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STATE ESTIMATION OF MULTIVARIABLE SYSTEMS USING THE DUALITY CONCEPT

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14. Abstract/Notes *This work presents the development of a new state estimator for dynamic systems based upon the duality concept between state estimation and optimal control problems. This leads to an estimator algorithm similar to the Extended Kalman Filter for use in real time nonlinear systems, which can recursively be approximated by linear systems. The duality concept is used to transform the original estimation problem into an equivalent one of virtual control. This control problem is then used to generate an adaptive, locally convergent algorithm where, instead of full state estimation, one has to estimate just a control vector with smaller dimension than the state vector. The control formulation also allows the development of convergence acceleration criteria. Three convergence criteria, as well as adaptive noise methods to compensate for model errors, are then developed and used with the proposed estimator in the test problem created to validate the estimator. The algorithm is implemented in a digital computer to estimate the orbit of a low Earth orbit satellite under simulated conditions. Numerical results include test cases considering different initial values for the estimator and different standard deviations for the observations provided by three Earth stations, allowing a preliminary evaluation of the estimator characteristics.*

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1. INTRODUCTION

A very common situation arising when one has to estimate the state of a multivariable system is the problem of lack of knowledge about the system dynamics. In this case, one has to adopt a dynamic model for the estimator which is only a crude approximation of reality. Unmodelled dynamic effects, specially in the case of nonlinear systems, can cause divergence of the estimates.

In order to avoid divergence, error compensation techniques are usually employed with different versions of the Kalman Filter (e.g. Maybeck, 1979, Jazwinski, 1970; Gelb et al., 1974). The techniques usually explore the information given by the observation residues to either directly estimate the unmodelled effects (e.g.: Tapley and Ingram, 1973; Cruz and Rios Neto, 1980; Rios Neto and Cruz; 1985) or to condition the state error covariance matrix to keep the capabitility of the estimator to extract information from the new observations (e.g. Jazwinski, 1969; Rios Neto and Kuga, 1981, 1982, 1985). In the resulting procedures there is always an increase in the number of variables to be estimated.

This work presents a new alternative scheme for the multivariable state estimation problem, specially for nonlinear systems. The duality concept between the estimation and optimal control problems is explored to transform the original estimation problem into one of tracking the observations with a virtual control. The key idea is that, independently of the dynamic model for the estimator, if the system is completely controllable and observable, one can choose a desirable control action to drive the system towards a region defined by the observations, in a finite time interval. Whithout the need of increasing the number of estimated variables, the observation residues are used to estimate the virtual control necessary to update the estimate of the state. Besides that, the virtual nature of the tracking control allows the possibility of choosing the control actions adequate to better extract the information contained in the observations (Rios Neto and Fleury, 1984; Fleury, 1985).

Therefore, the proposed estimator is indicated for situations where there is a great lack of knowledge about the system dynamics but there is a high local level of information in the observations.

The virtual formulation also allows to get some advantages from the system structure. As will be explained in item 3.2, one can define a control matrix correction in order to improve convergence.

Preliminary tests of the proposed estimator were done under digitally simulated conditions for the problem of real time orbit estimation of a low altitude artificial satellite. Results are shown to be satisfactory for this exploratory phase of the procedure.

2. PROPOSED PROCEDURE

The problem to be solved is the state estimation of a multivariable dynamic system of the type:

$$\dot{x} = f(x, t) + f^n(x,t) + \overline{G}(t)\; w(t), \tag{1}$$

$$y(t_k) = h_k\;(x(t_k),\; t_k) + v(t_k),\; k = 1,\, 2,\, \ldots \tag{2}$$

were x is the nx1 state vector; $w(t)$ and $v(t_k)$ are mx1 and rx1 independent Gaussian white noises with the usual hypothesis of $w(t)$ being independent of the past state and $v(t_k)$ being independent of the state, with distributions defined by zero means and covariances:

$$E[w(t)\; w^T(\tau)] = Q(t)\; \delta(t-\tau), \tag{3}$$

$$E[v(t_k)\; v^T(t_j)] = R(k)\; \delta_{kj}, \tag{4}$$

where $\delta(t-\tau)$ is the Dirac delta function and δ_{kj} is the Kronecker symbol.

In this problem, the term $f^n(x,t)$ in Equation (1) represents the unknown part of the dynamical model, which usually cannot be included in the estimator model because of lack of knowledge about the system dynamics. Consider now a typical discretization interval, $(t_k,\, t_{k+1})$. In the prediction phase of an extended Kalman filter (e.g. Jazwinski, 1970), a nominal trajectory is generated by:

$$\dot{\bar{x}} = f(\bar{x},t) \quad ; \quad \bar{x}(t_k) = \hat{x}(t_k|t_k) \tag{5}$$

where $\hat{x}(t_k|t_k)$ is the estimate in t_k.

Linearizing the system given in Equation (1) around the nominal trajectory and defining:

$$\Delta x(t) = x(t) - \bar{x}(t), \tag{6}$$

one obtains a first order approximation of the propagated error as given by:

$$\Delta x(t_{k+1}) = \Phi(t_{k+1},t_k)\ \Delta x(t_k) + \int_{t_k}^{t_{k+1}} \Phi\ (t_{k+1,s})\ \bar{G}(s)\ w(s)\ d(s). \tag{7}$$

In Equation (7), $\Phi(.,.)$ is the state transition matrix associated to the linear system:

$$\Delta\dot{x}(t) = f_x\ (\bar{x},t)\ \Delta x(t) + \bar{G}(t)\ w(t), \tag{8}$$

that is:

$$\dot{\Phi}(t,t_k) = f_x(\bar{x},t)\ \Phi(t,\ t_k) \quad ; \quad \Phi(t_k,\ t_k) = I \tag{9}$$

where the subindex x indicates a partial derivation with respect to the state.

The propagated error can be regarded as the a priori information in t_{k+1} which is the information based on previously processed observations:

$$\Delta x(t_{k+1}) = \Delta\hat{x}(t_{k+1}|t_k) + \eta(k+1|k). \tag{10}$$

Since the nominal trajectory was taken with the initial value in t_k equal to the estimated value at that time, then the propagated estimate in t_{k+1} is necessarily zero. Therefore, if the approximations of Equations (5) and (7) are assumed, there results:

$$\Delta x(t_{k+1}) = 0 + \eta(k+1|k), \tag{11}$$

where $\eta(k+1|k)$ is zero mean conditioned on the observations already processed with covariances given by:

$$E[\eta(k+1|k)\ \eta^T(k+1|k)] \triangleq P(k+1|k) =$$

$$\Phi(t_{k+1,t_k})\ P(k|k)\ \Phi^T(t_{k+1,t_k}) + \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},s)\ \overline{G}(s)\ Q(s)\ G^T(s)$$

$$\Phi^T(t_{k.1,s})\ ds. \tag{12}$$

In the proposed procedure the state estimation of the system in Equation (1) is approximated by the estimation of a "virtual control". Defining:

$$\dot{x}_c = f(x_{c,t}) + G(t)\ u(t), \tag{13}$$

the control vector $u(t)$ is to be estimated in order to force the controlled state x_c to be a good approximation of the true state x by tracking the observations given by Equation (2). From an heuristic point of view, the idea is quite simple.

Consider a typical interval (t_k, t_{k+1}). Since the model of the estimator is not a good approximation to the real system,within the interval t_k to t_{k+1},the propagated trajectory will deviate relative to the true one. A control action, based on the observations residues, is then calculated in t_{k+1} to update the estimate of the state vector. This action changes the initial condition for the nominal trajectory in t_{k+1} up to a point closer to the true trajectory than the original one. In some sense, this procedure resembles the extended Kalman filter but it must be pointed out that the virtual control scheme requires a number of estimated control components just equal to the number of degrees of freedom of the system (controllability) and also requires a number of observations with enough level of information which guarantees the calculation of the control action in t_{k+1} (observability). There results an adaptive procedure that forces the dynamic model in the estimator towards the true trajectory.

To obtain the control $u_{(t)}$ in the interval t_k to t_{k+1}, one assumes u as a first order perturbation and takes the controlled trajectory as:

$$\dot{\overline{x}}_c = f(\overline{x}_c,\ t) \quad , \ \overline{x}_c(t_k) = \hat{x}(t_k|t_k). \tag{14}$$

Using the same steps given by Equations (6) and (7):

$$\Delta\dot{x}_c = f_x(\overline{x}_c,\ t)\ \Delta x_c + G(t)\ u(t) \quad , \quad \Delta x_c(t_k) = 0, \tag{15}$$

where u(t) is modelled as a step process and calculated to satisfy:

$$\Delta x_c(t_{k+1}) \cong \Delta x(t_{k+1}), \tag{16}$$

$$y(t_{k+1}) = h_{k+1}(\overline{x}_c(t_{k+1}) + \Delta x_c(t_{k+1}),\ t_{k+1}) + v(t_{k+1}). \tag{17}$$

From Equations (11) and (16) there results:

$$0 = \Delta x_c(t_{k+1}) + \eta(k+1|k). \tag{18}$$

But from Equation (15) one obtains:

$$\Delta x_c(t_{k+1}) = \left(\int_{t_k}^{t_{k+1}} \Phi_c\ (t_{k+1},s)\ G(s)\ ds \right) u(t_k) \triangleq \gamma(k+1,k)\ u(t_k) \tag{19}$$

where $\Phi_c(t,t_k)$ is the transition matrix associated to Equation (15). From the linearization of Equation (17) one gets:

$$\Delta y(t_{k+1}) = \frac{\partial}{\partial x_c}\ h_{k+1}\ (\overline{x}_c(t_{k+1}),\ t_{k+1})\ \Delta x_c(t_{k+1}) + v(t_{k+1}) \tag{20}$$

where high order terms have been disregarded.

Finally, combining the results of Equations (18), (19) and (20), the following problem of parameter estimation results:

$$0 = \gamma(k+1,k)\ u(t_k) + \eta(k+1|k), \tag{21}$$

$$\Delta y(t_{k+1}) = H(k+1)\ \gamma(k+1,k)\ u(t_k) + v(t_{k\ 1}), \tag{22}$$

where

$$H(k+1) \triangleq \frac{\partial}{\partial x_c} h_{k+1}(\bar{x}_c\ (t_{k+1}),\ t_{k+1}). \tag{23}$$

Using a Gauss-Markov minimum variance estimator (e.g. Liebelt, 1967 and Maybeck, 1979), estimates of $u(t_k)$ are obtained:

$$\hat{u}(t_k) = P_u(k+1|k)\ \gamma(k+1,\ k)\ H^T(k+1)\ R^{-1}(k+1)\ \Delta y(t_{k+1}) \tag{24}$$

$$P_u(k+1|k) = [\gamma^T(k+1,\ k)\ P^{-1}(k+1|k)\gamma(k+1,k) + \gamma^T(k+1,\ k)\ .$$

$$H^T(k+1)\ R^{-1}(k+1)\ H(k+1)\ \gamma(k+1,k)], \tag{25}$$

where $R(k+1)$ and $P(k+1|k)$ are as defined in Equations (4) and (12).

To recover the estimate of the state, one shall combine Equations (19) and (24) to get:

$$\Delta\hat{x}_c(t_{k+1}) = \gamma(k+1,\ k)\ \hat{u}(t_k) \tag{26}$$

and take the approximation:

$$\hat{x}(t_{k+1}|t_{k+1}) = \bar{x}(t_{k+1}|t_k) + \Delta\hat{x}_c(t_{k+1}). \tag{27}$$

To recover the covariance matrix of the error in the estimate, it is only necessary to consider Equations (16), (19) and (27) to get:

$$e(t_{k+1}|t_{k+1}) \triangleq x(t_{k+1}) - \hat{x}(t_{k+1}|t_{k+1}) = \Delta x - \Delta\hat{x}_c \qquad (28)$$

and

$$P(k+1|k+1) \triangleq E[e(t_{k+1}|t_{k+1})\ e^T(t_{k+1}|t_{k+1})] = \gamma(k+1,k)\ P_u(k+1|k).$$
$$\gamma^T(k+1,\ k). \qquad (29)$$

3. APPLICATION

The procedure was tested under digital simulation (Burroughs B6800) for the case of real time orbit determination of a low altitude satellite with:

zero excentricity, 42^o inclination, 250 km altitude,
A/m = 0,00076 m²/kg, area over mass ratio,
C_D = 2.0, drag coefficient.

To simulate the observations, a true orbit was generated by numerical integration, using a dynamic model including the influence of gravity (up to J_6 zonal and C_{44}, S_{44} tesseral coefficients), atmospheric drag and perturbations of Sun and Moon. The model adopted for $f(x_c,t)$, in Equation (13), only included the gravitacional effects up to J_2, thus characterizing a situation of lack of knowledge quite serious in the dynamics of the system. It is shown in Kuga (1982) that the application of the Extended Kalman Filter to the same problem, whithout any error compensation technique, cause divergence of the estimates.

Bearing in mind the approximations taken in the proposed procedure, one must expect some difficulties in dealing with this type of problem. Therefore, error compensation techniques are employed.

3.1 - ADAPTIVE STATE NOISE ESTIMATION

The overcome the ill-conditioning on the state error covariance matrix due to the approximation of Equation (7) for the propagation of the error and due to the nonlinearities in the observations, within the applications both $P(k+1|k)$ and $P_u(k+1|k)$ are adapted by using and Adaptive State Noise Estimation technique (Rios Neto and Kuga, 1981, 1982, 1985). This technique is the generalization of a procedure by Jazwinski (1969) and consists of adding noise to the system through the state noise covariance matrix Q.

Consider the typical interval (t_k, t_{k+1}) and assume that $Q(k)$ for this interval is diagonal or can be diagonalized:

$$Q(k) = \begin{bmatrix} q_{1k} & & Q \\ & \ddots & \\ 0 & & q_{mk} \end{bmatrix}. \tag{30}$$

Defining the observation residue, $r(k+1|k)$, and the true residue $r_v(k+1|k)$, as:

$$r(k+1|k) = y(t_{k+1}) - \bar{y}(t_{k+1}), \tag{31}$$

$$r(k+1|k) = r_v(k+1|k)+v_{k+1}, \tag{32}$$

where $\bar{y}(t_{k+1})$ is the observation vector calculated with the propagated state; $r_v(k+1|k)$ is the observation residue which would be obtained if the observations were not contaminated by noise.

Now, the parameters $q_{i,k}$, $i= 1,\ldots, m$ must be chosen so as to maximize the probability of occurrence of the true residue $r_v(k+1|k)$, that is, the criterion to calculate $q_{i,k}$, $i = 1,\ldots, m$ is stated by:

$$\max_{q\geqq 0} p\,[r_v(k+1|k)]. \tag{33}$$

It can be shown (Jazwinski, 1969; Rios Neto and Kuga, 1985) that this criterion leads to:

$$(r_v(k+1|k))_0^2 = E[r_v^2\ (k+1|k)], \tag{34}$$

where the subindex is to mean the occurred value of $r_v(k+1|k)$.

Developing both sides of Equation (34), accounting for Equation (32) and manipulating them properly, there results an expression called

Pseudoobservations Equation, which allows to calculate the parameters $q_{i,k}$, in the form:

$$y'(t_{k+1}) = H'_{(k+1)}\, q_k + v'_{k+1}, \tag{35}$$

where $y'(t_{k+1})$ is the r x 1 pseudoobservations vector, q_k is the m x 1 vector which contains the parameters $q_{i,k}$, to be estimated and v'_{k+1} is a r x 1 Gaussian white noise (see Appendix for the necessary steps to obtain this equation).

The solution for q_k can now be obtained through the use of a linear Kalman Filter.

Employing the technique described above, $P_u(k+1|k)$ is substituted for:

$$P_u^c\,(k+1|k) = P_u(k+1|k) + Q_u(k+\ \ |k) \tag{36}$$

in Equation (29), while $P(k+1|k)$ is generated by:

$$P(k+1|k) = \Phi(k+1,k)\; P(k|k)\; \Phi^T(k+1,\, k) + \bar{\gamma}(k)\; Q(k)\; \bar{\gamma}^T(k), \tag{37}$$

where

$$\bar{\gamma}\,(k) \triangleq \int_{t_k}^{t_{k+1}} \Phi(t_{k+1})\; \bar{G}(s)\; ds$$

Matrices $Q_u(k+1|k)$ and $Q(k)$ are adapted in each step.

3.2 - CONTROL MATRIX CORRECTION

Generally, when one is faced with the problem of state estimation of mechanical systems, it is usual to have the matrix $\bar{G}$ in:

$$\dot{x} = f(x,t) + \bar{G}w(t) \tag{38}$$

as:

$$\overline{G} = \begin{bmatrix} 0(n-m)xm \\ \hdashline I\ mxm \end{bmatrix}, \tag{39}$$

where 0 and I are the null and the identity matrices, respectively.

This means that the coefficients in $\overline{G}$ just make the coupling between the state variables and noises.

In the "virtual control" context it is possible to define the control matrix G in Equation (13) in order to augment the coupling between estimated controls and state variables and thus accelerate the procedure convergence. In this paper, three types of corrections for the control matrix G are proposed. In what follows, a forced variable is defined as a state variable which is directly excited by noise in Equation (38). The other state variables are called nonforced variables.

3.2.1 - DIRECT CORRECTION

The simplest form to define a control matrix with the characteristic described before is to consider:

$$G = \begin{bmatrix} C_p\ I_{n-m} \\ \hdashline C_v\ I_m \end{bmatrix}, \tag{40}$$

where C_p and C_v are positive coefficients, held fixed during the time propagation.

Despite the advantage given by the simple form, this correction technique has the drawbacks of using a trial and error method to choose C_p, C_v and of adding the corrections to the state variables with the same sign.

3.2.2 - RESTORATION

The idea of restoration was first used in numerical gradient-like methods to solve optimal control problems, where one has to deal with different requirements of searching a gradient step towards the extremal and satisfying the terminal constraints (Miele, 1975).

In the present case, restoration is used to modify C_p and C_v in Expression (40), thus giving priority of convergence to either the forced state variables or the nonforced state variables, depending on the system behavior. In order to make a decision to change C_p and C_v a convergence measure must be available. Estimated errors, e_p for the nonforced variables and e_v for the forced ones, and defined by:

$$e_p(k) = \frac{1}{(n-m)\sigma_r} \left[\sum_{i=1}^{n-m} P_{ii}(k|k) \right]^{1/2} , \tag{41}$$

$$e_v(k) = \frac{1}{m\sigma_v} \left[\sum_{i=n-m+1}^{m} P_{ii}\ (k|k) \right]^{1/2} , \tag{42}$$

can be used to indicate convergence. In Equations (41) and (42), P_{ii} are the elements on the diagonal of the $P(k|k)$ matrix and σ_r, σ_v represent the expected standard deviation of the errors between estimated and true variables after convergence of the problem. With these definitions, e_p, $e_v \cong 1$ mean convergence to the true trajectory. The technique can now be implemented through the following may: the procedure is initialized with the Direct Correction; after $\bar{n}$ steps, a test is done. If $e_p > 2$ and $e_v > 2$, C_p and C_v are not changed, if $0 < e_p < 2$ and $e_v > 2$, only C_v is changed; if $0 < e_v < 2$ and $e_p > 2$, only C_p is changed and finally, if $0 < e_p < 2$ and $0 < e_v < 2$, both coefficients are changed. Propagation is started again with the new coefficients and after $\bar{n}$ steps a new test is done.

With this technique, besides the simple form, coefficients C_p and C_v are changed during the time interval of interest. However, the same disadvantages described for the Direct Correction are still present.

3.2.3 - AUTOMATIC CORRECTION

In the automatic correction technique, matrix G is generated at each step inside the algorithm, reinforcing the adaptive characteristic of the proposed procedure. Determination of the elements in G is done by solving, step by step, a deterministic linear optimal control problem. This control problem arises if one considers the estimation procedure as a controller that must drive the system from a given point in t_k to satisfy the observations in t_{k+1}, as mentioned before. Consider, for

(t_k, t_{k+1}), the propagation of the first order perturbation Δx:

$$\Delta x\,(t_{k+1}) = \Phi(k+1,\,k)\,\Delta x(t_k) + \gamma(k+1,\,k)\,u(t_k) \tag{43}$$

Assuming that the G matrix is constant in this interval:

$$\gamma(k+1,k) = \int_{t_k}^{t_{k+1}} \Phi(t_k,\,s)\,G(s)\,ds = \left[\int_{t_k}^{t_{k+1}} \Phi(t_k,s)\,ds\right] G_k \triangleq \bar{B}_k G_k \tag{44}$$

and using Equation (44) in (43):

$$\Delta x(t_{k+1}) = \Phi(k+1,\,k)\,\Delta x(t_k) + \bar{B}_k G_k\,u(t_k). \tag{45}$$

Consider now that all terms in the right-hand side of Equation (45) are known except the G_k matrix. In t_{k+1}, it is expected that $\Delta x(t_{k+1})$ is calculated to satisfy the observation residues $\Delta y(t_{k+1})$ and this corresponds to minimize a quadratic criterion given by:

$$J = [\Delta y(t_{k+1}) - H_{k+1}\,\Delta x\,(t_{k+1})]^T[\Delta y(t_{k+1}) - H_{k+1}\Delta x(t_{k+1})] \tag{46}$$

subject to the constraint given by Equation (45).

Defining G_k as:

$$G_k = \left[\begin{array}{c} G_1,\,k \\ \hline G_2,\,k \end{array}\right] = \begin{bmatrix} g_{\bar{n}} \;\; \cdots & 0 \\ 0 & \cdots\, g_{n-m,m} \\ g_{1},_{n-m+1} & 0 \\ 0 & \cdots\, g_{n,m} \end{bmatrix} \tag{47}$$

and making:

$$\frac{\partial J}{\partial G_k} = 0 \tag{48}$$

there results a linear system to calculate the nonnull elements of the G_k matrix in the form:

$$D_{k+1}\, g_k = \Delta_y(t_{k+1}) - H_{k+1}\, \Phi(k+1,\, k)\, \Delta x(t_k), \tag{49}$$

where g_k is the nx1 vector of elements g_{ij}, as in Equation (47).

It is important to point out that the D_{k+1} matrix in Equation (49) is (rxn) dimensional and, thus, the linear system shall be solved by a deterministic least squares method or by and equivalent recursive method, if $r \neq n$.

The automatic correction technique has the distinguished feature of providing coefficients for G_k adjusted at each step, with variable signs and modules. However, an one-step lag has to be imposed to G_k since it was admitted that terms of Equation (45) were already known. This means that matrix G_k, which will be used in $(t_k,\, t_{k+1})$ is calculated with the values obtained in $(t_{k+1},\, t_k)$. Another problem to mention is the fact that one can only use estimator propagated states in the technique and this represents only an approximation to the true states.

All the techniques to correct matrix G were applied to the orbit estimation problem. Results, as will be seen, were very satisfactory. As a final remark, it must be said that these techniques were developed for the general case and can be used with the proposed procedure in different problems (Fleury, 1985).

3.3 - OBSERVATIONS

In the test problem of orbit estimation of a low altitute satellite, observations were generated combining the true orbit data with the location of 3 fictitious symmetric topocentric tracking stations, to get at each 1 second range and range-rate data, contaminated by white Gaussian noise with standard deviations:

$$\sigma_\rho = 10.0 \text{ m}; \quad \sigma_{\dot{\rho}} = 0{,}1 \text{ m/s}.$$

4. RESULTS

Results for the two cases, where different initial conditions are taken for the proposed estimator, are shown in this section. The virtual control u(t) in Equation (13) is a 3x1 vector since it is possible to "control an orbit" using three independet forces, one for each axis. Parameters for analysis are true and estimated position errors, true and estimated velocity errors and range and range-rate normalized residues given, respectively by:

$$\Delta r(k) = \{\sum_{i=1}^{3} [x_i(k) - \hat{x}_i(k)]^2\}^{1/2} \qquad k = 1, 2, \ldots \tag{50}$$

$$\Delta \hat{r}(k) = \{\sum_{i=1}^{3} P_{ii}(k|k)\}^{1/2} \qquad k = 1, 2, \ldots \tag{51}$$

$$\Delta v(k) = \{\sum_{r=4}^{6} [x_i(k) - \hat{x}_i(k)]^2\}^{1/2} \qquad k = 1, 2, \ldots \tag{52}$$

$$\Delta \hat{v}(k) = \{\sum_{i=4}^{6} P_{ii}(k|k)\}^{1/2} \qquad k = 1, 2, \ldots \tag{53}$$

$$r_\rho(k) = \frac{1}{n_e\ \sigma_\rho} \{\sum_{i=1}^{n_e} [y_\rho(k) - \hat{y}_\rho(k)]_i\} \qquad k = 1, 2, \ldots \tag{54}$$

$$r_{\dot\rho}(k) = \frac{1}{n_e\ \sigma_{\dot\rho}} \{\sum_{i=1}^{n_e} [y_{\dot\rho}(k) - \hat{y}_{\dot\rho}(k)]_i\} \qquad k = 1, 2, \ldots \tag{55}$$

where $y_{\dot\rho}$, $y_{\dot\rho}$ are the actual (simulated) observations of range and range-rate respectively and $\hat{y}_\rho$, $\hat{y}_{\dot\rho}$ are the estimated observations of range and range-rate respectively. In the definitions (54) and (55) n_e means number of fictitious stations observing the satellite.

To guarantee convergence the true and estimated errors must converge to the same small values and the range and range-rate normalized residues must lie between the limits $\pm 3\sigma$.

4.1 - NORMAL CASE

In this case, the estimator is initialized with errors of 80m in position and 0,6 m/s in velocity. This corresponds to a situation where a good a priori estimate of the orbit is available as a result of preliminary orbit determination. Figures 1 and 2 show the results obtained with the use of the Direct Correction with coefficients $C_p = 4.0$, $C_v = 10.0$

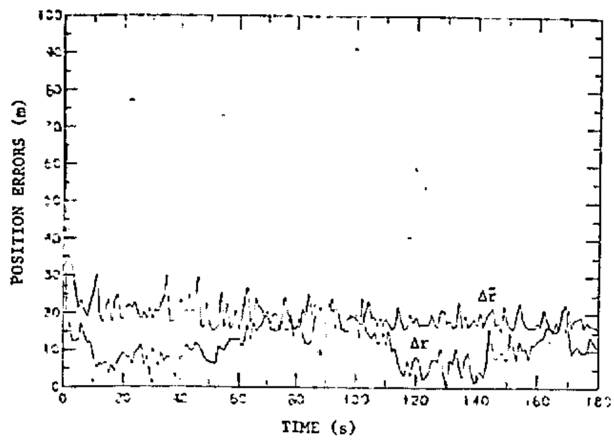


Fig. 1 - True and estimated position errors: Normal case, Direct correction.

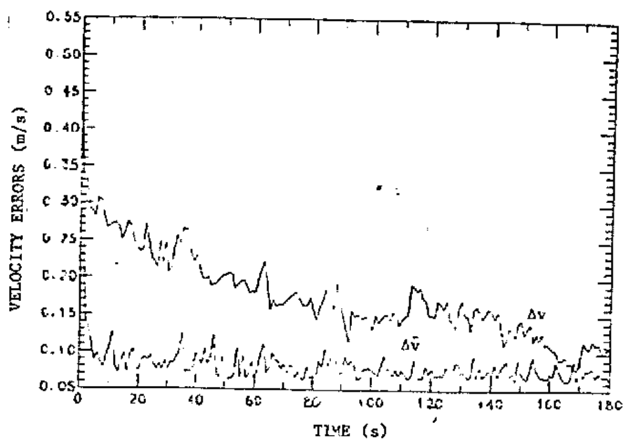


Fig. 2 - True and estimated velocity errors: Normal case, Direct correction.

Figures 3 and 4 show the application of the Restoration Correction to the same problem. Coefficients were chosen as one among the values 30.0, 1.0 and 0. for C_p; and as one among the values 3.0, 4.0 and 1.0 for C_v.

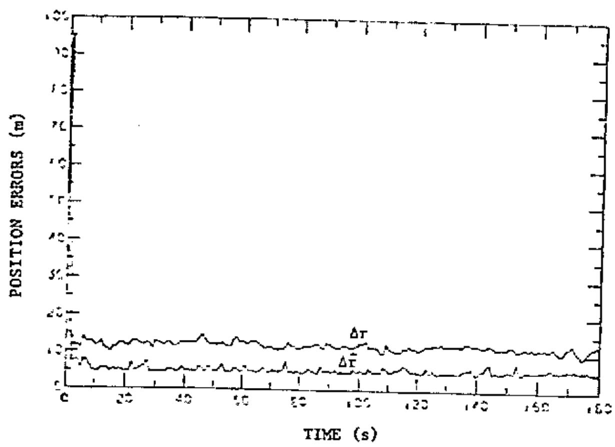


Fig. 3 - True and estimated position errors: Normal case, Restoration.

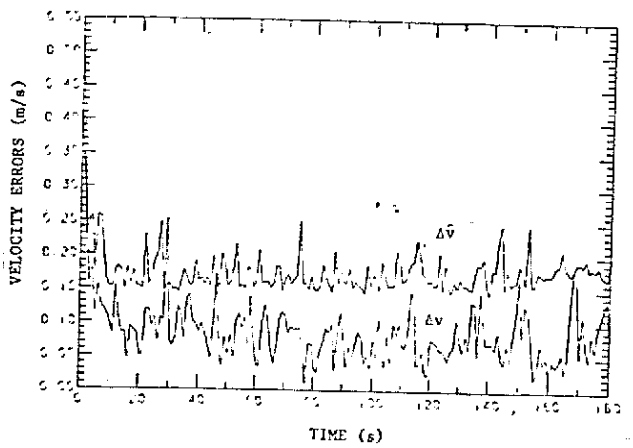
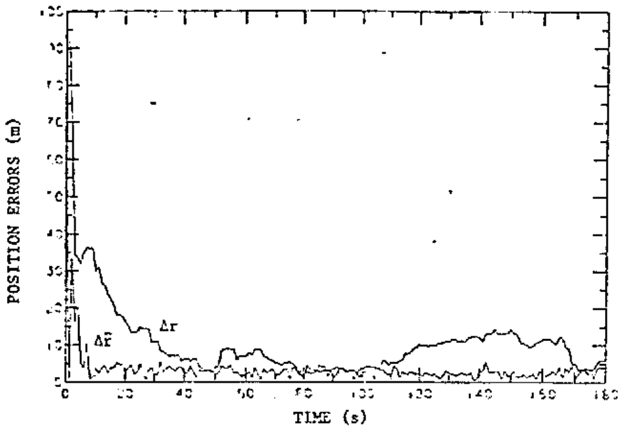


Fig. 4 - True and estimated velocity errors: Normal case, restoration.

Finally, Figures 5 and 6 present the results obtained with the use of the Automatic Correction.



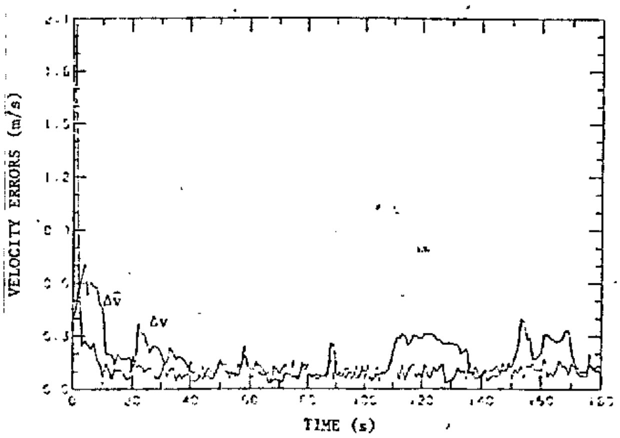


Fig. 5 - True and estimation position errors: Normal case, Automatic correction.

Fig. 6 - True and estimated velocity errors: Normal case, Automatic correction.

The analysis of Figures 2 to 7 shows that one can attain good results using any of the correction techniques, since convergence to values close to the true orbit occurs for the three methods. Best performance is achieved using Automatic Correction, as expected, although this is the most time consuming of the proposed techniques (Fleury, 1985). Another important feature which can be extracted comparing Figures 1 and 2 with Figures 3 and 4 is that an improvement in position convergence is obtained when Restoration is used instead of Direct Correction.

4.2 - CRITICAL CASE

In this case, the estimator is initialized with errors of 1000m in position and 10 m/s in velocity. This corresponds to a situation where the initial error are larger than the usual initial errors obtained in the preliminary orbit determination.

The results obtained with the Direct Correction technique are shown in Figures 7, 8, 9 and 10

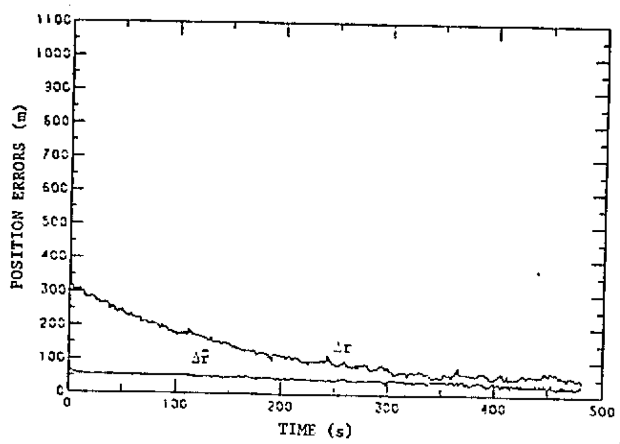


Fig. 7 - True and estimation position errors: Critical case, Direct correction.

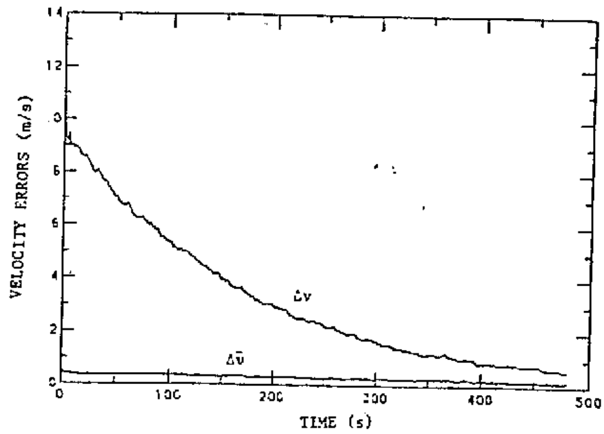


Fig. 8 - True and estimated velocity errors: Critical case, Direct correction.

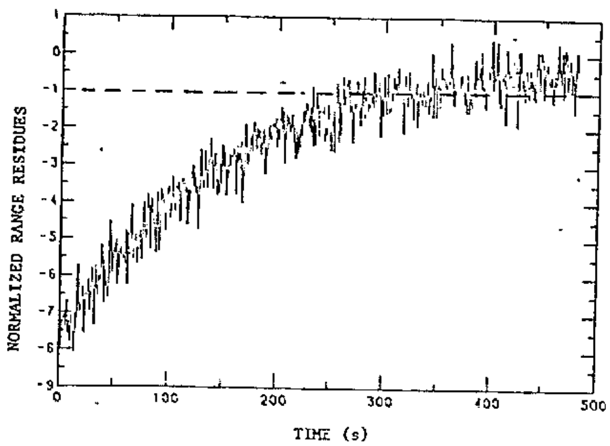


Fig. 9 - Range normalized residues: Critical case, Direct correction.

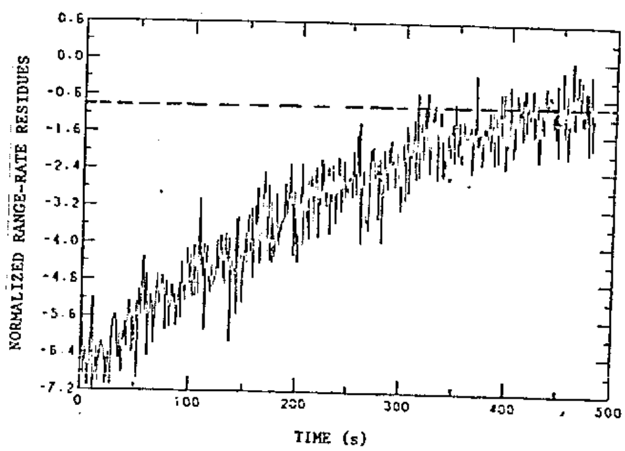


Fig. 10 - Range-rate normalized residues: critical case, Direct correction

Figures 11, 12, 13 and 14 present the results accomplished with the proposed procedure aided by the Automatic Correction technique.

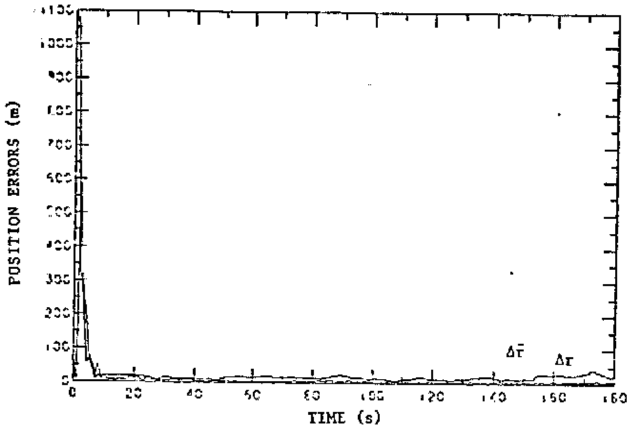


Fig. 11 - True and estimated position errors: Critical case, Automatic correction.

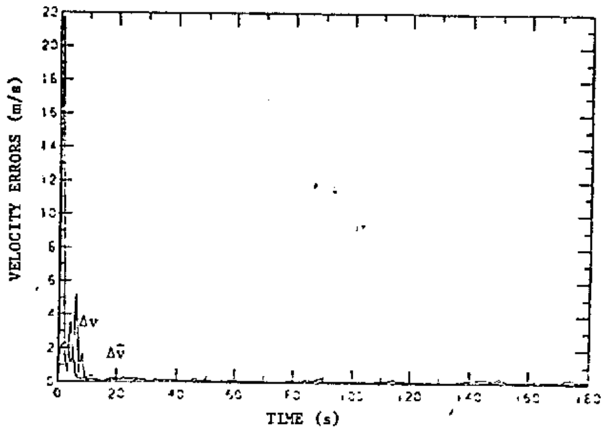


Fig. 12 - True and estimated velocity errors: Critical case, Automatic correction.

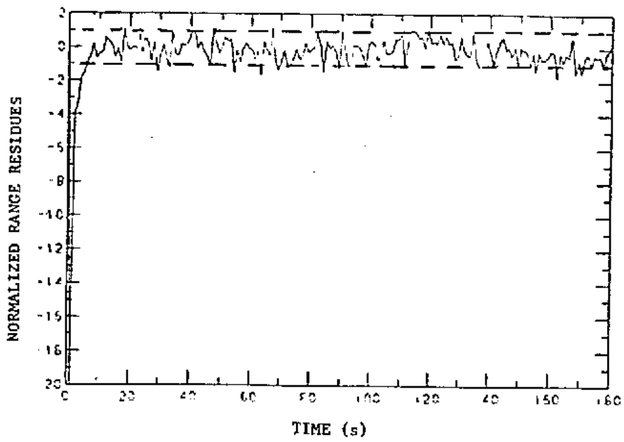


Fig. 13 - Range normalized residues: Critical case, Automatic correction.

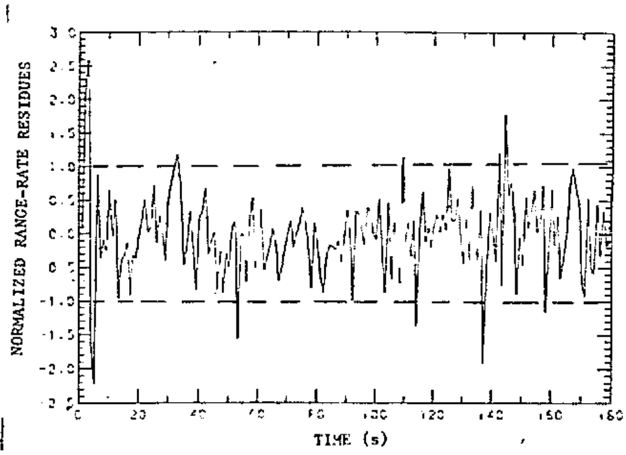


Fig. 14 - Range-rate normalized residues: Critical case, Automatic correction.

The analysis of Figures 7 to 14 shows very good results for the Automatic Correction and a satisfactory convergence with the use of the Direct Correction. In this last case, the 1000m and 10 m/s initial errors are reduced to approximately 30 m and 0,4 m/s. Although the final errors are reasonably small, the time interval for convergence is quite long.

Notice also that the estimated errors in the transient phase of convergence are very optimistic when compared to the real ones. These characteristics show that there are still further efforts to be done in this Direct Correction technique to reach a better performance.

On the other hand, results obtained with the Automatic Correction scheme are comparable to results presented by other techniques usually employed in nonlinear system estimation such as the Extended Kalman Filter aided by Model Compensation Technique (e.g. Kuga, 1982).Convergence is very fast and normalized residues are mantained between the limites of $\pm$ 1σ during all time propagation.

5. CONCLUSIONS

A new approach to state estimation of nonlinear systems has been presented, where the concept of duality was explored to transform the estimation problem into one of determining a virtual control. As indicated by the tests done in this exploratory phase, it is expected to be a valid alternative for the case where a great lack of knowledge in the dynamics exists, but a good level of information is locally provided by the observations.

In this work, one only started to explore the possibilites opened by the virtual nature of control in Equation (13). Presently, efforts are being done to further explore these possibilities, mostly in terms of the structure of the virtual control u(t) and of the structure of G(t), seeking for other forms of automatic and adaptive generation of this matrix. Besides this, one is also paying efforts in trying to tune the filter with constant levels of noise to avoid extra efforts imposed by adaptive estimation of these noises.

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APPENDIX - ADAPTIVE STATE NOISE ESTIMATION: PSEUDOOBSERVATION EQUATION

As seen in the text, to estimate the diagonal elements of $Q(k)$, one imposes consistency between the observation residues and their statistics (Jazwinski, 1969). This corresponds to impose that the diagonal elements of $Q(k)$ must assume values that maximize the probability of occurrence of the true residue of each observation (Rios Neto and Kuga, 1982). To understand the resulting procedure, particularly the pseudoobservation equation, the key steps are reproduced in this Appendix.

The observation residue $r(k+1|k)$ is defined as in Equation (31) by:

$$r(k+1|k) = y(t_{k+1}) - \bar{y}(t_{k+1}) = y(t_{k+1}) - H_{(k+1)}\, \bar{x}(t_{k+1}) \qquad \text{(A.1)}$$

and the true residue (Equation 32) by:

$$r_v(k+1|k) = r(k+1|k) - v_{k+1}. \qquad \text{(A.2)}$$

Under the hypothesis of having normal probability distributions the criterion of statistical consistence is realized by imposing:

$$(r_v(k+1|k))_0^2 = E\,[r_v^2(k+1|k)]. \qquad \text{(A.3)}$$

where the subindex $_0$ is to mean to occurred value of $r_v(k+1|k)$. Equation (A.3) is the condition to maximize the probability of occurence of the value $(r_v(k+1|k))_0$. Using Equation (A.2) and Equation (8) of Section 2 in both sides of Equation (A.3), there results after some algebraic manipulations:

$$(r^2(k+1|k) - 2\, r(k+1|k)\, v(t_{k+1}) + v^2(t_{k+1}))_0 =$$

$$= H_{k+1}\, \Phi(t_{k+1}, t_k)\, P(k|k)\, \Phi^T(t_{k+1}, t_k)\, H_{k+1}^T + H_{k+1} \bar{\gamma}_r Q_r \bar{\gamma}_r^T\, H_{k+1}^T. \qquad \text{(A.4)}$$

Define a noise v'_{k+1} as:

$$v'_{k+1} = -2(r(k+1|k))_c \; v(t_{k+1}) + v^2(t_{k+1}) - R_{k+1} \qquad (A.5)$$

Then

$$E[v'_{k+1}] = 0, \qquad (A.6)$$

$$E[(v'_{k+1})^2] = 4(r^2(k+1|k))_c \; R_{k+1} + 2R^2_{k+1}, \qquad (A.7)$$

where $(r(k+1|k))_c$ is the calculated value, corresponding to the occured value of the random variable $r(k+1|k)$.

With this definition and using the fact that $Q(k)$ is a diagonal matrix, it is reasonable to adopt, from Equation (A4), the Pseudoobservation equation in the form:

$$y'(t_{k+1}) = H'(k+1) \; q_k + v'_{k+1}, \qquad (A.8)$$

where

$$y'(t_{k+1}) = (r^2(k+1|k))_c + R(k+1) - H_{k+1} \; \Phi(t_{k+1}, t_k) \; P(k|k).$$

$$\Phi^T(t_{k+1}, t_k) \; H^T_{k+1}, \qquad (A.9)$$

$$H'(k+1) = [(\sum_{j=1}^{n} Hij.\bar{\gamma}j1)^2; \; (\sum_{j=1}^{n} Hij \; \bar{\gamma}j_2)^2; \; \ldots; (\sum_{j=1}^{n} Hij \; \bar{\gamma}jm)^2], \quad (A.10)$$

and $q_k = \{q_{k1}, \ldots, q_{km}\}$ is the m x 1 vector which contains the parameters to be estimated.

To estimate the vector q_k, the following algorithm is implemented:

- Prediction or propagation phase

$$\hat{q}(k+1|k) = \hat{q}(k|k), \qquad (A.11)$$

$$P^q(k+1|k) = P^q(k|k) + W^q(k|k), \qquad (A.12)$$

where $W^q(k|k)$ is a diagonal matrix of small constants which is added to $P^q(k|k)$ to guarantee that $P^q(k+1|k)$ is not smaller than the computer numerical zero.

- Filtering phase: follows the same steps of the linear Kalman Filter (e.g. Kuga, 1982; Maybeck, 1979).