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14. Abstract/Notes

This paper presents the development of a new estimator for the approximate estimation of the state of nonlinear dynamic systems. This estimator uses the duality between state estimation and optimal control problems to create an adaptive algorithm. This algorithm is then implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results are presented and discussed, as well as the theoretical aspects of the estimation algorithm.

15. Remarks

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STATE ESTIMATION PROCEDURE BASED UPON THE DUALITY CONCEPT

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ABSTRACT

This paper presents the development of a new estimator for the approximate estimation of the state of nonlinear dynamic systems. This estimator uses the duality between state estimation and optimal control problems to create an adaptive algorithm. This algorithm is then implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results are presented and discussed, as well as the theoretical aspects of the estimation algorithm.

1 - INTRODUCTION

In problems of state estimation, a common situation is that of having to adopt a dynamic model for the estimator which is only a crude approximation of reality. To avoid divergence, specially in the case of nonlinear dynamic systems, extended and adaptive versions of the Kalman filter have been used (e.g.: Jazwinski, 1970; Maybeck, 1979; Gelb et alii, 1974). Usually one explores the information given by the observations residues to either directly estimate the unmodelled effects (Tapley and Ingram, 1973; Cruz and Rios Neto, 1980) or to condition the state error covariance matrix to keep the capability of the estimator to extract information of new observations (e.g.: Jazwinki, 1969; Rios Neto and Kuga, 1981, 1982, 1985). In the procedures resulting from these approaches there is always an increase in the number of variables to be estimated.

The proposed procedure presents an alternative scheme for the state estimation problem, specially for nonlinear systems. The concept of duality between the estimation and the optimal control problems is used to transform the original estimation problem in one of tracking the observations with a virtual control. Without the need of increasing the number of estimated variables, the observations residues are used to estimate the virtual control necessary to update the estimate of the state. The virtual nature of the tracking control allows the possibility of choosing control actions adequate to better extract the information contained in the observations (Rios Neto and Fleury, 1984; Fleury, 1985). This makes the procedure specially indicated for situations where there is a great lack of system dynamics knowledge, but there is a high local level of information in the observations.

A preliminary test of a first version of the procedure was done under digitally simulated conditions. This was done for the problem of real time orbit determination of a low altitude artificial satellite and the results are shown to be satisfactory.

2 - PROPOSED PROCEDURE

Consider the state estimation of a dynamic system of the type:

$$\dot{x} = f(x,t) + f^{n}(x,t) + \bar{G}(t)\, w(t) \,, \qquad (1)$$

$$y(t_k) = h_k(x(t_k),t_k) + v(t_k),\; k = 1,2,\ldots, \qquad (2)$$

where x is the $n \times 1$ state vector; $w(t)$ and $v(t_k)$ are $r \times 1$ and $m \times 1$ independent Gaussian white noises with the usual hypothesis of $w(t)$ being independent of past states and $v(t_k)$ being independent of the state, with distributions defined by zero means and covariances:

$$E[w(t)w^{T}(\tau)] = Q(t)\delta(t-\tau);\; E[v(t_k)v^{T}(t_j)] = R(k)\delta_{kj} \,, \qquad (3)$$

where $\delta(t-\tau)$ is the Dirac delta function and δ_{kj} is the Kronecker Symbol.

In this problem, $f^{n}(x,t)$ represents the unknown part of the dynamical model, which usually can not be included in the model of the estimator. In a typical discretization interval, (t_k,t_{k+1}), the prediction phase of an extended Kalman filter (e.g. Jazwinski, 1970), a nominal trajectory is generated by:

$$\dot{\bar{x}} = f(\bar{x},t) \quad,\quad \bar{x}(t_k) = \hat{x}(t_k/t_k) \,, \qquad (4)$$

where $\hat{x}(t_k/t_k)$ is the estimate in t_k, and a first order approximation of the propagated error is given as:

$$\Delta x(t_{k+1}) = \Phi(t_{k+1},t_k)\,\Delta x(t_k) + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},s)\bar{G}(s)w(s)ds \,, \qquad (5)$$

where

$$\dot{\Phi}(t,t_k) = f_x(\bar{x},t)\Phi(t,t_k),\ \Phi(t_k,t_k) = I \quad (6)$$

and the subindex x is to indicate a partial derivation with respect to the state.

The propagated error can be regarded as the a priori information in t_{k+1}, which is the information based on previously processed observations:

$$\Delta x(t_{k+1}) = \Delta\hat{x}(t_{k+1}/t_k) + \eta(k+1/k), \quad (7)$$

and if the approximation of Equations (4) and (5) are assumed, there results:

$$\Delta x(t_{k+1}) = 0 + \eta(k+1/k), \quad (8)$$

where $\eta(k+1/k)$ is zero mean conditioned on the observations already processed with covariances given by:

$$E[\eta(k+1/k)\ \eta^T(k+1/k)] \triangleq P(k+1/k) =$$

$$\Phi(t_{k+1},t_k)\ P(k/k)\ \Phi^T(t_{k+1},t_k) +$$

$$+ \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},s)\bar{G}(s)Q(s)G^T(s)\Phi(t_{k+1},s)ds. \quad (9)$$

In the proposed procedure the state estimation of the system in Equation (1) is approximated by the estimation of a "virtual control", defining:

$$\dot{x}_c = f(x_c,t) + G(t)\ u(t). \quad (10)$$

The control $u(t)$ is to be estimated to force the controlled state x_c to be a good approximation of x by tracking the observations of Equation (2). This is done by assuming the control to be a first order perturbation and taking in a typical (t_k,t_{k+1}):

$$\dot{\bar{x}}_c = f(\bar{x}_c,t), \quad \bar{x}_c(t_k) = \hat{x}(t_k/t_k), \quad (11)$$

$$\Delta\dot{x}_c = f_x(\bar{x}_c,t)\Delta x_c + G(t)u(t),\ \Delta x_c(t_k) = 0, \quad (12)$$

where $u(t)$ is modelled as a step process and calculated to satisfy:

$$\Delta x_c(t_{k+1}) \equiv \Delta x(t_{k+1}), \quad (13)$$

$$y(t_{k+1}) = h_{k+1}(\bar{x}_c(t_{k+1}) + \Delta x_c(t_{k+1}),t_{k+1}) +$$

$$+ v(t_{k+1}). \quad (14)$$

From Equations (8) and (13), there results:

$$0 = \Delta x_c(t_{k+1}) + \eta(k+1/k). \quad (15)$$

But from Equation (12) one obtains:

$$\Delta x_c(t_{k+1}) = (\int_{t_k}^{t_{k+1}} \Phi_c(t_{k+1},s)G(s)ds)\ u(t_k)$$

$$\triangleq \gamma\ (k+1,k)\ u(t_k), \quad (16)$$

where $\Phi_c(t,t_k)$ is the transition matrix associated to Equation (14). From the linearization of Equation (14) one gets:

$$\Delta y(t_{k+1}) = \frac{\partial}{\partial x_c} h_{k+1}(\bar{x}_c(t_{k+1}),t_{k+1})\ .$$

$$.\ \Delta x_c(t_{k+1}) + v(t_{k+1}), \quad (17)$$

where high order terms have been disregarded. Finally, combining the results of Equations (15), (16) and (17), the following problems of parameter estimation result:

$$0 = \gamma(k+1,k)\ u(t_k) + \eta(k+1/k), \quad (18)$$

$$\Delta y(t_{k+1}) = H(k+1)\gamma(t_{k+1},t_k)u(t_k) + v(t_{k+1}), \quad (19)$$

where

$$H(k+1) \triangleq \frac{\partial}{\partial x_c} h_{k+1}(\bar{x}_c(t_{k+1}),t_{k+1}).$$

Using a Gauss-Markov minimum variances estimator (e.g. Liebelt, 1967 and Maybeck, 1979), estimates of $u(t_k)$ are obtained:

$$\hat{u}(t_k) = P_u(k+1/k).\gamma^T(k+1,k).H^T(k+1)\ .$$

$$.\ R^{-1}(k+1)\ \Delta y(t_{k+1}), \quad (20)$$

$$P_u(k+1/k) = (\gamma^T(k+1,k)P^{-1}(k+1/k)\gamma(k+1,k)$$

$$+ \gamma^T(k+1,k)H^T(k+1)R^{-1}(k+1)H(k+1)\gamma^T(k+1,k))^{-1}, \quad (21)$$

where $R(k+1)$ and $P(k+1/k)$ are as defined in Equations (3) and (9).

To recover the estimate of the state, one combines Equations (16) with (20) to get:

$$\Delta\hat{x}_c(t_{k+1}) = \gamma(k+1,k)\ \hat{u}(t_k) \quad (22)$$

and takes the approximation:

$$\hat{x}(t_{k+1}/t_{k+1}) = \hat{x}(t_{k+1}/t_k) + \Delta\hat{x}_c(t_{k+1}). \quad (23)$$

To recover the covariance matrix of the error in the estimate, it is only necessary to consider Equations (13), (16) and (23) to get:

$$e(t_{k+1}/t_{k+1}) \triangleq x(t_{k+1}) - \hat{x}(t_{k+1}/t_{k+1}) = \Delta x_c - \Delta\hat{x}_c, \quad (24)$$

and

$$P(k+1/k+1) \triangleq E[e(t_{k+1}/t_{k+1})e^T(t_{k+1}/t_{k+1}))$$

$$= \gamma(k+1/k)P_u(k+1/k)\gamma^T(k+1/k) \quad . \quad (25)$$

To overcome the ill-conditioning in the state error covariance matrix due to the approximation of Equation (5) in the propagation of the error and due to the nonlinearities in the observations, in the applications both $P(k+1/k)$ and $P_u(k+1/k)$ are adapted using the procedure by Rios-Neto and Kuga (1985, see elsewhere in these proceedings).

3 - APPLICATION

The procedure was tested under digital simulation (Burroughs 6800) for the case of real time orbit determination of a low altitude satellite with:

zero excentricity, 42^o inclination, 250 km altitude,

$A/m = 0.00076 m^2/kg$, area over mass ratio,

C_D = 2,0 drag coefficient.

To simulate the observations, a true orbit was generated by numerical integration, using a dynamic model including the influence of: gravity (up to J_6 zonal and C_{44}, S_{44} tesseral coefficients), atmospheric drag, and perturbations of Sun and Moon. The observations were generated combining this orbit data with the locations of 3 fictitious symmetric topocentric tracking stations, to get at each 1 second range and range-rate values, contaminated by white Gaussian noise with standards deviations:

$$\sigma_{\rho} = 10.0m \quad \sigma_{\dot{\rho}} = 0.1m/s \ .$$

To initialize the estimator, errors of 1000 m and 10m/s were considered in position and velocity, respectively, with a state error covariance matrix with variances of $9\times10^5 m^2$ and $9\times10^2 m^2/s^2$, respectively.

The model adopted for $f(x_c,t)$, in Equation (10), only included the gravitational effects up to J_2, thus characterizing a situation of lack of knowledge quite serious in the dynamics of the system.

The virtual control in Equation (10) is a 3×1 vector since it is possible to "control an orbit" using three independent forces, one for each axis. For the matrix $G(t)$ the following structure is used:

$$G(t) = \begin{bmatrix} C_p I_3 \\ C_v I_3 \end{bmatrix} , \quad (26)$$

where the scalars were taken $C_p = 5.0$; $C_v = 2.0$.

Note that the matrix $G(t)$ has to guarantee the necessary controllability conditions over the typical interval (t_k, t_{k+1}).

The results obtained are shown in Figs. 1 and 2, where the following residues are defined for position and velocity:

$$\Delta v(t_k) \triangleq (\sum_{i=1}^{3} [x_i(t_k) - \bar{x}_i(t_k/t_k)]^2)^{1/2} \ ,$$

$$\Delta\bar{r}(t_k) \triangleq (\sum_{i=1}^{3} P_{ii}(k/k))^{1/2} \ ; \quad (27)$$

$$\Delta v(t_k) \triangleq (\sum_{i=4}^{6} [x_i(t_k) - \bar{x}_i(t_k/t_k)]^2)^{1/2} \ ,$$

$$\Delta\bar{v}(t_k) \triangleq (\sum_{i=4}^{6} P_{ii}(k/k))^{1/2} \ , \quad (28)$$

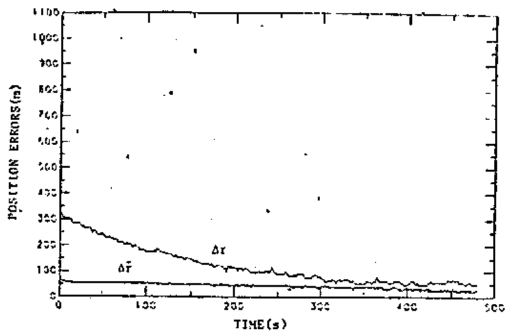

Fig. 1 - Estimated and actual errors in position.

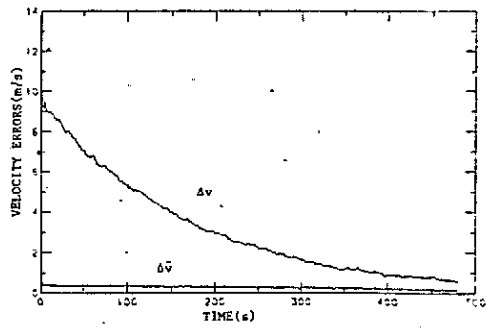

Fig. 2 - Estimated and actual errors in velocity

The analysis of these figures shows that the state estimatives converge on values close to the true orbit. The 1000 m and 10m/s initial errors in position and velocity are reduced to approximately 30m and 0.4m/s. Although the final errors are reasonably small, the interval of time for convergence is quite long; notice also that the estimated errors in the transient phase of convergence are very optimistic when compared to the real ones. These characteristics show that there are still further efforts to be done in order to get values for the parameters in matrix G(see Eq. (26)) to reach a better performance.

4 - CONCLUSIONS

A new approach to state estimation has been presented, where the concept of duality was explored to transform the problem of estimation into one of determing a virtual control. As indicated by the preliminary tests, it is expected to be a valid alternative for the case where a great lack of knowledge in the dynamic exists, but a good level of information is locally provided by the observations.

In this work, one only starded to explore the possibilities opened by the virtual nature of control in Equation (10). Presently, efforts are being done to further explore these possibilities, mostly in terms of the structure of G(t) and of treating the determination of this matrix automactically and adaptively (Fleury, 1985).

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