sum_{j=1}^{p} f_j(t) \ \delta(r - r_j) \ . \qquad (13)$$

From (9), the input matrix B according to (1) is given by:

$$B = [0 \quad B_A]^T \quad \text{with} \ (B_A)_{ij} = \phi_i(r_j). \qquad (14)$$

Finally, if there are q displacement sensors placed at the positions $r_j (j = 1,\ldots,q)$, the output matrix C in (2) can be obtained from (9) written as:

$$C = [0 \quad C_S] \quad \text{with} \ (C_S)_{ji} = \phi_i(r_j). \qquad (15)$$

Systems whose plant matrices have the form given by (12) are said to be given in modal description form. These systems have some distinguishing features: the matrices Λ and V are known in advance, and V_{12} and V_{21} are null matrices.

Note that in the problem here addressed, B and C depend on the type of actuators and sensors and, in some sense, are not constant matrices since they vary if the positions of those elements are changed.

4 - ORDER REDUCTION METHODS

This section is divided into three parts. The first one discusses some techniques for selection of critical modes. In the second one, methods to obtain reduced models are presented. Finally a comparison of order reduction procedures is presented, specifically considering the control of flexible structures.

4.1 - SELECTION OF CRITICAL MODES

Bonvin and Mellichamp [1] proposed the Generalized Structural Dominance Method to obtain quantitative measures that can be used to select the critical modes as well as the most sensitive state variables and the most effective inputs. They consider the response of the i-th state variable

$$x_j(t) = \sum_{j=1}^{n} \sum_{i=1}^{p} p_{sji} \int_0^t \exp[\lambda_j(t-\tau)] \ u_i(\tau) \ .$$

$$. \ (-\lambda_j \ d\tau) \ , \qquad (16)$$

where

$$p_{sji} = - \ v_{sj} \frac{1}{\lambda_j} \sum_{k=1}^{n} u_{jk} b_{ki} \ . \qquad (17)$$

Equation (16) holds true for any type of input and the coefficient p_{sji} (independent from the input) measures the importance of the i-th input/j-th mode/s-th state variable coupling. For a fixed input, the analysis of p_{sji} values will permit to select the critical modes and associated state variables.

Litz and Roth [2] define a dominance measure that can be used to rank the modes considering not only the position of the eigenvalues in the complex plane, but also observability and controllability properties. Considering the output response

$$y_{kj}(t) = \sum_{i=1}^{n} h_{ki}\, g_{ij}[\exp(\lambda_i t)-1]/\lambda_i, \quad (18)$$

a dominance measure is defined for the i-th mode:

$$R(\lambda_i) = \frac{1}{pq}\sum_{k=1}^{q}(1/\mu_k)\sum_{j=1}^{p}\left|\frac{h_{ki}\, g_{ij}}{\lambda_i}\right|\gamma_j, \quad (19)$$

where μ_k and γ_j are normalizing factors. Eigenvalues with larger indices $R(\lambda_i)$ are the dominant ones.

Lastman et al. [3]presented a method to select the most significant states to be retained in a reduced model via the evaluation of the contribution of each state to the total output energy of the system. The mode selection procedure utilizes the A,B, and C matrices, taking into account the relative values of the coupling of each state to the outputs and inputs. Taking the response of the i-th component of the state vector to an unit impulse at the k-th component of the input vector

$$x_i(t) = v_i \exp(\Lambda t)\; b^k, \quad (20)$$

where b^k denotes the k-th column of B, the impulse energy at the output is defined as

$$E_k = \int_0^\infty y^T y\,dt = \sum_{j=1}^{n}\sum_{i=1}^{n} d_{ij}\int_0^\infty x_i(t)x_j(t)dt \quad (21)$$

where d_{ij} are the elements of $D = C^T C$. Making the integration indicated in (21), the W matrix is defined by

$$w_{ij} = \int_0^\infty x_i(t)x_j(t)dt = (b^k)^T U^T\left[\frac{v_{i\mu}\; v_{i\nu}}{\lambda_\mu + \lambda_\nu}\right]Ub^k, \quad (22)$$

If more than one input is to be considered at one time, b^k should be replaced by the weighted sum of the columns of B. The total output energy is given by

$$E = tr[DW] = tr[P]. \quad (23)$$

The most significant states correspond to the elements of large magnitude of the output energy integral participation matrix $P = DW$.

Likins et al [4] used definitions of the degrees of observability and controllability of the original system as criteria for mode truncation. Defining

$$Q_c = [B\; AB \dots A^{n-1}B]\; ;\; Q_o = [C^T A^T C^T \dots (A^T)^{n-1} C^T] \quad (24)$$

they suggest that coordinate truncation should be accomplished so as to minimally influence the trace of $(Q_c Q_c^T)^{-1}$, which is a measure of the degree of controllability of the original system. Another criterion proposed in [4] is based on the scalars

$$\delta_c = \det[Q_c\; Q_c^T] \quad ; \quad \delta_o = \det[Q_o\; Q_o^T]\;. \quad (25)$$

stating that for a given degree of truncation, coodinates should be preserved which maximize the ratio δ_o/δ_c. The scalars δ_c and δ_o are measures of the degree of controllability and observability respectively. It should be stressed that these indices have to be calculated for two systems: "with" and "without" the mode candidate for neglection.

Hughes and Skelton [5] proposed a method for selection of critical modes by taking into account the contribution of each mode to the total linear and angular momentum of the vehicle. The formalism is developed for a spacecraft composed by a rigid central part having flexible appendages attached to it. Hybrid coordinate description is used with "rigid body" and "elastic" modes of vibration. The method requires the evaluation of coefficients of linear and angular momentum related to the i-th vibration mode of the appendage, and given by:

$$P_i = \int_E \phi_i dm \;;\; H_{xi} = \int_E y\,\phi_i dm \;;\; H_{yi} = \int_E x\,\phi_i dm. \quad (26)$$

The reader is addressed to the work of Hablani [10] for more details. There are identities which provide relations between the coefficients P_i, H_{xi} and H_{yi}, and inertial properties of the appendage like its principal moments of inertia I_{ex}, I_{ey} and its mass m_e. The following identities hold true if all modes are considered.

$$\sum_{i=1}^{\infty} P_i^2 = m_e \;;\; \sum_{i=1}^{\infty} H_{xi}^2 = I_{ex} \;;\; \sum_{i=1}^{\infty} H_{yi}^2 = I_{ey}. \quad (27)$$

The modes to be retained in the reduced system are those which more strongly contribute to the sums in (27). This method is based solely on dynamical considerations, not taking into account actuators and sensors.

4.2 - REDUCED MODEL EVALUATION

Davison [8] proposed a technique to obtain a reduced model in which only the dynamics of the dominant modes is included and a compensation is introduced to eliminate steady state errors.

The dynamic model without compensation is:

$$\dot{x}^*_{1i} = V_{11}\,\Lambda_1\,V_{11}^{-1}\,x^*_{1i} + V_{11}\,g_1^i\,u_i . \tag{28}$$

The essential and nonessential state variables are evaluated from using compensating matrices D_1 and D_2 that ensure steady state agreement when unit steps are applied at the inputs:

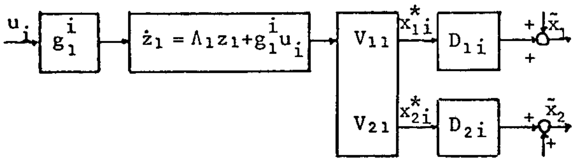
$$\tilde{x}_1 = \sum_{i=1}^{p} D_{1i}\,x^*_{1i} \quad ; \quad \tilde{x}_2 = \sum_{i=1}^{p} D_{2i}\,V_{21}\,V_{11}^{-1}\,x^*_{1i} , \tag{29}$$

where

$$(d_{1i})_j = \frac{(V_{11}\,\Lambda_1^{-1} g_1^i)_j + (V_{12}\,\Lambda_2^{-1}\,g_2^i)_j}{(V_{11}\,\Lambda_1^{-1} g_1^i)_j} ,$$

$$(d_{2i})_j = \frac{(V_{21}\,\Lambda_1^{-1}\,g_1^i)_j + (V_{22}\,\Lambda_2^{-1} g_2^i)_j}{(V_{21}\,\Lambda_1^{-1} g_1^i)_j} . \tag{30}$$

The signal flow diagram corresponding to Davison's Method is:



The main idea behind this method is, knowing that the nondominant modes are also excited by the inputs, to make corrections in the dominant ones. Since the objective is steady state agreement, transient errors can be expected.

Litz[7] presented a technique in which the nonsignificant modes are expressed as a linear combination of the significant ones. This combination is optimal in the sense that it minimizes the differences between original and reduced systems. Writing an approximation to the noncritical modes in the form

$$\tilde{z}_2 = E\,z_1 , \tag{31}$$

where z_1 are the critical ones and defining the error ε_j assuming unit step responses

$$\varepsilon_j(t) = z_{2j} - \tilde{z}_{2j} , \tag{32}$$

the E matrix can be obtained so as to minimize the cost:

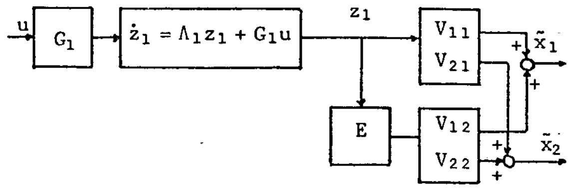
$$J = \sum_{j=1}^{p} q_j \int_0^{\infty} (\varepsilon_j)^T(\varepsilon_j)\,dt \; ; \; q_j = \text{weighting factors.} \tag{33}$$

The reduced model equations are given by

$$\dot{\tilde{x}}_1 = M\,\Lambda_1 M^{-1}\tilde{x}_1 + M\,G_1\,u ,$$

$$\tilde{x}_2 = N\,M^{-1}\,\tilde{x}_1 , \tag{34}$$

where $M = V_{11} + V_{12}E$ and $N = V_{21} + V_{22}E$. Note that here again, only the dynamics of the critical modes is taken into consideration. The signal flow diagram corresponding to Litz method is:



This method clearly implies a new eigenvector orientation. The final reduced model strongly depends on the choice of the input weighting factors q_j.

In the Singular Perturbation Method, the original system is decomposed into two parts, slow and fast ones. A reduced model, obtained by considering only the slow part, is improved via the introduction of corrections for the effect of the neglected part. The original system equations are written in the form:

$$\begin{Bmatrix} \dot{x}_1 \\ \varepsilon\dot{x}_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \; ;$$

x_1: slow part

x_2: fast part (35)

$$y = [C_1 \;\; C_2] \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} . \tag{36}$$

Georgakis and Bauer [8] suggested a method to derive a reduced model, taking into account a linear transformation:

$$v_1 = x_1 \; ; \; v_2 = x_2 + L\,x_1 , \tag{37}$$

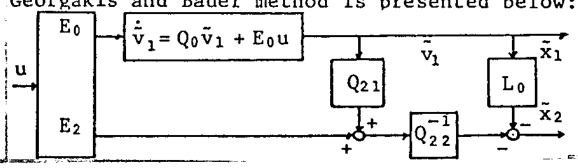
where $L = A_{22}^{-1} A_{21} + \varepsilon N$. Note that A_{22}^{-1} must exist. The essential idea of the method is to obtain a transformation matrix N such that the influence of v_1 over v_2 is eliminated. The final form of the reduced model is:

$$\dot{\tilde{v}}_1 = Q_0\,\tilde{v}_1 + E_0 u ,$$

$$\tilde{v}_2 = -\,Q_{22}^{-1}\,(Q_{21}\,\tilde{v}_1 + E_2 u) ,$$

$$y = (C_1 - C_2 L_0)\tilde{v}_1 + C_2\,\tilde{v}_2 . \tag{38}$$

The reader is addressed to [9] for more details. The signal flow diagram for the Georgakis and Bauer method is presented below:



The Singular Perturbation Method presents the interesting feature of preserving the physical structure of the original system. Since the dynamics of the fast modes is neglected, one may expect transient problems.

4.3 - COMPARISON OF DIFFERENT ORDER REDUCTION PROCEDURES

The control of flexible structures has distinguishing characteristics that influence the choice of an order reduction technique. Dynamic models derived by either modal analysis or by the Finite Element Method imply modal coordinates. Whichever the approach, the accuracy of the higher modes is poor, so the lower ones in some sense are more critical. The input and output matrices are not "constant", since they depend on the placement of actuators and sensors which can be changed.

Under this point of view, methods that consider the system dynamics as well as observability and controllability measures are more suitable. The Dominance [1] and Participation [3] Methods fulfill these requirements.

For structure dynamic models, the reduced model obtained by Davison's Method does not succed as a consequence of being based on the coupling between critical and noncritical modes. The Litz's Method seems to be an interesting approach due to the involved optmization. The use of the Singular Perturbation Method in this case might give rise to transient problems because the slow and fast eigenvalues may be close.

5 - ACTUATORS AND SENSORS ARRANGEMENT

Since the lower modes are the critical ones under a dynamical point of view (they are determined with higher accuracy), the objective here is, using the Dominance Method [1] previously discussed, to determine actuators and sensors configurations for which these modes present high dominance levels and the higher modes show less importance (with respect to that criterion). This section is devided into three parts presenting: modal data for the investigated structures, dominance measure distributions and discussion of the results.

5.1 - INVESTIGATED STRUCTURES

In order to obtain mode dominance distributions for several configurations of actuators and sensors, two structures have been investigated [9]: i) a free-free uniform beam; ii) a completely free rectangular plate. The modal data for both structures are presented following:

TABLE 1

FREQUENCIES OF THE LOWEST EIGHT FLEXIBLE MODES OF THE f-f BEAM

Mode No. (k)	ω_k (rd/s)	ω_k/ω_1 -
1	.6283	1.0000
2	1.7321	2.7568
3	3.3954	5.4041
4	5.6128	8.9333
5	8.3846	13.3449
6	11.7107	18.6387
7	15.5911	24.8147
8	20.0259	31.8732

- Free-free beam characteristics:
  - . length = 100m
  - . width = 1m
  - . thickness = 0.2m
  - . material = - steel m = 1540 kg/m
    - aluminium m = 540 kg/m
    - for both materials

$\sqrt{E/\rho} = 5000$ m/s.

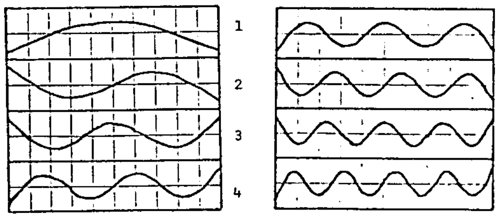


Fig. 1 - First flexible modes for the f-f beam.

TABLE 2

FREQUENCIES OF THE FIRST NINE FLEXIBLE MODES OF THE COMPLETELY FREE RECTANGULAR PLATE

Mode No.	ω_k (rd/s)	ω_k/ω_f -	ω_k/ω_1 -
1	0.01195	21.3750	1.000
2	0.01831	32.7536	1.532
3	0.03321	59.4173	2.779
4	0.03938	70.3789	3.295
5	0.06486	116.0441	5.428
6	0.06578	117.6891	5.505
7	0.07808	139.6880	6.534
8	0.08519	152.4128	7.129
9	0.10007	179.0224	8.374

- Completely free rectangular plate characteristics:
  - . flexural rigidity D: 20×10^8 N m
  - . Mass per unit area: 0,2622 kg/m^2
  - . Length: 12,5 km
  - . Width: 5.0 km

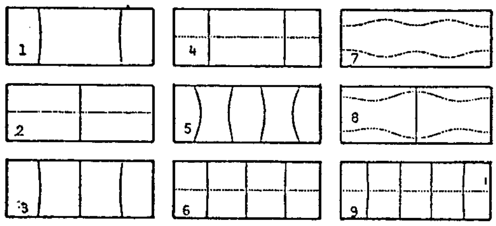


Fig. 2 - First nine flexible modes of the completely free rectangular plate.

5.2 - DOMINANCE MEASURES DISTRIBUTIONS

Two types of colocated (placed at the same position) actuators and sensors have been investigated to evaluate mode dominance distributions: concentrated force actuators with displacement sensors and concentrated torquers with slope sensors. For the plate, pairs of torquers and slope sensors (x,y directions) have been utilized.

Dominance distributions are given in form of histograms, each one having a sketch beside illustrating the position of colocated actuators and sensors. The dominance values are nondimensionalized such that the maximum value is always equal to 100.

Figure 3 shows mode dominance distributions for the f-f beam with 1,3 and 5 force actuators and displacement sensors.

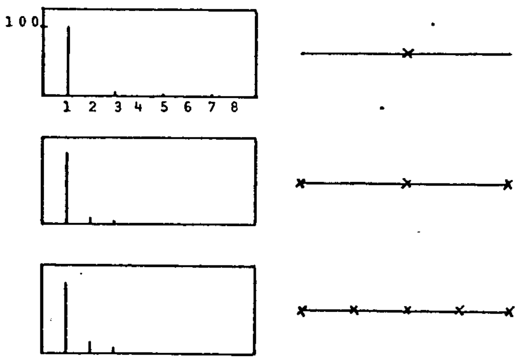


Fig. 3 - Dominance distribution for the f-f beam with force actuators and displacement sensors.

Note that when solely one force actuator is placed at the center of the beam, only modes no. 1,3,5,7 are excited as can be seen from Figure 1.

Changing the type of actuators and sensors, Figure 4 illustrates dominance distributions for the f-f beam with 1,3 and 5 torques colocated with slope sensors.

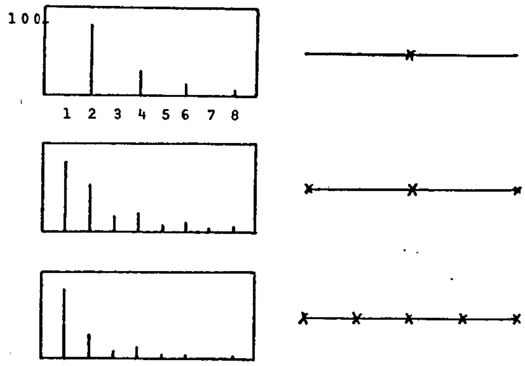


Fig. 4 - Dominance distributions for the f-f beam with torquers and slope sensors.

For one torquer placed at the center of the beam, only modes no. 2,4,6,8 have dominance indices different from zero because they present slope different from zero at that point (see Figure 1).

Considering the completely free rectangular plate, Figure 5 presents mode dominance values for 1,3 and 5 force actuators placed at the center and corners of the plate.

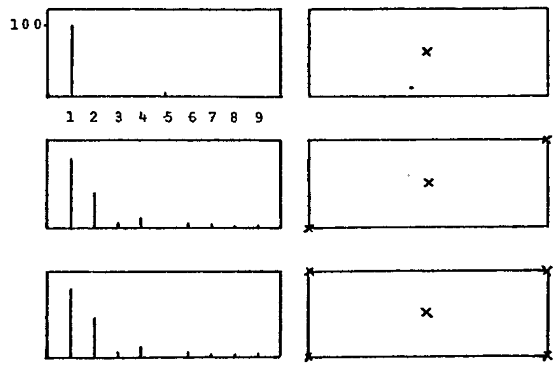
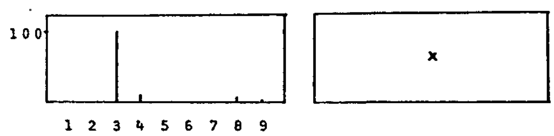


Fig. 5 - Dominance distribution for the completely free plate with force actuators and displacement sensors.

Since only modes no. 1,5 and 7 have displacement different from zero at the center of the plate, these are the modes with dominance indices in Figure 5 when one force actuator is positioned at that point.

Figure 6 presents dominance distributions for the completely free plate with pairs of torquers colocated with slope sensors at the same positions of Figure 5.



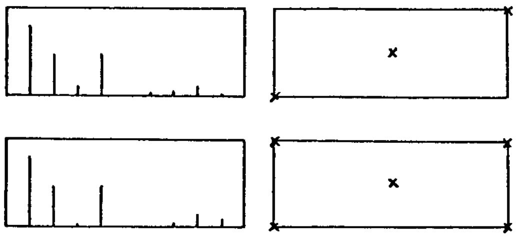


Fig. 6 - Dominance distribution for the completely free plate with pairs of torquers and slope sensors.

From Figure 2, it can be seen that only mades no. 3,4,8 and 9 have slope not null at the center of the plate in either x or y directions. These are the modes with dominance indices different from zero if only one pair of torquers is placed at the center of the plate.

In all the cases presented here, the relative importance of the modes decreases as their eigenfrequencies increase but Figures 4 and 6 clearly indicated that mode dominance distributions are strongly affected by the number, position and type of actuators and sensors used.

5.3 - DISCUSSION OF NUMERICAL RESULTS

The placement of actuators and sensors at points where either the displacement or slope are maximum (corresponding to force actuators and torquers respectively) for the majority of the modes, often imply a dominance distribution for which the lowest modes have the highest dominance levels.

For the beam, dominance distributions indicate that the first two modes are the critical ones in most of the cases. It is interesting to observe that torquers and slope sensors tend to produce higher dominance indices, when compared with force actuators with displacement sensors.

For the completely free rectangular plate, the obtained dominance distributions show that the determination of how many and which modes are the most significant ones depends strongly on the actuators/sensors configuration. In a rough sense, the importance of the modes decreases as modal frequencies increase, but some unexpected behaviour could be observed [9].

6 - CONCLUSIONS

In retrospect, this work considered two problems related to the dynamics and control of flexible structures: the evaluation of a reduced model and the definition of a configuration of actuators/sensors.

From the analysis and comparison of some order reduction procedures here investigated, the general conclusions are:

. selection of significant modes should be based on indices that take into account the system dynamics and measures of observability and controllability.

. reduced order models generated by modal approaches or by the Singular Perturbation Method are particularly convenient because they provide an explicit formulation for the reduced model.

. for systems in modal description form, such as vibrating structures, the methods of Litz and singular perturbation seem to be promising.

From the numerical tests performed with the two structures here presented, varying the type, number and position of actuators/ sensors, it can be stated that:

. the dominance measure [1] is an interesting tool that can handle selection of critical modes and definition of actuators/sensors configurations.

. care must be exercised while specifying those configurations because sensitivity analyses indicate that some modes are much more influenced than others by placement changing.

Finally considering the control of other distributed parameter systems, different from vibrating structures, the results from this work remain useful requiring only the system equations to be discretized and written in standard state variable form.

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