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14. Abstract/Notes

For an Earth orbiting satellite to accomplish its objectives, it is of main concern that it keeps a certain attitude in relation to the planet. Among the existing attitude control actuators, the gas jet is the one with the largest number of application because of its large range of torque generation capability and because of its independence in relation to other kinds of systems. To be able to use a gas jet system, however, it is fundamental to know its time performance. This work proposes a mathematical modelling for the thrust of a basic gas jet system, including on-off transient times. The model considers two valves in the line: a pressure regulating one and a control one. The latter is modelled in details and its on and off times are obtained analytically. Expressions for the thrust and mass flow with time are also obtained. Transient times and thrust are calculated numerically and considered coherents with prevision.

15. Remarks

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MODELLING OF A COLD GAS JET SYSTEM FOR SATELLITE ATTITUDE CONTROL

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ABSTRACT

For an Earth orbiting satellite to accomplish its objectives, it is of main concern that it keeps a certain attitude in relation to the planet. Among the existing attitude control actuators, the gas jet is the one with the largest number of application because of its large range of torque generation capability and because of its independence in relation to other kinds of systems. To be able to use a gas jet system, however, it is fundamental to know its time performance. This work proposes a mathematical modelling for the thrust of a basic gas jet system, including on-off transient times. The model considers two valves in the line: a pressure regulating one and a control one. The latter is modelled in details and its on and off times are obtained analytically. Expressions for the thrust and mass flow with time are also obtained. Transient times and thrust are calculated numerically and considered coherents with prevision.

1 - INTRODUCTION

For an earth orbiting satellite, the attitude control system is of main concern because the ability to orient the satellite normally determines the mission success. One method of accomplishing the control is through the use of cold gas jets, so as to provide corrective torques.

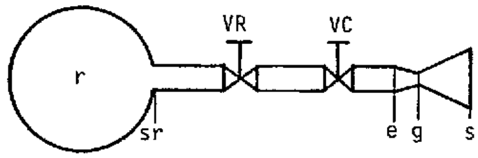
Since the necessary pointing accuracy is often very high, the on-off transient times of the jets become of vital importance. The components that determine these times are the control valve, gas storage tankage, supply lines and pressure regulation valves. In this work, we first deal with a general mathematical model of the entire system and then proceed to a detailed investigation of the most important component, the control valve.

2 - GENERAL SYSTEM MODELLING

A scheme of a satellite attitude control system is shown in figure 1.

The main simplifying assumptions that can be made are:

- the flow is isentropic throughout the system, except in the pressure regulating valve, where it is isenthalpic;
- all the system dimensions are known;
- the gas is considered perfect;
- the pressure regulating valve is perfect i.e., the outlet stagnation pressure is constant, being independent of flow rate and inlet pressure.



r - reservoir
sr - reservoir exit
VR - pressure regulating valve
VC - control valve
e - nozzle inlet
g - nozzle throat
s - nozzle exit

Fig. 1 - Scheme of a cold gas jet satellite attitude control system.

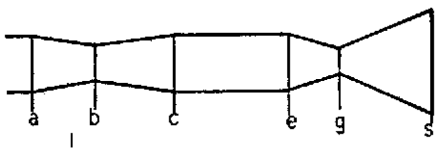
For this model, the control valve is treated as a time varying throat area nozzle (see Fig. 2). The time equation of this area will be shown in the next section.

As the flow is isentropic, the stagnation pressure (p_o) and temperature (T_o) after the pressure regulator remains constant during the actuation time, that is:

$$p_o = \text{const}$$

$$T_o = \text{const}$$

with both being known.



a - valve inlet,
b - valve passage area (variable with time),
c - valve exit,
e - thrust nozzle inlet,
g - thrust nozzle throat area,
s - thrust nozzle exit.

Fig. 2 - Control Valve Flow Model.

In our approach, based on the equations of compressible fluid flow theory [4], it is assumed that the Mach number in section b is unity, when section b is less than the thrust nozzle throat area. As soon as section b becomes greater than the latter, the unity Mach number instantly moves to section g.

The well-known nozzle thrust equation is:

$$F = \dot{m} \cdot v_s + (p_s - p_{at})A_s \,, \qquad (1)$$

where: $\dot{m}$ - mass flow rate;
v_s - gas velocity at nozzle exit;
p_s - gas pressure at nozzle exit;
p_{at} - ambient pressure;
A_s - nozzle exit area.

From these, only p_{at} and A_s are known. To obtain the remaining variables, the following procedure is used [4]:

i) Flow rate ($\dot{m}$):

$$\frac{1}{M_c}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M_c^2\right)\right]^{\frac{k+1}{2(k-1)}} = \frac{A_c}{A_b(t)}, \qquad (2)$$

$$\frac{p_o}{p_c} = \left(1+\frac{k-1}{2}M_c^2\right)^{\frac{k}{k-1}}, \qquad (3)$$

$$\frac{T_o}{T_c} = 1+\frac{k-1}{2}M_c^2 \,. \qquad (4)$$

In these equations, k is the relation between the specific heats of the gas; $A_b(t)$ is the time varying area of section b and M_c, A_c, p_c and T_c are, respectively, the Mach number, the area, the pressure and the temperature at section c.

From equation 2 M_c is obtained and applied in 3 giving p_c, and in 4 giving T_c.

All the values in section c are the same of section e, since the flow is isentropic. Then, from the perfect gas law:

$$\rho_e = \frac{p_c}{RT_c}\,, \qquad (5)$$

where ρ_e is the gas density at section e and R is the gas constant.

From the definition of Mach number, it is possible to get the gas velocity at e:

$$v_e = M_e\sqrt{kRT_e} \,. \qquad (6)$$

Now one can write the mass flow rate as:

$$\dot{m} = \rho_e \cdot A_e \cdot v_e \,. \qquad (7)$$

ii) Thrust nozzle exit gas velocity:

The Mach number at section s is given by:

$$\frac{M_s}{M_e}\left(\frac{1+\frac{k-1}{2}M_e^2}{1+\frac{k-1}{2}M_s^2}\right)^{\frac{k+1}{2(k-1)}} = \frac{A_e}{A_s} \qquad (8)$$

and the temperature by:

$$\frac{T_s}{T_e} = \frac{1+\frac{k-1}{2}M_e^2}{1+\frac{k-1}{2}M_s^2} \,. \qquad (9)$$

The gas velocity at the exit section is:

$$v_s = M_s \cdot \sqrt{kRT_s} \,. \qquad (10)$$

iii) Thrust nozzle exit pressure:

$$\frac{p_s}{p_e} = \left(\frac{1+\frac{k-1}{2}M_e^2}{1+\frac{k-1}{2}M_s^2}\right)^{\frac{k}{k-1}} . \qquad (11)$$

Equations 7, 10 and 11, combined in 1, provide the thrust as a function of the control valve passage area. This, as a function of time, is derived from a more detailed consideration of the control valve as shown in the next section.

3 - CONTROL VALVE MODELLING

In the previous section it was shown a model for the mass flow rate through the control valve, in which there was a time varying area $A_b(t)$. In this section, the procedure to obtain a mathematical model for this passage area is described. Figure 3 shows a scheme of the valve.

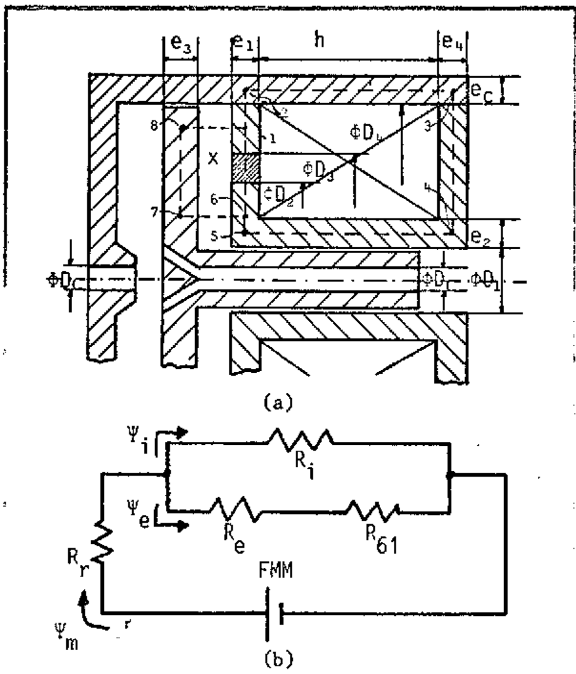


Fig. 3 - a) schematic of magnetic flux path; b) equivalent electric circuit.

The symbols in the figure are defined by:

R_i - insulator reluctance;

R_e - iron gap reluctance;

R_{61} - material reluctance in path 6-7-8-1 excluding the iron gap;

R_r - reluctance of the remaing circuit;

ψ_m - total magnetic flux;

ψ_e - iron gap magnetic flux;

ψ_i - insulator magnetic flux;

FMM - magnetomotive force.

It can be seen that:

$$Ab(t) = \pi \cdot D_c \cdot (x_{max} - x(t)), \qquad (12)$$

where $x(t)$ is the displacement of the valve actuator, and x_{max} is its maximum valve.

Making a force balance on the actuator, one gets [2]:

$$\frac{d^2x}{dt^2} = \frac{1}{m_c}(F_p - F_c - F_m), \qquad (13)$$

being: F_p - pressure force;

F_c - contact force;

F_m - magnetic force;

m_c - actuator mass.

The pressure force is given by:

$$F_p = p_a \cdot \frac{\pi D_i^2}{4} - p_c \cdot \frac{\pi D_c^2}{4}. \qquad (14)$$

Before turning on the valve, the contact force equals the pressure force. After this, F_c decreases at the same rate as F_m increases, until the latter equals F_p, when F_c vanishes.

When the valve is totally open, the contact force reappears in the oposite direction. An analogous reasoning is valid for the closing of the valve.

The magnetic force is given by [3]:

$$F_m = \frac{\psi_e^2}{2\mu_o S_{eq}}, \qquad (15)$$

where S_{eq} is the equivalent gap transversal area, μ_o is the magnetic permeability of the air (gap) and ψ_e is the magnetic flux through the gap.

The area S_{eq} is calculated as:

$$S_{eq} = \frac{\pi}{4}\,\frac{(D_2^2 - D_1^2)(D_4^2 - D_3^2)}{D_2^2 - D_1^2 + D_4^2 - D_3^2}. \qquad (16)$$

The total magnetic flux in the circuit of figure 3 is:

$$\psi_m = \frac{NI}{R_{eq}}, \qquad (17)$$

where N is the number of turns of the coil, I is the current and R_{eq} is the equivalent reluctance of the whole circuit. As can be seen in figure 3, this reluctance can be obtained from [1]:

$$R_{eq} = R_r + \frac{R_i(R_e + R_{61})}{R_i + R_e + R_{61}} \qquad (18)$$

and

$$R_r = \frac{K_r}{\mu_m}, \qquad (19)$$

$$R_i = \frac{1}{2\pi\mu_o e_1}\,\ell n\,\frac{D_3}{D_2}, \qquad (20)$$

$$R_e = \frac{x}{\mu_o S_{eq}}, \qquad (21)$$

$$R_{61} = \frac{K_{61}}{\mu_{62}}. \qquad (22)$$

The subscripts in the variable μ refer to the region where it is calculated, and the constants K_r and K_{61} are geometric factors of the valve, calculated as:

$$K_r = \frac{1}{2\pi}\left\{\frac{1}{e_1}\ln\frac{(D_1+D_2)(D_4+e_c)}{(D_1+e_2)(D_3+D_4)}+\frac{1}{e_4}\ln\frac{D_4+e_c}{D_1+e_2}+\right.$$

$$\left.+(2h+e_1+e_4)\left[\frac{1}{e_c(D_4+e_c)}+\frac{1}{e_2(D_1+e_2)}\right]\cdot\right\}, \quad (23)$$

$$K_{61} = \frac{1}{2}\left(\frac{e_1+e_3}{S_{eq}}+\frac{\ln\frac{D_3+D_4}{D_1+D_2}}{\pi e_3}\right). \quad (24)$$

The current I is obtained from the electric circuit of the valve. It is a simple R-L circuit with variable inductance. Its equation is:

$$V = RI + \frac{d(LI)}{dt}, \quad (25)$$

where: V - applied electrical tension;
R - coil resistance;
L - inductance;

and the inductance is given by:

$$L = \frac{N^2}{R_{eq}}. \quad (26)$$

The set of equations obtained as far as here describes the behaviour of the proposed model, since the following assumptions are taken into account [1]:

- there are no leakages throughout the magnetic circuit;
- the magnetic fluxes follow the paths as showed in figure 3.a;
- the magnetic permeability is supposed to have a mean value in each region, i.e., there is a value for path 1-2-3-4-5-6 and another for path 6-7-8-1.

4 - EXAMPLE OF RESOLUTION OF A CASE

To check the coherence of the equations above, a generic case is solved. One must take into account that the dimensions and characteristics of the system are not optimized, as well as the material of the valve. The main data used are: $D_g = 1$ mm, $D_s = 30$ mm, $P_0 = 5$ atm, $T_0 = 300$ K, $N = 3000$, $D_1 = 3$ mm, $D_2 = 10$ mm, $D_3 = 15$ mm, $D_4 = 30$ mm. The gas used is Nitrogen and the material of the valve is stainless steel. Figure 4 shows the opening and figure 5 shows the closing of the valve.

Although the valve of the given example is not optimized, one can notice that the transient times obtained are in good agreement with testing data of existing valves.

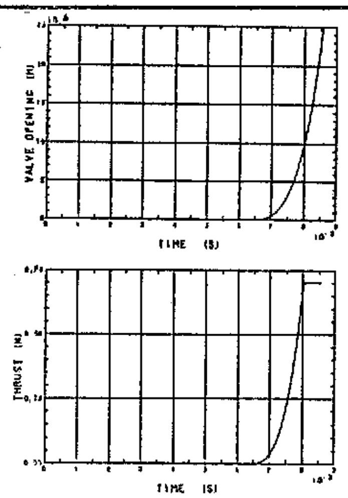


Fig. 4 - Valve opening and thrust during on transient.

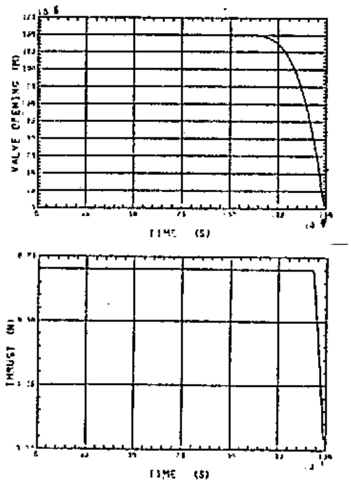


Fig. 5 - Valve opening and thrust during off-transient.

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