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AN ANALYTICAL METHOD FOR THE DETERMINATION OF SOLAR CELL MODEL PARAMETERS

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14. Abstract/Notes

A method to determine the parameters of an electrotechnical model of the current-voltage characteristics of a solar cell is presented. It is shown that all the parameters can be obtained analytically from the experimental data by solving a single parameter transcendental equation.

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AN ANALYTICAL METHOD FOR THE DETERMINATION OF SOLAR CELL MODEL PARAMETERS

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A method to determine the parameters of an electrotechnical model of the current-voltage characteristics of a solar cell is presented. It is shown that all the parameters can be obtained analytically from the experimental data by solving a single parameter transcendental equation.

An electrotechnical model is a phenomenological description of the current-voltage characteristics of a solar cell. It describes the solar cell by an equivalent circuit of a photocurrent source in parallel with one or two diodes together with shunt and series resistances. Mathematically it is expressed as[1]

$$I = I_{ph} - I_1\left(e^{\frac{V+IR_S}{V_T}} - 1\right) - I_n\left(e^{\frac{V+IR_S}{nV_T}} - 1\right) - \frac{V+IR_S}{R_{SH}}\,, \qquad (1)$$

where I and V are output current and voltage, respectively, I_{ph} is the photocurrent, V_T is the thermal potential, and R_S and R_{SH} are the series and shunt resistances, respectively. The second and third terms on the right hand side of eqn. 1 correspond to two diodes with reverse saturation currents I_1 and I_n and diode quality factors 1 and n, respectively. The quantities I_1, I_n, R_S, R_{SH}, and n are the solar cell model parameters which are related to various physical phenomena. A simple method for determining these parameters from experimental data is quite valuable in the design and evaluation of solar cells for research, development and production.

The traditional method[2] of determining the parameters is based on fitting the eqn. 1 at three points of the current-voltage (I - V) curve, namely the open circuit voltage (V_{OC}), the short circuit current (I_{SC}), and the maximum power point voltage and current (V_m, I_m); and at two slopes of I - V curve at the open circuit and short circuit conditions:

$$R_{S_0} = -\left(\frac{\partial V}{\partial I}\right)_{V=V_{OC}} \quad ; \quad R_{SH_0} = -\left(\frac{\partial V}{\partial I}\right)_{I=I_{SC}} \quad .$$

These five fitting conditions give the following five equations in terms of six experimentally measured quantities V_{OC}, I_{SC}, V_m, I_m, R_{S_0}, and R_{SH_0}.

$$I_{ph} = I_1\left(e^{\frac{V_{OC}}{V_T}} - 1\right) + I_n\left(e^{\frac{V_{OC}}{nV_T}} - 1\right) + \frac{V_{OC}}{R_{SH}} , \tag{2}$$

$$I_{ph} = I_{SC} + I_1\left(e^{\frac{I_{SC}R_S}{V_T}} - 1\right) + I_n\left(e^{\frac{I_{SC}R_S}{nV_T}} - 1\right) + \frac{I_{SC}R_S}{R_{SH}} , \tag{3}$$

$$I_m = I_{ph} - I_1\left(e^{\frac{V_m+I_mR_S}{V_T}} - 1\right) - I_n\left(e^{\frac{V_m+I_mR_S}{nV_T}} - 1\right) - \frac{V_m+I_mR_S}{R_{SH}}\ , \qquad (4)$$

$$\frac{1}{R_{S_0}-R_S} = \frac{1}{R_{SH}} + \frac{I_1}{V_T}\,e^{\frac{V_{OC}}{V_T}} + \frac{I_n}{nV_T}\,e^{\frac{V_{OC}}{nV_T}}\ , \qquad (5)$$

$$\frac{1}{R_{SH_0}-R_S} = \frac{1}{R_{SH}} + \frac{I_1}{V_T}\,e^{\frac{I_{SC}R_S}{V_T}} + \frac{I_n}{nV_T}\,e^{\frac{I_{SC}R_S}{nV_T}}\ . \qquad (6)$$

The first three eqns. 2-4 are combined into the following two equations by eliminating I_{ph} from them.

$$I_{SC} = I_1\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{I_{SC}R_S}{V_T}}\right) + I_n\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{I_{SC}R_S}{nV_T}}\right) + \frac{V_{OC} - I_{SC}R_S}{R_{SH}}\ , \qquad (7)$$

$$I_m = I_1\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{V_m+I_mR_S}{V_T}}\right) + I_n\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{V_m+I_mR_S}{nV_T}}\right) + \frac{V_{OC}-V_m-I_mR_S}{R_{SH}}\ . \qquad (8)$$

The solution of the nonlinear eqns. 5-8 can give us any four out of five parameters I_1, I_n, R_S, R_{SH} and n. For a double exponential model ($I_1 \neq 0$, $I_n \neq 0$), normally n is taken as a known parameter. However, for a single exponential model ($I_1 = 0$), there are only four parameters

I_n, R_S, R_{SH} and n and so all of them can be determined. These nonlinear equations are usually solved by numerical methods which require extensive computation and good initial guesses for the iteration to converge. In many cases, there are difficulties in determining these guesses in order to solve these equations.

In this paper we show how by adding an equation corresponding to the maximum power condition, it is possible to replace the extensive and complicated numerical solution of the nonlinear equations by the solution of a single parameter transcendental equation which can be easily solved by either iterative or graphical methods. The solution of this transcendental equation determines all the solar cell parameters analytically. The maximum power condition can be obtained from eqn. 1. If we put $\overline{V} \equiv V + IR_S$ in eqn. 1, the power $P = IV = I(\overline{V} - IR_S)$ becomes a function of $\overline{V}$ and, consequently, the maximum power condition $\frac{\partial P}{\partial \overline{V}} = 0$ is obtained as

$$\frac{I_m}{V_m - I_m R_S} = \frac{1}{R_{SH}} + \frac{I_1}{V_T} e^{\frac{V_m + I_m R_S}{V_T}} + \frac{I_n}{nV_T} e^{\frac{V_m + I_m R_S}{nV_T}} . \tag{9}$$

Now we shall show how a simple algebra can give rise to a single parameter transcendental equation from the solution of the coupled nonlinear eqns. 5-9. First we obtain two equations containing the same exponential factors as that of eqns. 7-8. Substracting eqns. 6 and 10 from eqn. 5, we get

$$\frac{1}{R_{S_0}-R_S} - \frac{1}{R_{SH_0}-R_S} = \frac{I_1}{V_T}\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{I_{SC}R_S}{V_T}}\right) + \frac{I_n}{nV_T}\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{I_{SC}R_S}{nV_T}}\right), \quad (10)$$

$$\frac{1}{R_{S_0}-R_S} - \frac{I_m}{V_m-I_mR_S} = \frac{I_1}{V_T}\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{V_m+I_mR_S}{V_T}}\right) + \frac{I_n}{nV_T}\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{V_m+I_mR_S}{nV_T}}\right). \quad (11)$$

We should note that each equation in pairs of eqns. 7, 10 and 8, 11 contains the same exponential factor. The solution of these pairs of equations for I_1 and I_n give the following equations:

$$(n-1)\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{I_{SC}R_S}{V_T}}\right)I_1 = -I_{SC} + \frac{V_{OC}-I_{SC}R_S}{R_{SH}} + nV_T\left(\frac{1}{R_{S_0}-R_S} - \frac{1}{R_{SH_0}-R_S}\right), \quad (12)$$

$$\frac{n-1}{n}\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{I_{SC}R_S}{nV_T}}\right)I_n = I_{SC} - \frac{V_{OC} - I_{SC}R_S}{R_{SH}} - V_T\left(\frac{1}{R_{S_0}-R_S} - \frac{1}{R_{SH_0}-R_S}\right), \quad (13)$$

$$(n-1)\left(e^{\frac{V_{OC}}{V_T}} - e^{\frac{V_m+I_mR_S}{V_T}}\right)I_1 = -I_m + \frac{V_{OC}-V_m-I_mR_S}{R_{SH}} + nV_T\left(\frac{1}{R_{S_0}-R_S} - \frac{I_m}{V_m-I_mR_S}\right), \quad (14)$$

$$\frac{n-1}{n}\left(e^{\frac{V_{OC}}{nV_T}} - e^{\frac{V_m+I_mR_S}{nV_T}}\right)I_n = I_m - \frac{V_{OC}-V_m-I_mR_S}{R_{SH}} - V_T\left(\frac{1}{R_{S_0}-R_S} - \frac{I_m}{V_m-I_mR_S}\right). \quad (15)$$

By eliminating I_1 and I_n from eqns. 12-15, we obtain

$$F_1(R_S) = \frac{(I_m - f_n(R_S))R_{SH} - V_{OC} + V_m + I_m R_S}{(I_{SC} - g_n(R_S))R_{SH} - V_{OC} + I_{SC}R_S}\,, \tag{16}$$

$$F_n(R_S) = \frac{(I_m - f_1(R_S))R_{SH} - V_{OC} + V_m + I_m R_S}{(I_{SC} - g_1(R_S))R_{SH} - V_{OC} + I_{SC}R_S}\,, \tag{17}$$

where

$$F_p(R_S) = \frac{e^{\frac{V_{OC}}{pV_T}} - e^{\frac{V_m + I_m R_S}{pV_T}}}{e^{\frac{V_{OC}}{pV_T}} - e^{\frac{I_{SC}R_S}{pV_T}}}\,, \tag{18}$$

$$f_p(R_S) = pV_T\left(\frac{1}{R_{S_0} - R_S} - \frac{I_m}{V_m - I_m R_S}\right)\,, \tag{19}$$

$$g_p(R_S) = pV_T\left(\frac{1}{R_{S_0} - R_S} - \frac{1}{R_{SH_0} - R_S}\right)\,, \tag{20}$$

and $p = 1$ or n.

Finally we eliminate R_{SH} from eqns. 16-17 and get the desired single parameter transcendental equation as

$$\frac{V_{OC} - V_m - I_m R_S}{V_{OC} - I_{SC} R_S} =$$

$$= \frac{F_1(R_S)F_n(R_S)[g_n(R_S)-g_1(R_S)]+f_1(R_S)[f_1(R_S)-I_m)+F_n(R_S)[I_m-f_n(R_S)]}{F_n(R_S)[I_{SC}-g_1(R_S)]+F_1(R_S)[g_n(R_S)-I_{SC}]+f_1(R_S)-f_n(R_S)} . \quad (21)$$

An iterative or graphical solution of eqn. 21 provides us the parameter R_S which in turn can be used for the analytical determination of R_{SH} from eqns. 16 or 17. After knowing R_S and R_{SH}, we can obtain remaining parameters I_1 and I_n from eqns. 12-15. We should mention that in eqn. 21, it is assumed that n is known. In case n is not known but R_S is known from some experiment measurement[3,4], eqn. 21 can be used to determine n and subsequently other parameters from eqns. 12-17 as described above. In case neither n nor R_S is known but instead R_{SH} is known, we solve eqn. 16 for n and then substitute this value of n in eqn.17 to get another transcendental equation in R_S.

For the single exponential model ($I_1 = 0$), the transcendental equation can be obtained by solving eqns. 12 and 14 for $1/R_{SH}$ and n. The solution of these equations give

$$\frac{1}{R_{SH}} = \frac{I_{SC} f_1(R_S) - I_m g_1(R_S)}{(V_{OC} - I_{SC} R_S) f_1(R_S) - (V_{OC} - V_m - I_m R_S) g_1(R_S)} , \quad (22)$$

$$n = \frac{(I_m - I_{SC}) V_{OC} + I_{SC} V_m}{(V_{OC} - I_{SC} R_S) f_1(R_S) - (V_{OC} - V_m - I_m R_S) g_1(R_S)} . \quad (23)$$

Substituting these values of $\frac{1}{R_{SH}}$ and n in eqn. 17, we get the transcendental equation in R_S (we should note that in this case eqn. 16 does not exist because $I_1 = 0$).

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