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14. Abstract/Notes

Here we study the problem of state estimation and parameter identification of semilinear systems of the type $\dot{z} = Az + Nz$ ($A = A(\alpha)$ linear, $N(\alpha)$ nonlinear, $\alpha \in R^p$ being the p parameters to be identified) with an observation $y = Cz$ (C linear). It is assumed that the dynamic of the linear part (i.e., $\dot{z} = Az$) can be described in terms of a strongly continuous semigroup on an appropriate Banach space Z, so that this formulation includes distributed parameter systems and delay systems as well as lumped parameter systems. First of all we develop some theoretical results by using fixed pint theorems with some mappings $F:X \to X$, where X is a space of functions from the interval $[0,T]$ to Z (the possible trajectories of the system). However, these results are obtained by imposing some conditions on the space of output functions, which restrict the class of systems studied. Therefore we also consider an approach involving the pair $(z_0, z(.))$ consisting of the initial state and the trajectory in a space M which is the cross product Z x X between the state space Z and the space of trajectories X. We show that the solution of the problems of state estimation and parameter identification can be obtained via the fixed point of a variety of maps $F:M \to M$.

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STATE ESTIMATION AND PARAMETER IDENTIFICATION FOR SEMILINEAR DISTRIBUTED PARAMETER SYSTEMS

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Abstract

Here we study the problem of state estimation and parameter identification of semilinear systems of the type $\dot{z} = Az + Nz$ ($A = A(\alpha)$ linear, $N(\alpha)$ nonlinear, $\alpha \in \mathbb{R}^p$ being the p parameters to be identified) with an observation $y = Cz$ (C linear). It is assumed that the dynamics of the linear part (i.e., $\dot{z} = Az$) can be described in terms of a strongly continuous semigroup on an appropriate Banach space Z, so that this formulation includes distributed parameter systems and delay systems as well as lumped parameter systems. First of all we develop some theoretical results by using fixed point theorems with some mappings $F:X \rightarrow X$, where X is a space of functions from the interval $[0,T]$ to Z (the possible trajectories of the system). However, these results are obtained by imposing some conditions on the space of output functions, which restrict the class of systems studied. Therefore we also consider an approach involving the pair $(z_0, z(\cdot))$ consisting of the initial state and the trajectory in a space M which is the cross product $Z \times X$ between the state space Z and the space of trajectories X. We show that the solution of the problems of state estimation and parameter identification can be obtained via the fixed point of a variety of maps $F:M \rightarrow M$.

Keywords: Distributed parameter systems; nonlinear systems; observability, parameter identification; semigroup; state estimation.

1. INTRODUCTION

In this paper linear systems of the type

$$\dot{z} = Az + A_1\alpha, \quad z(0) = z_0 \tag{1}$$

(A and A_1 linear on Z) and semilinear systems of the type

$$\dot{z} = Az + A_1\alpha + N(z,\alpha), \quad z(0) = z_0 \tag{2}$$

(N nonlinear) are considered. It is assumed that A generates a strongly continuous semigroup S(t) on Z (the state space).

The observation process is described by

$$y = Cz \tag{3}$$

($C:Z \rightarrow Y$ linear).

The joint problem of state estimation and parameter identification is to construct the state $z(t)$, $t \in [0,T]$ and identify the parameter $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_p)$ $\mathbb{R}^p$ when the output observation $y(\cdot) \in \mathcal{Y}$ is given.

If we define $\bar{Z} = Z \times \mathbb{R}^p$ and $\bar{A}$, $\bar{z}$, $\bar{N}$ and $\bar{C}$ by

$$\bar{A} = \begin{bmatrix} A & A_1 \\ 0 & 0 \end{bmatrix}, \quad \bar{z} = \begin{bmatrix} z \\ \alpha \end{bmatrix},$$

$$\bar{N}\bar{z} = \begin{bmatrix} N(z,\alpha) \\ 0 \end{bmatrix} \text{ and } \bar{C} = [C \quad 0],$$

then (1) may be written in the form

$$\dot{\bar{z}} = \bar{A}\bar{z}, \quad \bar{z}(0) = \bar{z}_0, \tag{4}$$

(2) may be written as

$$\dot{\bar{z}} = \bar{A}\bar{z} + \bar{N}\bar{z}, \quad \bar{z}(0) = \bar{z}_0, \tag{5}$$

and the observation (3) as

$$y = \bar{C}\bar{z}. \tag{6}$$

So, the joint problem of state estimation and parameter identification for the linear problem (1), with state space Z and observation (3), is equivalent to just state estimation of system (4), with the enlarged state space $\bar{Z}$ and observation (6). Similarly, the joint problem of state estimation and parameter identification for the nonlinear problem (2), with state space Z and observation (3), is equivalent to just state estimation of system (5), with the enlarged state space $\bar{Z}$ and observation (6).

For the single problem of state estimation consider the system

$$\dot{z} = Az, \quad z(0) = z_0$$
$$y = Cz \tag{7}$$

for the linear case, and the system

$$\dot{z} = Az + Nz, \quad z(0) = z_0$$
$$y = Cz \tag{8}$$

for the nonlinear case.

2. EXAMPLES

As an example of a the joint problem of state and parameter estimation, consider the wave equation

$$\frac{\partial^2 w}{\partial t^2}(x,t) = \frac{\partial^2 w}{\partial x^2}(x,t) + \alpha w(x,t) \tag{9}$$

with boundary conditions

$$w(0,t) = 0, \quad w(1,t) = 1,$$

and initial conditions

$$w(x,0) = w_0(x), \quad w_t(x,0) = w_1(x).$$

The observation is given by

$$y(t) = \int_0^1 c(x)w(x,t)dx.$$

Assume the initial guess

$$\bar{\alpha} = 0$$

for the parameter α and

$$\bar{w}_0(x) = x, \quad \bar{w}_1(x) = 0$$

for the initial state $w_0(x)$, $w_1(x)$. Then, solving the above equations yield

$$\bar{w}(x,t) = x.$$

Setting $w = \bar{w} + w'$ and $x = \bar{\alpha} + \alpha'$ give

$$w'_{tt} = w'_{xx} + x\alpha' + \alpha' w',$$

$$w'(0,t) = 0, \quad w'(1,t) = 0,$$

$$y(t) = \int_0^1 xc(x)dx + \int_0^1 c(x)w'(x)dx.$$

The above system can be expressed in the form (6) by setting

$$\bar{z} = \begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \\ \bar{z}_3 \end{pmatrix} = \begin{pmatrix} w' \\ w'_t \\ \alpha' \end{pmatrix}, \quad \bar{A} = \begin{bmatrix} 0 & I & 0 \\ -A & 0 & x \\ 0 & 0 & 0 \end{bmatrix},$$

and

$$\overline{Nz} = \begin{bmatrix} 0 \\ \bar{z}_1\bar{z}_3 \\ 0 \end{bmatrix},$$

where $Aw = -w_{xx}$ with

$$\mathcal{D}(A) = H^2(0,1) \cap H_0^1(0,1).$$

Now let

$$\bar{y}(t) = y(t) - \int_0^1 xc(x)dx$$

and $\overline{Cz} = \langle c, \bar{z}_1 \rangle$. Then the observation $\bar{y}$ can be written in the form (6) (i.e., $\bar{y} = \overline{Cz}$).

As an example of the problem of state estimation, consider the nonlinear wave equation

$$\frac{\partial^2 w}{\partial t^2}(x,t) = \frac{\partial^2 w}{\partial x^2}(x,t) + N'(w,w_t) \tag{10}$$

with the same boundary and initial conditions of the previous example, the observation given by

$$y(t) = \int_0^1 c(x)w_t(x,t)dx$$

and N' being a nonlinear operator wuch as $N'(w,w_t) = ww_t,\ w_x w_t,\ w_x w_t^2$ etc.

Setting A as in the previous example, this system can be expressed in the form (8) with

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} w \\ w_t \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix},$$

$$z = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}, \quad N = \begin{bmatrix} N' \\ 0 \end{bmatrix},$$

and

$$Cz = \int_0^1 c(x)w_t(x,t)dx = \langle c, z_2 \rangle.$$

If $\mathcal{D}(A) = \mathcal{D}(A) \times \mathcal{D}(A^{1/2})$, then A generates a strongly continuous semigroup $S(t)$ on $\mathcal{D}(A^{1/2}) \times L^2(0,1)$ (Curtain and Pritchard, 1978) given by

$$S(t)\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{\infty} \left[\cos n\pi t \langle z_1, \phi_n \rangle + \frac{1}{n\pi} \sin n\pi t \langle z_2, \phi_n \rangle\right]\phi_n \\ \sum_{n=1}^{\infty} [-n\pi \sin n\pi t \langle z_1, \phi_n \rangle + \cos n\pi t \langle z_2, \phi_n \rangle]\phi_n \end{bmatrix},$$

where $\phi_n(x) = \sqrt{2} \sin n\pi x$, and the inner is in $L^2(0,1)$.

3. LINEAR OBSERVABILITY

Equation (7), for the problem of linear state estimation, is to be interpreted in the mild sense

$$z(t) = S(t)z_0, \quad z(0) = z_0, \qquad y(t) = CS(t)z_0, \tag{11}$$

for $t \in [0,T]$.

If it is assumed that the output function $y(\cdot)$ belongs to a reflexive Banach space $\mathcal{Y}$, and if the operator $H: Z \to \mathcal{Y}$ is defined by

$$Hz_0 = CS(\cdot)z \ ,$$

then y in (11) may be written

$$y = Hz_0. \tag{12}$$

In the sequel it will be assumed that Range (H) is closed in $\mathcal{Y}$. If this is not the case for a particular pair $(Z,\mathcal{Y})$, then it is necessary to enlarge the state space Z or to restrict the space of output functions $\mathcal{Y}$. Pritchard (1981) and Carmichael, Pritchard and Quinn (1982) showed that it is always possible to put a Banach space topology on Range (H) so that if $\mathcal{Y}$ = Range (H) clearly the above assumption is satisfied. Later Felippe De Souza (1983a; Chapter 5 of 1983c) showed that Z and/or $\mathcal{Y}$ can always be adjusted with Hilbert space topologies such that Range (H) is closed in $\mathcal{Y}$. Furthermore, Felippe De Souza (1983b) presents an iterative procedures for this adjustment to be done.

If Z and $\mathcal{Y}$ are Hilbert spaces, since Range (H) is closed, the generalized inverse $H^\dagger$ defined by

$$H^\dagger z_0 = (H/_{[\text{kernel}(H)]^\perp})z_0, \quad z_0 \in \text{Range (H)}$$
$$= 0, \quad z_0 \in [\text{Range H})]^\perp$$

is bounded (or continuous). For a given output observation $y \in \mathcal{Y}$, the state

$$z_0 = H^\dagger y$$

is the state of minimum norm, that is

$$\| z_0 \|_Z$$

is minimum. Moreover, this state z_0 also minimizes the functional

$$\| y - Hz_0 \|_{\mathcal{Y}}$$

over all $z_0 \in Z$ which satisfy (12).

4. FIXED POINT THEOREMS

Fixed point theorems have been used since the beginning of the century to show local existence theorems for differential equations. Recently, several papers (Carmichael, Pritchard and Quinn, 1981, 1982; Carmichael and Quinn, 1982; Pritchard, 1981, 1982; Felippe De Souza, 1983c, 1984a, 1984b, 1984c) have been published using fixed point theorems to provide existence of solutions of the problem of nonlinear observability. These papers usually provide mappings ϕ or F defined on some space X of functions from [0,T] to Z (e.g., $X = C(0,T;Z)$ or $X = L^p(0,T;Z)$ for some $p \geq 1$, etc.), and the solutions of the problems of nonlinear controllability and state estimation are obtained via the fixed points of such mappings.

So, the existence of a solution for the problem of nonlinear controllability or state estimation is transformed into the existence of fixed points of these mappings.

Types of Fixed Point Theorems

The papers referred to above used fixed point theorems of contractive type, topological type or contractive type with perturbations.

A classical example of a contractive type fixed point theorem is the Banach contraction principle (Banach, 1922). Other fixed point theorems of this type may be found in Belluce and Kirk (1969) or Nashed and Wong (1969). For topological fixed point theorems, we refer to Brouwer's fixed point theorems (Dunford and Schwartz, 1963) and Schauder fixed point theorem (Leray and Schauder, 1934). Finally we have fixed point theorems for mappings F of the type

$$F = F_1 + F_2,$$

where F_1 is a contractive type (such as a contraction) and F_2 is either compact or completely continuous. Among the numerous more recent papers which develop fixed point theorems of this type we mention Nussbaum (1969), Belluce and Kirk (1969) and Petryshyn (1973).

5. STATE ESTIMATION

First assume that the linearized system (7) is continuous initially observable. Then, the mapping $[CS(\cdot)]: Z \to \mathcal{Y}$ has a continuous inverse $[CS(\cdot)]^{-1}$ (Curtain and Pritchard, 1978). Now let X be the space of the trajectories (e.g., $X = C(0,T;Z)$ or $X = L^p(0,T;Z)$ for some $p \geq 1$). The mild form of system (8) can be expressed by

$$z(\cdot) = S(\cdot)z_0 + L(\cdot)Nz(\cdot), \quad z(0) = z_0, \tag{13}$$

where L(t), defined on $\bar{X} \supseteq X$ for each $t \in [0,T]$, is the linear operator defined as before. So, the observation y in (8) may be written as

$$y = CS(\cdot)z_0 + CL(\cdot)Nz(\cdot),$$

thus,

$$CS(\cdot)z_0 = (y - CL(\cdot)Nz(\cdot)),$$

hence,

$$z_0 = [CS(\cdot)]^{-1}(y - CL(\cdot)Nz(\cdot))$$
$$= H^{-1}(y - CL(\cdot)Nz(\cdot)),$$

and, therefore, if $z^*(\cdot)$ is the actual state of the system observed, we can write, by (13),

$$z^*(\cdot) = S(\cdot)H^{-1}(y - CL(\cdot)Nz^*(\cdot)) + CL(\cdot)Nz^*(\cdot).$$

Clearly, $z^*(\cdot)$ is a fixed point of the mapping $\phi: X \to X$ defined by

$$(\phi z(\cdot))(t) = S(t)H^{-1}(y - CL(\cdot)Nz(\cdot)) + L(t)Nz(\cdot). \qquad (14)$$

That is, $z^*(\cdot) = \phi(z^*(\cdot))$. The problem of state estimation of system (1) with the observation given by (2) is transformed into finding a fixed point for ϕ.

This approach was used in Carmichael, Pritchard and Quinn, 1981, 1982) and Pritchard (1981). In fact, Pritchard (1981) and Carmichael, Pritchard and Quinn (1982), using contractive type fixed point theorems, showed that the state of the hyperbolic system (10) with the nonlinearity $Nw = w^2$ can be estimated as long as the observation y lies in the ball of radius r in (r being dependent on the norms on X of $S(\cdot)z_1$, $z_1 \in Z$ and $Nz(\cdot)$, $z(\cdot) \in X$).

Carmichael and Quinn (1982) used ϕ in (14) in an attempt to construct an algorithm for solving the problem of state estimation. The assumption of continuously initial observability for (7) is a bit strong since it eliminates cases such as, for example, systems described by partial differential equations of parabolic type.

Pritchard (1982) and Felippe De Souza and Pritchard (1985) used mappings ϕ with $H^\dagger$ instead of $[CS(\cdot)]^{-1}$.

Unfortunately, however, the new ϕ with $H^\dagger$ does not have neither necessary nor sufficient condition for obtaining state estimation via its fixed points. An estimation of the state is not necessarily a fixed point of ϕ. A fixed point of ϕ will be the estimated state only if a check of consistency presented in Pritchard (1982) is satisfied.

A more sophisticated approach to solve the problem of state estimation of (8) was introduced in Felippe De Souza (1982). This new approach involved a pair $z = (z_0, z(\cdot))$ consisting of the initial state $z_0 \in Z$ and the trajectory $z(\cdot) \in X$. Instead of $\phi: X \to X$, the mapping used was $F: M \to M$, M here being the cross product between Z and X.

Let M be the normed space

$$M = Z \times X$$

with norm given by

$$\|(z_0, z(\cdot))\|_M = \|z_0\|_Z + \|z(\cdot)\|_X,$$

and define the operators $\mathcal{S}: Z \to M$ and $\mathcal{L}: X \to M$ by

$$\mathcal{S}z = (z_0, S(\cdot)z_0),$$
$$\mathcal{L}x(\cdot) = (0, L(\cdot)x(\cdot)).$$

Now, system (13) can be written in the compact form

$$z = \mathcal{S}z_0 + \mathcal{L}Nz(\cdot), \quad z(0) = z_0, \qquad (15)$$

and the output equation in (8) can be written as

$$y(\cdot) = Cz(\cdot). \qquad (16)$$

The mapping $F: M \to M$ used in Felippe De Souza (1982) was

$$F(z) = \gamma + (I-P)\mathcal{L}Nz(\cdot) + Pz - \mathcal{S}[CS(\cdot)]^\dagger Cz(\cdot), \qquad (17)$$

where $\gamma = \mathcal{S}[CS(\cdot)]^\dagger y \in M$ is a fixed element, I is the identity on M and P is any continuous projection onto Range $(\mathcal{S})$.

Felippe De Souza (1982, 1983c) showed that if $z^* = (z_0^*, z^*(\cdot))$ is a fixed point of F, then $z^*(t)$, $t \in [0,T]$ is the trajectory of the system and in particular $z^*(0) = z_0^*$. In other words, F was the first mapping with sufficient condition to obtain state estimation via its fixed points. This was possible because of the approach of the pair $(z_0, z(\cdot))$ consisting of the initial state and the trajectory.

Note that no condition on Range $(CS(\cdot))$ was imposed. Also, F in (17) is in fact a family of mappings, since P is any continuous projection onto Range $(\mathcal{S})$. Several examples of continuous projections onto Range $(\mathcal{S})$ have been shown in Chapter 4 of Felippe De Souza (1983c). For example, let $P: M \to M$ be

$$P(z_0, z(\cdot)) = (z_0, S(\cdot)z_0).$$

Clearly P is a continuous projection onto Range $(\mathcal{S})$. It is also shown that for a particular projection $P = \Pi$, where $\Pi = \Pi_1 + \Pi_2$ was defined in Felippe De Souza (1982), F in (15) becomes

$$F(z) = ((\phi z(\cdot)(0), \phi z(\cdot)) + \Pi_2(z - LNz(\cdot)),$$

where ϕ is as before. So the approach of ϕ was incorporated in the mappings F. Also $\Pi_2(z - LNz(\cdot))$ is the missing term in ϕ which gives sufficiency to F.

Later Felippe De Souza (1984a, 1984b, 1984c, 1985, 1986) developed mappings $F: M \to M$ which provide necessary and sufficient conditions for their fixed points to the estimated states.

The following theorems present two mappings $F: M \to M$ with considerable advantages over the mappings $\phi: X \to X$ in (14) and $F: M \to M$ in (17).

<u>Theorem 1:</u>

Let $\bar{x}$ be any fixed element of M which satisfies

$$\bar{x} \in \text{Range } (\mathcal{S}) \text{ and } \bar{x} \neq 0,$$

q be the functional on X given by

$$q(z(\cdot)) = \|y(\cdot) - Cz(\cdot)\|_Y$$

and $F:M \to M$ be the mapping

$$F(z) = Sz_0 + LNz(\cdot) + q(z(\cdot))\bar{x}$$

for $z = (z_0, z(\cdot)) \in M$. The pair $z^* = (z_0^*, z^*(\cdot)) \in M$ is a fixed point of F if and only if z^* satisfies both (15) and (16), that is, if and only if $z^*(0) = z_0^*$ (z_0^* is an estimation of the initial state) and $z^*(\cdot)$ is an estimation of the trajectory on $[0,T]$. □

Proof: Let $E = \text{Range }(S)$. Note that E is a closed subspace of M, and $P:M \to M$ given by

$$P(z_0, z(\cdot)) = Sz_0 \tag{18}$$

is a continuous projection onto E (i.e., $P^2 = P$ and Range $(P) = E$). Also note that

$$PLNz(\cdot) = 0. \tag{19}$$

If $z^* = (z_0^*, z^*(\cdot)) = F(z^*)$, then

$$(z^* - LNz^*(\cdot)) = Sz_0^* + q(z^*(\cdot))\bar{x}$$

and hence, since $\bar{x} \in E$, one has that $(z^* - LNz(\cdot)) \in E$. Thus, using (12),

$$P(z^* - LNz^*(\cdot)) = (z^* - LNz(\cdot)) = Sz_0^* \tag{20}$$

and therefore z^* satisfies (15). Moreover, by (20), since $\bar{x} \neq 0$, one has that $q(z^*(\cdot)) = 0$ which implies that $z^*(\cdot)$ satisfies (16). This proves necessity.

Now if $z^* = (z_0^*, z^*(\cdot))$ satisfies both (15) and (16), then one can easily verify that $z^* = F(z^*)$. So, sufficiency also holds. This concludes the proof. □

Theorem 2:

Let q be again the functional on X given by

$$q(z(\cdot)) = \|y(\cdot) - Cz(\cdot)\|_Y$$

and $F:M \to M$ be the mapping

$$F(z) = q(z(\cdot))Sz_0 + LNz(\cdot),$$

$$z = (z_0, z(\cdot)) \in M.$$

The pair $z^* = (z_0^*, z^*(\cdot)) \in M$ is a fixed point of F if and only if z^* satisfies both (15) and (16), that is, if and only if $z^*(0) = z_0^*$ (z_0^* is an estimation of the initial state) and $z^*(\cdot)$ is an estimation of the trajectory on $[0,T]$. □

Proof: Necessity: if $z^* = (z_0^*, z^*(\cdot)) = F(z^*)$, then

$$z^* - LNz^*(\cdot) = q(z^*(\cdot))Sz_0^*. \tag{21}$$

Thus, since $f(z^*(\cdot)) \in \mathbb{R}$,

$$(z^* - LNz^*(\cdot)) \in \text{Range }(S)$$

and, therefore, since P defined in (18) is a continuous projection onto Range (S),

$$P(z^* - LNz^*(\cdot)) = z^* - LNz^*(\cdot). \tag{22}$$

Clearly, $P(z^*) = Sz_0^*$ and $P(LNz^*(\cdot)) = 0$ and hence, since P is linear,

$$P(z^* - LNz^*(\cdot)) = Sz_0^*. \tag{23}$$

By (22) and (23),

$$z^* - LNz^*(\cdot) = z_0^* \tag{24}$$

and therefore (15) holds. By (21) and (24)

$$q(z^*(\cdot)) = 1$$

and therefore (16) also holds. This proves necessity.

Sufficiency: Immediate. If $z^* = (z_0^*, z^*(\cdot))$ satisfies both (15) and (16) one can easily verify that $z^* = F(z^*)$. This concludes the proof. □

Remark

If $z^* = (z_0^*, z^*(\cdot))$ is a fixed point of either the mapping F given in Theorem 1 or the mapping F given in Theorem 2, then

$$z^*(0) = z_0^*.$$

This shows that if we can find a fixed point $z^* = (z_0^*, z^*(\cdot))$ of F, then *$z^*(\cdot)$ is an estimation of the state and $z^*(0) = z_0^*$ is an estimation of the initial state.*

Furthermore, any pair $(z_0^*, z^*(\cdot) \in Z \times X = M$, which satisfies the dynamic equation (15), and the output equation (16) will be a fixed point of both F given in Theorem 1 and F given in Theorem 2.

So, similarly to the papers mentioned in the beginning of this section, the task of finding an estimation of the state $z(t)$, $t \in [0,T]$ is converted into finding a fixed point of a mapping.

6. THE JOINT STATE AND PARAMETER ESTIMATION

The mild version of (5) is

$$\bar{z}(t) = \bar{S}(t)\bar{z} + \int_0^t \bar{S}(t-\tau)\bar{N}\bar{z}(\tau)d\tau,$$

where $\bar{S}(\cdot)$ is the semigroup generated by $\bar{A}$. Felippe De Souza (1983c, 1984b) has shown that $\bar{A}$ generates the strongly continuous semigroup $\bar{S}(\cdot)$ on $\bar{Z}$ given by

$$\bar{S}(t)\bar{z}_0 = \begin{bmatrix} S(t) & L(t)A \\ 0 & I \end{bmatrix} \begin{bmatrix} z_0 \\ \alpha \end{bmatrix}.$$

So the joint problem of state estimation and parameter identification of (2) is transformed into states estimation of (5), and the techniques of the previous section will then have an immediate extension for problems of state and parameter estimation of systems of the type (2).

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