1. Publication Nº	2. Version	3. Date	5. Distribution
INPE-3855-PRE/919		*April, 1986*	☐ Internal ☒ External
4. Origin *DMC/DGC* — Program *CONTAT*			☐ Restricted

6. Key words - selected by the author(s)

CONTROLLABILITY — *PARAMETER ESTIMATION*
DISTRIBUTED PARAMETER SYSTEMS — *SEMIGROUP*
OBSERVABILITY — *STATE ESTIMATION*

7. U.D.C.: *681.5.015*

8. Title *INPE-3855-PRE/919*

CONTROL, STATE ESTIMATION AND PARAMETER IDENTIFICATION OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS USING FIXED POINT TECHNIQUES: A SURVEY

10. Nº of pages: *7*

11. Last page: *6*

12. Revised by

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14. Abstract/Notes

Fixed point theorems have been an important tool used by mathematicians in the study of the existence of a solution of nonlinear differential equations, and nonlinear operator equations in general, since the beginning of the century. In the last seven years we have seen the appearance of some papers which extended the basic idea of this tool to solve problems of control, state estimation and parameter identification of nonlinear systems, including distributed parameter systems. The present paper attempts to survey the fixed point techniques used in nonlinear distributed parameter systems in papers already published on this subject. We shall also point out the fact that most of these papers concentrate on theoretical results. Though several of them bring creative ideas, they are poor in practical results or concrete examples of applications

15. Remarks

This paper will be presented at the IV IFAC Symposium on Control of Distributed Parameter Systems to be held in Los Angeles, California, USA, from Jane 29 to July 3, 1986.

CONTROL, STATE ESTIMATION AND PARAMETER IDENTIFICATION OF NONLINEAR DISTRIBUTED PARAMETER SYSTEMS USING FIXED POINT TECHNIQUES: A SURVEY

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Abstract. Fixed point theorems have been an important tool used by mathematicians in the study of the existence of a solution of nonlinear differential equations, and nonlinear operator equations in general, since the beginning of the century. In the last seven years we have seen the appearance of some papers which extended the basic idea of this tool to solve problems of control, state estimation and parameter identification of nonlinear systems, including distributed parameter systems. The present paper attempts to survey the fixed point techniques used in nonlinear distributed parameter systems in papers already published on this subject. We shall also point out the fact that most of these papers concentrate on theoretical results. Though several of them bring creative ideas, they are poor in practical results or concrete examples of applications.

Keywords. Controllability; distributed parameter systems; nonlinear systems; observability; parameter estimation; semigroup; state estimation.

INTRODUCTION

In this paper semilinear systems of the type

$$\dot{z} = Az + Nz + Bu, \quad z(0) = z_0 \tag{1}$$

(A linear on Z, N nonlinear, $B: U \to Z$ linear and u being a control) are considered for the case of controllability, and systems with an observation

$$\dot{z} = Az + Nz, \quad z(0) = z_0 \tag{2}$$

$$y = Cz \tag{3}$$

(A linear, N nonlinear and $C: Z \to Y$ linear) for the case of state estimation. It is assumed that the dynamics of the linearized system

$$\dot{z} = Az, \quad z(0) = z_0 \tag{4}$$

can be described in terms of a strongly continuous semigroup S(t) on an appropriate Banach space Z (the state space).

The problem of controllability is to find a control $u(\cdot) \in \mathcal{U}$ (where $\mathcal{U}$ is a space of functions from [0,T] to the input space U of the system) which drives system (1) from z_0 at $t = 0$ to a given desired state $z_d \in Z$ at $t = T$. The problem of state estimation is to construct the state z(t), $t \in [0,T]$ of system (2) when the output observation $y(\cdot) \in \mathcal{Y}$ (where $\mathcal{Y}$ is a space of functions from [0,T] to the output space Y of the system) is given by (3).

We also consider the joint problem of state and parameter estimation of system of the type

$$\begin{aligned} \dot{z}(t) &= Az(t) + A_1\alpha + N(z(t),\alpha), \quad z(0) = z_0, \\ y(t) &= Cz(t), \end{aligned} \tag{5}$$

where A is a linear operator which generates a strongly continuous semigroup on a Banach space Z (the state space), $A_1 : \mathbb{R}^p \to Z$m N a nonlinear operator from $Z \times \mathbb{R}^p$ to Z, and C a linear operator from the state space Z to $\mathcal{Y}$, a space of output functions from [0,T] to Y (the output space).

The joint problem of state and parameter estimation is to construct the state z(t), $t \in [0,T]$ and identify the parameter $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_p) \in \mathbb{R}^p$ for system (1) when the output observation $y(\cdot) \in \mathcal{Y}$ is given.

In order to draw comparisons we shall try to maintain the notation introduced here even when results are quoted from papers which adopt different terminology.

Examples

An an example of the problem of control consider the following diffusion process on $Z = L^2(0,1)$

$$\begin{aligned} &z_t = z_{xx} + Nz + bu, \\ &z_x(0,t) = z_x(1,t) = 0, \quad z(x,0) = z_0(x), \end{aligned} \tag{6}$$

where $b(\cdot) \in L^2(0,1)$ and N is a nonlinear operator on $L^2(0,1)$ such as $Nz = z^4$, zz_x, z_x^2, $z^2 z_x$, etc. The above system can be expressed in the form (1) by setting $U = \mathbb{R}$, $B: \mathbb{R} \to L^2(0,1)$ given by $Bu = b(x)u$ and A the linear operator on $L^2(0,1)$, $A = z_{xx}$, with domain

$$\mathcal{D}(A) = \{z \in L^2(0,1) : \frac{\partial^2 z}{\partial x^2} \in L^2(0,1), \frac{\partial z}{\partial x} = 0 \text{ at } x = 0,1\}.$$

It can be shown (see p.46 of Curtain and Pritchard, 1978) that A generates a strongly continuous semigroup S(t) on Z.

As an example of a problem of state estimation consider the nonlinear hyperbolic system

$$w_{tt} = w_{xx} + Nw, \tag{7}$$

$$w(0,t) = w(1,t) = 0, \; w(x,0) = w_0(x), \; w(x,0) = \bar{w}_0(x),$$

$$y(t) = \int_0^1 c(x) \frac{\partial w}{\partial t}(x,t)dx,$$

where $c(\cdot) \in L^2(0,1)$ and N is a nonlinear operator on $L^2(0,1)$ such as $Nw = \sin w$, $-|w|^p w$, w^3. By setting $\bar{w} = \dot{w}$ and A the operator from $\mathcal{D}(A) \subset L^2(0,1)$ to $L^2(0,1)$

$$Aw = -\frac{\partial^2 x}{\partial x^2}, \; \mathcal{D}(A) = H^2(0,1) \cap H_0^1(0,1),$$

then the above system can be written in the form (2)-(3) with $z = (w,\bar{w})$, $z_0 = (w_0,\bar{w}_0)$

$$Az = A\begin{bmatrix} w \\ \bar{w} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}\begin{bmatrix} w \\ \bar{w} \end{bmatrix},$$

$$Nz = N\begin{bmatrix} w \\ \bar{w} \end{bmatrix} = \begin{bmatrix} 0 \\ Nz \end{bmatrix},$$

the state space $Z = \mathcal{D}(A^{\frac{1}{2}}) \times L^2(0,1) = H_0^1(0,1) \times L^2(0,1)$,

$$\|z\|_Z^2 = \|(w.w)\|_Z^2 = \|w\|_{H_0^1(0,1)}^2 + \|\bar{w}\|_{L^2(0,1)}^2,$$

$Y = \mathbb{R}$ and $C: Z \to \mathbb{R}$

$$Cz = C(w,\bar{w}) = \langle c,\bar{w}\rangle_{L^2(0,1)}.$$

It can be shown (see p.24 of Curtain and Pritchard, 1978) that A generates a strongly continuous semigroup $S(t)$ on Z.

Finally, for the joint problem of state estimation and parameter identification consider the following diffusion process

$$z_t = z_{xx} + \alpha z,$$

$$z(0,t) = z(1,t) = 0, \quad z(x,0) = z_0(x),$$

with output equation

$$y(t) = \int_0^1 c(x)\; z(x,t)\; dx,$$

where $\alpha \in \mathbb{R}^1$ is the parameter to be identified and $c(\cdot) \in L^2(0,1)$.

Now, if we linearize the above system about $(\bar{z},\bar{\alpha})$ (Carmichael, Pritchard and Quinn, 1980; Pritchard, 1982; Felippe De Souza, 1984c) and set A as in the diffusion process in (6), $p = 1$ (i.e., one parameter $\alpha \in \mathbb{R}^1$), $A_1: \mathbb{R}^1 \to Z$ given by $A_1\alpha = \bar{z}\alpha$, the nonlinearity $N(z,\alpha) = \alpha z + \bar{z}_{xx}$ and $C: Z \to \mathbb{R}$

$$Cz = \langle c(\cdot), z(\cdot)\rangle_{L^2(0,1)},$$

then, we can express it in the form (5).

FIXED POINT THEOREMS

Fixed point theorems have been used since the beginning of the century to show local existence theorems for differential equations. Recently several papers have been published using fixed point theorems to provide existence of the solutions of the problems of controllability and state estimation. These papers usually provide mappings ϕ or F defined on some space X of functions from $[0,T]$ to Z (e.g., $X = C(0,T;Z)$ or $X = L^p(0,T;Z)$ for some $p \geq 1$, etc.) and the solutions of the problems of nonlinear controllability and state estimation are obtained via the fixed points of such mappings.

So, existence of a solution for the problem of nonlinear controllability or state estimation is transformed into existence of fixed points of these mappings. The papers referred to above used fixed point theorems of contractive type, topological type or contractive type with perturbations.

A classical example of a contractive type fixed point theorem is the Banach contraction principle (Banach, 1922).

Among the several topological fixed point theorems, we refer to Brouwer's fixed point theorems (Dunford and Schwartz, 1963) and Schauder fixed point theorem (Leray and Schauder, 1934).

Among the numerous more recent papers which develop fixed point theorems of the last type we mention Nussbaum (1969) and Petryshyn (1973).

CONTROL PROBLEM

First let X denote the space of the trajectories (e.g., $X = C(0,T;Z)$ or $X = L^p(0,T;Z)$ for some $p \geq 1$, etc.) and $L(t)$ be the linear operator defined on $\tilde{X} \supseteq X$ for each $t \in [0,T]$ by

$$L(t)x(\cdot) = \int_0^t S(t-s)x(s)ds.$$

Now let $G: \mathcal{U} \to Z$ be the operator

$$Gu = \int_0^T S(T-s)Bu(s)ds = L(T)Bu$$

and set $\tilde{X}$ = kernel (G) and $\tilde{G}: \mathcal{U}/\tilde{X} \to Z$ defined by $\tilde{G}[u] = Gu$ for all equivalence classes $[u] \in \mathcal{U}/\tilde{X}$ so that $\tilde{G}^{-1}$ always exists.

Suppose now that we know an actual trajectory $z^*(t)$, $t \in [0,T]$ which takes system (1) from the initial state $z_0 = 0$ (the origin) to the desired state z_d on the interval $[0,T]$. Thus, it is easy to verify that $u^* \in \mathcal{U}$ defined by

$$\begin{aligned} u^* &= \tilde{G}^{-1}\left[z_d - \int_0^T S(T-\rho)Nz^*(\rho)d\alpha\right] \\ &= \tilde{G}^{-1}[z_d - L(T)Nz^*(\cdot)] \end{aligned} \tag{8}$$

is the wanted control which steers system (1) from the origin to z_d on $[0,T]$. This follows since the mild form of (1) can be expressed as

$$z(\cdot) = S(\cdot)z_0 + L(\cdot)Nz(\cdot) + L(\cdot)Bu, \; z(0) = z_0. \tag{9}$$

So, the wanted control u^* as given in (8) depends on the knowledge of the actual trajectory $z^*(\cdot)$ of the system. Substituting $z(\cdot)$ and u in (9) by respectively $z^*(\cdot)$ and u^* given in (8), we obtain

$$z^*(\cdot) = L(\cdot)B\tilde{G}^{-1}[z_d - \{L(T)Nz^*(\cdot)\}] + L(\cdot)Nz^*(\cdot),$$

which is an expression of the actual trajectory depending on itself. Clearly, $z^*(\cdot)$ is a fixed point of the mapping $\phi: X \to X$ defined by

$$\phi(z(\cdot) = L(\cdot)B\tilde{G}^{-1}[z_d - L(T)Nz(\cdot)] + L(\cdot)Nz(\cdot). \tag{10}$$

That is, $z^*(\cdot) = \phi(z^*(\cdot))$. The problem of controllability of system (1) (in other words, the problem of existence of a wanted control u^*) is transformed into existence of a fixed point for ϕ. Moreover, the problem of finding a wanted control u^* is transformed into finding a fixed point for ϕ. This approach was used in Magnusson, Pritchard and Quinn (1981); Carmichael, Pritchard and Quinn (1981a, 1981b) and Pritchard (1981). In fact, Magnusson, Pritchard and Quinn (1981) and Carmichael, Pritchard and Quinn (1981a) using respectively a contraction mapping theorem and a fixed point theorem for mappings of contractive type with perturbations, have shown that the diffusion process (6) with the non-linearity $Nz=z^2$ is controllable to any z_d within a ball of radius r in Z (r being dependent on the norms on X of $S(\cdot)z_1$, $z_1 \in Z$ and $Nz(\cdot)$, $z(\cdot)\in X$).

Actually, the work of Ichikawa and Pritchard (1979) was one of the first to use a similar approach to solve existence and uniqueness of evolution equations. Mappings similar to ϕ in (10) were used in Carmichael and Quinn (1982a) to solve an optimal control problem and also in Kassara an El Jai (1982) in an attempt to construct an algorithm for solving the control problem.

Felippe De Souza (1982, 1983c) showed that the mapping ϕ in (10) does not have sufficient conditions for us to obtain the wanted controls u* which drive the system from $z_0=0$ to z_d in the interval [0,T]. That is, there may be wanted controls u* which can never be found via the fixed points $z^*(\cdot)$ of ϕ. Pritchard (1982) found that the mapping ϕ of (10) could be simplified by using $G^\dagger$ (the generalized inverse of G) instead of G^{-1}, as long as Range (G) is closed in Z. Moreover, Pritchard (1982) assumed that U and/or Z could be adjusted in order to the operator G to have closed range in Z. Later Felippe De Souza (1983a, chapter 5 of 1983c) showed that this adjustment of U and Z is always possible. Furthermore, Felippe De Souza (1983b) presents an interative procedure for this adjustment to be done.

The mapping ϕ with $G^\dagger$ was an improvement in the simplicity but, on the other hand, it does not have neither necessary nor sufficient conditions for us to obtain the wanted controls u*. Actually, a fixed point $z^*(\cdot)$ of ϕ must satisfy

$$[z_d - L(T)Nz^*(\cdot)] \in \text{Range } (G)$$

in order to u* given by (8) be a wanted control for system (1). This conditions was called "check of consistencey" in Pritchard (1982). Check of consistency has also been used in Felippe De Souza and Pritchard (1985a) with similar mappings ϕ for which sufficient conditions for us to obtain a wanted control u* does not hold either. Felippe De Souza (1985) developed similar mappings ϕ which admitted cases where $z_0 \neq 0$.

A more sophisticated approach to solve the control problem for system (1) was introduced in Felippe De Souza (1982). This new approach involved a pair $z=(z(\cdot),z_f)$ consisting of the trajectory $z(\cdot)\in X$ and the final state $z_f \in Z$. Instead of $\phi: X \to X$, the mapping used was $F: M \to M$, M being the cross product between X and Z.

Assume that the state space Z is a Hilbert space and the trajectory $z(\cdot)$ on [0,T] lies in X

$$X = L^2(0,T;Z).$$

Define the Hilbert space

$$M = X \times Z$$

with the inner-product given by

$$<\begin{pmatrix} z(\cdot) \\ z_f \end{pmatrix}, \begin{pmatrix} z'(\cdot) \\ z_f' \end{pmatrix}>_M = <z(\cdot),z'(\cdot)>_X + <z_f,z_f'>_Z.$$

Define the operators $S: Z \to M$ and $L: X \to M$ by

$$Sz_f = (S(\cdot)z_f,\ S(T)z_f)$$

$$Lx(\cdot) = (L(\cdot)x(\cdot),\ L(T)x(\cdot)).$$

Now system (9) can be represented in the compact form

$$z = Sz_0 + LNz(\cdot) + LBu(\cdot),\ z(0) = z_0. \qquad (11)$$

It is assumed that Range (LB) is closed in M. If this is not the case for a particular choice (U,Z) of space of input functions U and state space Z it is necessary to reframe the system in a restricted state space Z' and/or an enlarged space of input functions U' such that the assumption will hold for the choice (U',Z'). Here again the work of Felippe De Souza (1983a, 1983b, 1983c) on the adjustment of the topology of the spaces can be applied.

The mapping $F: M \to M$ used in Felippe De Souza was

$$F(z) = \gamma + (I-P)LNz(\cdot) + Pz - LBG^\dagger z_f, \qquad (12)$$

where $\gamma = [LBG^\dagger z_d + (I-P)Sz_0] \in M$ is a fixed element, I is the identity on M and P is any continuous projection onto Range (LB).

Felippe De Souza (1982) showed that if there is a wanted control u* in U which drives the system from z_0 at t=0 to the desired state z_d at t=T, then u* can be obtained via the fixed points of F. In other words, F was the first mapping with sufficient conditions for us to obtain the wanted controls u*. This was possible only because of the approach of the pair $(z(\cdot),z_f)$ consisting of the trajectory and final state.

Also note that F in (12) is in fact a family of mappings, since P is any continuous projections onto Range (LB). Several examples of continuous projections onto Range (LB) are shown in chapter 4 of Felippe De Souza (1983c). It is also shown that when $z_0=0$ and for a particular projection $P=\Pi$, where $\Pi = \Pi_1 + \Pi_2$ was defined in Felippe de Souza (1982), F in (12) becomes

$$F(z) = (\phi z(\cdot), (\phi z(\cdot))(T)) + \Pi_2(z - LNz(\cdot)),$$

where ϕ is as before. That is, ϕ can be regarded as the particular case of F when $P=\Pi_1$. Also, $\Pi_2(z - LNz(\cdot))$ is the missing term in ϕ which gives sufficiency to F.

Later, Felippe De Souza (1983c, 1984a) developed mappings $F: M \to M$ which have necessary and sufficient conditions. That is: if z* is a fixed point of F, then we can obtain a wanted control u*, and if u* is a wanted control, then it can be obtained using a fixed point z* of F.

Let $\xi: M \to M$ be the mapping

$$\xi(z) = z - Sz_0 - LNz(\cdot),$$

$\bar{x}$ be any fixed element of M which satisfies

$$\bar{x} \in \text{Range (LB) and } \bar{x} \neq 0,$$

q be the functional on Z given by

$$q(z_f) = \|z_d - z_f\|_Z,$$

and $F: M \to M$ be the mapping

$$F(z) = Sz_0 + LNz(\cdot) + P\xi(z) + q(z_f)\bar{x} \qquad (13)$$

for $z = (z(\cdot),z_f) \in M$.

Felippe De Souza (1983c) showed that: If $z^* = (z^*(\cdot),z_f^*) \in M$ is a fixed point of F, then the control $u^* \in U$ given by

$$u^* = (LB)^{\dagger}\xi(z^*) \qquad (14)$$

drives system (1) from z_0 at $t=0$ to z_d at $t=T$.

Moreover, any control $u^* \in U$ which drives system (1) to z_d in time T has the form (14) for some fixed point z^* of F.

STATE ESTIMATION

First assume that the linearized system (4) is continuous initially observable. Then the mapping $[CS(\cdot)]:Z \to Y$ has a continuous inverse $[CS(\cdot)]^{-1}$ (Curtain and Pritchard, 1978).

Now let X be the space of the trajectories (e.g., $X = C(0,T;Z)$ or $X = L^p(0,T;Z)$ for some $p \geq 1$). The mild form of system (2) can be expressed by

$$z(\cdot) = S(\cdot)z_0 + L(\cdot)Nz(\cdot), \quad z(0) = z_0, \qquad (15)$$

where $L(t)$, defined on $\bar{X} \supseteq X$ for each t [0,T], is the linear operator defined as before. So, the observation y in (3) may be written as

$$y = CS(\cdot)z_0 + CL(\cdot)Nz(\cdot),$$

thus,

$$CS(\cdot)z_0 = (y - CL(\cdot)Nz(\cdot)),$$

hence,

$$z_0 = [CS(\cdot)]^{-1}(y - CL(\cdot)Nz(\cdot)),$$

and therefore, if $z^*(\cdot)$ is the actual state of the system observed, we can write, by (15),

$$z^*(\cdot) = S(\cdot)[CS(\cdot)]^{-1}(y - CL(\cdot)Nz^*(\cdot)) + CL(\cdot)Nz^*(\cdot).$$

Clearly, $z^*(\cdot)$ is a fixed point of the mapping $\phi: X \to X$ defined by

$$(\phi z(\cdot))(t) = S(t)[CS(\cdot)]^{-1}(y - CL(\cdot)Nz(\cdot)) + L(t)Nz(\cdot). \qquad (16)$$

That is, $z^*(\cdot) = \phi(z^*(\cdot))$. The problem of state estimation of system (1) with the observation given by (2) is transformed into finding a fixed point for ϕ.

This approach was used in Carmichael, Pritchard and Quinn (1980, 1981a, 1981b, 1982), Carmichael, Pritchard and Fincham (1981) and Pritchard (1981). In fact, Pritchard (1981) and Carmichael, Pritchard and Quinn (1982), using contractive type fixed point theorems, showed that the state of the hyperbolic system (7) with the nonlinearity $Nw = w^2$ can be estimated as long as the observation y lies in the ball of radius r in Y (r being dependent on the norms on X of $S(\cdot)z_1$, $z_1 \in Z$ and $Nz(\cdot)$, $z(\cdot) \in X$).

Carmichael and Quinn (1982b) used ϕ in (16) in an attempt to construct an algorithm for solving the problem of state estimation. The assumption of continuously initial observability for (4) is a bit strong since it eliminates cases such as, for example, systems described by partial differential equations of parabolic type.

Pritchard (1982) and Felippe De Souza and Pritchard (1985b) used mappings ϕ with $[CS(\cdot)]^{\dagger}$ instead of $[CS(\cdot)]^{-1}$. It was assumed that Y and/or Z could be adjusted in order to the operator $CS(\cdot)$ to have closed range in Y. Later Felippe De Souza (1983a, chapter 5 of 1983c) showed that this adjustment of Y and Z is always possible. Moreover, Felippe De Souza (1983b) presents an iterative procedure for this adjustment to be done. This approach can deal with cases left out earlier because of the continuously initial observability assumption for (4). Unfortunately, however, the new ϕ with $[CS(\cdot)]^{\dagger}$ does not have neither necessary nor sufficient condition for obtaining state estimation via its fixed points. An estimation of the state is not necessarily a fixed point of ϕ. A fixed point of ϕ will be the estimated state only if a check of consistency presented in Pritchard (1982) is satisfied.

A more sophisticated approach to solve the problem of state estimation of (2) with observation given by (3) was introduced in Felippe De Souza (1982). This new approach involved a pair $z = (z_0, z(\cdot))$ consisting of the initial state $z_0 \in Z$ and the trajectory $z(\cdot) \in X$. Instead of $\phi: X \to X$, the mapping used was $F: M \to M$, M here being the cross product between Z and X.

Let M be the normed space

$$M = Z \times X$$

with norm given by

$$\|(z_0, z(\cdot))\|_M = \|z_0\|_Z + \|z(\cdot)\|_X,$$

and define the operators $S: Z \to M$ and $L: X \to M$ by

$$Sz_0 = (z_0, S(\cdot)z_0),$$

$$Lx(\cdot) = (0, L(\cdot)x(\cdot)).$$

Now system (15) can be written in the compact form

$$z = Sz_0 + LNz(\cdot), \quad z(0) = z_0, \qquad (17)$$

and the output equation (3) can be written as

$$y(\cdot) = Cz(\cdot). \qquad (18)$$

The mapping $F: M \to M$ used in Felippe De Souza (1982) was

$$F(z) = \gamma + (I-P)LNz(\cdot) + Pz - S[CS(\cdot)]^{\dagger}Cz(\cdot) \qquad (19)$$

where $\gamma = S[CS(\cdot)]^{\dagger}y \in M$ is a fixed element, I is the identity on M and P is any continuous projection onto Range S.

Felippe De Souza (1982, 1983c) showed that if $z^* = (z_0^*, z^*(\cdot))$ is a fixed point of F, then $z^*(t)$, $t \in [0,T]$ is the trajectory of the system and in particular $z^*(0) = z_0^*$. In other words, F was the first mapping with sufficient condition to obtain state estimation via its fixed points. This was possible because of the approach of the pair $(z_0, z(\cdot))$ consisting of the initial state and the trajectory.

Note that no condition on Range $(CS(\cdot))$ was imposed. Also, F in (19) is in fact a family of mappings, since P is any continuous projection onto Range (S). Several examples of continuous projections onto Range (S) have been shown in Chapter 4 of Felippe De Souza (1983c). For example, let $P: M \to M$ be

$$P(z_0, z(\cdot)) = (z_0, S(\cdot)z_0).$$

Clearly P is a continuous projection onto Range (S). It is also shown that for a particular projection $P = \Pi$, where $\Pi = \Pi_1 + \Pi_2$ was defined in Felippe De

Souza (1982), F in (17) becomes

$$F(z) = ((\phi z(\cdot)(0),\ \phi z(\cdot)) + \pi_2(z - LNz(\cdot)),$$

where ϕ is as before. So the approach of ϕ was incorporated in the mappings F. Also $\pi_2(z - LNz(\cdot))$ is the missing term in ϕ which gives sufficiency to F.

Later Felippe De Souza (1984a, 1984b, 1984c) developed mappings $F: M \to M$ which have necessary and sufficient conditions for their fixed points to the estimated states.

The following result was proved in Felippe De Souza (1984a).

Let $\bar{x}$ be any fixed element of M which satisfies

$$\bar{x} \in \text{Range}\ (S) \text{ and } \bar{x} \neq 0,$$

q be the functional on X given by

$$q(z(\cdot)) = \|y(\cdot) - Cz(\cdot)\|_Y$$

and $F: M \to M$ be the mapping

$$F(z) = Sz_0 + LNz(\cdot) + q(z(\cdot))\bar{x} \tag{20}$$

for $z = (z_0, z(\cdot)) \in M$. The pair $z^* = (z_0^*, z^*(\cdot)) \in M$ is a fixed point of F if and only if z^* satisfies both (17) and (18), that is, if and only if $z^*(0) = z_0^*$, z_0^* is an estimation of the initial state and $z^*(\cdot)$ is an estimation of the trajectory on $[0,T]$.

Felippe De Souza (1984b) defined a functional f on X and showed a similar result for the mapping F given by

$$F(z) = f(z(\cdot))Sz_0 + LNz(\cdot). \tag{21}$$

THE JOINT STATE AND PARAMETER ESTIMATION

In this section we show that the joint problem of state estimation and parameter identification, as formulated for system (5) with state space Z, is equivalent to just state estimation of a system with an enlarged state space $Z \times \mathbb{R}^p$ which has the form

$$\begin{aligned} \dot{\bar{z}}(t) &= \bar{A}\bar{z}(t) + \bar{N}\bar{z}(t), \quad \bar{z}(0) = \bar{z}_0, \\ y(t) &= \bar{C}\bar{z}(t), \end{aligned} \tag{22}$$

where $\bar{A}$ generates a strongly continuous semigroup $\bar{S}(t)$ on $\bar{Z} = Z \times \mathbb{R}^p$.

Actually this approach has been used in Carmichael, Pritchard and Quinn (1982), Pritchard (1981, 1982) and Felippe De Souza (1982, 1984b, 1984c).

Since $\alpha \in \mathbb{R}^p$, $\dot{\alpha} = \frac{d\alpha}{dt} = 0$ and system (5) can be expressed as

$$\begin{aligned} \begin{bmatrix} \dot{z}(t) \\ \dot{\alpha} \end{bmatrix} &= \begin{bmatrix} A & A_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ \alpha \end{bmatrix} + \begin{bmatrix} N(z(t),\alpha) \\ 0 \end{bmatrix} \\ y(t) &= [C \quad 0] \begin{bmatrix} z(t) \\ \alpha \end{bmatrix} \end{aligned} \tag{23}$$

with initial condition $(z(0), \alpha(0)) = (z_0, \alpha)$.

System (23) was called the overall system for (5) in Felippe De Souza (1982, 1984c).

Now, if we define $\bar{z}_0 = (z_0, \alpha)$ and $\bar{z}(t) = (z(t), \alpha)$ for each $t \in [0,t]$, then we can write the overall system in the form (22) with

$$\bar{A} = \begin{bmatrix} A & A_1 \\ 0 & 0 \end{bmatrix}, \quad \bar{N} = \begin{bmatrix} N \\ 0 \end{bmatrix} \text{ and } \bar{C} = [C \quad 0].$$

So, the joint problem of state estimation and parameter identification of (5) is transformed into state estimation of (22) and the techniques of the previous section will then have an immediate extension for problems of state and parameter estimation of systems of the type (5).

Felippe De Souza (1983c, 1984c) also shows that $\bar{A}$ generates the strongly continuous semigroup $\bar{S}(t)$ on $\bar{Z}$ given by

$$\bar{S}(t) = \begin{bmatrix} S(t) & L_1(t) \\ 0 & I \end{bmatrix},$$

where

$$L_1(t)\alpha = \int_0^t S(\tau)A_1\alpha\, d\tau$$

and I is the identity on $\mathbb{R}^p$.

CONCLUSIONS

Theoretically, no doubt that the mappings F in (12) and (13) for the control problem and in (19), (20) and (21) for the problem of nonlinear state estimation have some advantages over the mapping ϕ in (10) and (16) respectively. Besides, they are more elegantly posed and their construction involved some nice creative ideas. In practice, however, it is much more difficult to apply fixed point theorem to the mappings F than to the mappings ϕ since F are defined in a larger space M. Nevertheless, since the mappings ϕ never give necessary and sufficient conditions to solve each of the problems of nonlinear controllability and state estimation, there are still some advantages to consider the mappings F.

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