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14. Abstract/Notes

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A VARIATIONAL PRINCIPLE FOR THE PLASMA CENTRIFUGE

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Summary: A variational principle is derived which describes the stationary state of the plasma column in a plasma centrifuge. Starting with the fluid equations in a rotating frame the theory is developed using the method of irreversible thermodynamics. This formulation easily leads to an expression for the density distribution of the ℓ-species at sedimentation equilibrium, taking into account the effect of the electric and magnetic forces. Assuming stationary boundary conditions and rigid rotation nonequilibrium states the condition for thermodynamic stability integrated over the volume of the system reduces, under certain restrictions, to the principle of minimum entropy production in the stationary state. This principle yields a variational problem which is equivalent to the original problem posed by the stationary fluid equations. The variational method is useful in achieving approximate solutions that give the electric potential and current distributions in the rotating plasma column consistent with an assumed plasma density profile.

1 Introduction

Rotation in a fully ionized plasma column is induced by crossed electric and magnetic fields. This concept led to the development of vacuum-arc centrifuges [3,4,6,9] in which it was shown that isotopic enrichment well in excess of those measured earlier in gas discharge centrifuges [2,8,10] can be attained. The advantage of a fully ionized plasma centrifuge over their partially ionized counterpart is that the former is not limited to values of the rotational velocity below the *Alfvén* critical velocity [1]. In the device under consideration a highly ionized plasma consisting of the cathode material is created by an arc discharge and propagates into a vacuum, streaming along the magnetic field lines. As a result of ambipolar diffusion across the magnetic field lines a self-consistent electric field is established in the cylindrical plasma column. This inward directed radial electric field, crossed with the axial magnetic field, determines the rotation frequency of the plasma, to lowest order. The negative radial pressure gradient and the centrifugal radial force, crossed with the axial magnetic field, produce an azimuthal drift of the positive ions in the direction opposite to the electromagnetic drift. Accordingly, the azimuthal velocity of the ions is somewhat slower than the $\vec{E}\times\vec{B}/B^2$ value. On the other hand, the diamagnetic drift of the electrons adds to the electromagnetic drift. Thus a net azimuthal current flows in the direction opposite to the plasma rotation. The resulting inward directed Lorentz force balances the outward forces and confines the plasma column. An interesting situation takes place for sufficiently large values of the rotation velocity; the radial electric field in the rotating frame points outward and the negatively charged plasma column can be electrostatically confined. Due to collisions between ions an azimuthal frictional force arises which, crossed with the axial magnetic field, produces an inward directed radial drift of the ions with smaller m/Z value and an outward drift of the ions with larger m/Z. Finally, a situation is reached in which the column rotates as a rigid rotor and each ion species attains a condition of dynamical equilibrium between sedimentation, diffusion and electromagnetic forces. Slow radial diffusion of the plasma column, constrained by charge neutrality, proceeds as a result of electron-ion collisions.

The above physical account basically describes the centrifuge process in a fully ionized plasma column. Due to the complexities of the processes at the cathode region, the analysis of the plasma centrifuge is usually restricted to the quasi-equilibrium steady-state in which the plasma column is uncoupled from the cathode. Two problems must be addressed in this analysis: the density distribution of the various ion species in sedimentation equilibrium, that is, the elementary separation factor that can be achieved in the plasma column; and the profiles of the various plasma parameters, particularly of the self-consistent radial electric field. In this work an analytical model for the quasi-equilibrium steady-state of the plasma centrifuge is presented using the method of irreversible thermodynamics.

2 Analytical model

In a reference frame rotating with the constant angular velocity Ω the one-fluid equations that describe a plasma with several components are

$$\partial(\rho_m c_\alpha)/\partial t + \nabla\cdot(\rho_m c_\alpha \vec{u}') = -\nabla\cdot\vec{\Gamma}_\alpha,$$

$$\rho_m(\partial\vec{u}'/\partial t+\vec{u}'\cdot\nabla\vec{u}') = -\nabla p-\nabla\cdot\overleftrightarrow{\pi}-2\rho_m\vec{\Omega}\times\vec{u}'+\rho_m\nabla(|\vec{\Omega}\times\vec{r}|^2/2)+\rho\vec{E}'+\vec{j}'\times(\vec{B}_0+\vec{B}),$$

$$\partial U/\partial t+\vec{u}'\cdot\nabla U+(U+p)\nabla\cdot\vec{u}' = -\nabla\cdot\vec{q}-\overleftrightarrow{\pi}:\nabla\vec{u}'+\vec{j}_c\cdot[\vec{E}'+\vec{u}'\times(\vec{B}_0+\vec{B})].$$

The species α is conveniently described by the concentration $c_\alpha = n_\alpha m_\alpha/\rho_m$ and the diffusion momentum $\vec{\Gamma}_\alpha = n_\alpha m_\alpha(\vec{u}'_\alpha-\vec{u}')$, which satisfy

$$\sum_\alpha c_\alpha = 1 \quad , \quad \sum_\alpha \vec{\Gamma}_\alpha = 0.$$

The mass conservation equations for each species add up to

$$\partial\rho_m/\partial t+\nabla\cdot(\rho_m\vec{u}') = 0,$$

where ρ_m is the total mass density. The primed quantities are measured in the rotating frame, so that $\vec{u}=\vec{u}'+\vec{\Omega}\times\vec{r}$ is the velocity of the center of mass in the inertial frame. The electric field transforms from one frame to the other according to

$$\vec{E} = \vec{E}'-(\vec{\Omega}\times\vec{r})\times(\vec{B}_0+\vec{B}),$$

where $\vec{B}_0$ denotes the external magnetic field and $\vec{B}$ the internal field. The *current density* $\vec{j}'$ *is given by the sum of the convection current density* $\rho\vec{u}'$ and the conduction current density $\vec{j}_c = \sum_\alpha(e_\alpha/m_\alpha)\vec{\Gamma}_\alpha$.

The local entropy density S satisfies the entropy balance equation

$$\partial S/\partial t+\nabla\cdot(S\vec{u}') = -\nabla\cdot\vec{J}_s+\sigma,$$

where $\vec{J}_s$ is the entropy flux, given in terms of the heat flux $\vec{q}$ and the thermodynamic potentials per unit mass of each species μ_α by

$$\vec{J}_s = T^{-1}(\vec{q}-\sum_\alpha\mu_\alpha\vec{\Gamma}_\alpha),$$

and σ is the rate of production of entropy, given in terms of the independent thermodynamic fluxes and forces for a compound system of ℓ components by

$$\sigma = \vec{q}\cdot\nabla T^{-1}+\sum_{\alpha=1}^{\ell-1}\vec{\Gamma}_\alpha\cdot(\vec{X}_\alpha-\vec{X}_\ell)-T^{-1}\overleftrightarrow{\pi}:\nabla\vec{u}'\geq 0.$$

The thermodynamic force $\vec{X}_\alpha$, associated with diffusion phenomena, is defined by

$$\vec{X}_\alpha = -\{\nabla(\mu_\alpha/T)-(e_\alpha/m_\alpha T)[\vec{E}'+\vec{u}'\times(\vec{B}_0+\vec{B})]\}.$$

The ions and electrons in a plasma comprise ideal mixtures, for which the following thermodynamic relation is valid

$$T\nabla(\mu_\alpha/T) = (kT/m_\alpha)[n_\alpha^{-1}\nabla n_\alpha+p^{-1}\nabla p+(5T/2)\nabla T^{-1}].$$

The pressure p and the temperature T satisfy the ideal gas law $p = nkT$, where $n = \sum_\alpha n_\alpha$ is the particle density, and $U = 3p/2$ is the internal energy density.

A basic inequality in the present formulation is the condition for intrinsic thermodynamic stability of a fluid mixture which may be written as [1;]

$$\frac{\partial}{\partial t}\left(\frac{1}{T}\right)\frac{\partial U}{\partial t} - \frac{\rho_m}{T}\left(\frac{\partial \vec{u}'}{\partial t}\right)^2 - \sum_\alpha \frac{\partial}{\partial t}\left(\frac{\mu_\alpha}{T}\right)\frac{\partial}{\partial t}(\rho_m c_\alpha) \leq 0.$$

Assume that the plasma column has reached a state of mechanical equilibrium with small velocity gradients. In this state the column rotates as a rigid body and not only the acceleration vanishes but also the viscous stress tensor π may be neglected. With the rigid rotor assumption ($\dot{\vec{u}}' = 0$, $\nabla\vec{u}' = 0$) the fluid equations in the rotating frame become

$$\rho_m \partial c_\alpha/\partial t = -\nabla\cdot\vec{\Gamma}_\alpha \qquad (d\rho_m/dt = 0),$$
$$0 = -\nabla p + \rho_m \nabla(|\vec{\Omega}\times\vec{r}|^2/2) + \rho\vec{E}' + \vec{J}_c\times(\vec{B}_0+\vec{B}),$$
$$\partial U/\partial t = -\nabla\cdot\vec{q} + \vec{J}_c\cdot\vec{E}',$$

and the rate of production of entropy is given by

$$\sigma = \vec{q}\cdot\nabla T^{-1} + \sum_{\alpha=1}^{\ell-1} \vec{\Gamma}_\alpha\cdot(\vec{X}_\alpha - \vec{X}_\ell) \geq 0,$$

where $\vec{X}_\alpha = -[\nabla(\mu_\alpha/T) - e_\alpha\vec{E}'/m_\alpha T]$.

Using standard procedures [5], it can be shown that at mechanical equilibrium

$$T\sigma = -\vec{J}_s\cdot\nabla T + T\sum_{\alpha=1}^{\ell-1} \vec{\Gamma}_{\alpha,0}\cdot\vec{X}_{\alpha,0},$$

with the independent thermodynamic forces given by (for $\alpha = 1, 2, \ldots, \ell-1$)

$$\vec{X}_{\alpha,0} = -\frac{1}{T}\sum_{\beta=1}^{\ell-1}\left(\delta_{\alpha\beta} + \frac{a_\alpha c_\beta}{a_\ell c_\alpha}\right)\left[(\nabla\mu_\beta)_p + \frac{S}{\rho_m}\nabla T - \left(1 - \frac{\rho_m}{nm_\beta}\right)\left(\Omega^2\vec{r}_\perp + \frac{\vec{J}_c\times(\vec{B}_0+\vec{B})}{\rho_m}\right) - \left(1 - \frac{\rho}{ne_\beta}\right)\frac{e_\beta}{m_\beta}\vec{E}'\right].$$

The diffusion momentum $\vec{\Gamma}_{\alpha,0} = n_\alpha m_\alpha(\vec{u}'_\alpha - \vec{u}'_0)$ is defined with respect to an arbitrary reference velocity given by $\vec{u}'_0 = \sum_\alpha a_\alpha \vec{u}'_\alpha$, where a_1, a_2, ... a_ℓ are the normalized weights ($\sum_\alpha a_\alpha = 1$). On a long enough time scale one can consider the situation of a fully ionized plasma mixture in which the ion components are in sedimentation equilibrium but not the electrons (the subscripts $\alpha = 1, 2, \ldots \ell-1$ indicate ion components and the subscript $\alpha = \ell$ indicates the electron gas). For an ion mixture in isothermal and sedimentation equilibrium the vanishing of the thermodynamic forces that act on the ions gives an explicit expression for $(\nabla\mu_\alpha)_{p,T}$ which can be substituted into the relation $(\nabla\mu_\alpha)_{p,T} = (kT/m_\alpha)n_\alpha^{-1}\nabla n_\alpha$. One then obtains an expression that relates the densities of two ion species at each position

$$\nabla\left(\ln\frac{n_\alpha}{n_\beta}\right) = \frac{1}{kT}\left[(m_\alpha - m_\beta)\left(\Omega^2\vec{r}_\perp + \frac{\vec{j}_c\times(\vec{B}_0+\vec{B})}{\rho_m}\right) + (e_\alpha - e_\beta)\vec{E}'\right].$$

From this expression the separation factor can be calculated if the other relevant plasma parameters are known. It must be pointed out that for a plasma (ions and electrons) in thermodynamic equilibrium the conduction current strictly vanishes. However, due to the smallness of the mass ratio m_e/m_i, one may assume that only the ion species are in sedimentation equilibrium on the time scale under consideration.

Using the conservation laws for diffusing components in a rotating frame the condition for intrinsic stability may be rearranged in the form

$$\nabla\cdot\left[-\vec{q}\frac{\partial}{\partial t}\left(\frac{1}{T}\right) + \rho_m\sum_\alpha c_\alpha\vec{u}'_\alpha\frac{\partial}{\partial t}\left(\frac{\mu_\alpha}{T}\right)\right] + \vec{q}\cdot\frac{\partial}{\partial t}\left(\nabla\frac{1}{T}\right) + \sum_{\alpha=1}^{\ell-1}\vec{\Gamma}_\alpha\cdot\frac{\partial}{\partial t}(\vec{X}_\alpha - \vec{X}_\ell) - \frac{\vec{j}_c}{T}\cdot\frac{\partial\vec{E}'}{\partial t} \leq 0.$$

Assuming quasi-stationary conditions for the electromagnetic field the last term in the above equation may be neglected. The electromagnetic field variables are self-consistently determined by Maxwell's equations neglecting the displacement current and introducing the electrostatic potential ϕ' so that $\vec{E}' = -\nabla\phi'$. Integrating the stability condition over the volume of the system and assuming stationary boundary conditions at the surface, one finds that

$$\int_V\left[\vec{q}\cdot\frac{\partial}{\partial t}\left(\nabla\frac{1}{T}\right) + \sum_{\alpha=1}^{\ell-1}\vec{\Gamma}_\alpha\cdot\frac{\partial}{\partial t}(\vec{X}_\alpha - \vec{X}_\ell)\right]d^3r \leq 0,$$

which has the general form

$$\int_V\left(\sum_i\vec{J}_i\cdot\partial\vec{X}_i/\partial t\right)d^3r \leq 0,$$

whereas the entropy production rate is given by

$\sigma = \sum_i \vec{J}_i \cdot \vec{X}_i \geq 0.$

The total entropy production P in the system is the volume integral of σ. Taking the differential with respect to time

$$\frac{\partial P}{\partial t} = \int_V \sum_i \vec{J}_i \cdot \frac{\partial \vec{X}_i}{\partial t} d^3r + \int_V \sum_i \frac{\partial \vec{J}_i}{\partial t} \cdot \vec{X}_i d^3r.$$

Hence, as long as the change of the entropy production is due to the rate of change of the thermodynamic forces, it will be negative. If the following conditions are satisfied: (1) linear phenomenological relations; (2) constant phenomenological transport coefficients; (3) Onsager reciprocal relations (valid for a weakly magnetized plasma), then it can be easily shown that [5]

$$\partial P/\partial t = 2\int_V (\sum_i \vec{J}_i \cdot \partial \vec{X}_i/\partial t) d^3r \leq 0.$$

The inequalities $P \geq 0$ and $\partial P/\partial t \leq 0$ imply the diminishing of the total entropy production until the stationary state is reached (Prigogine's principle of minimum entropy production). Therefore, the rigid-rotor equilibria for a weakly magnetized plasma with constant transport coefficients can be determined by the solution of the variational problem $\delta P = 0$ with fixed boundary conditions. Conversely, it can be shown that the variational principle $\delta P = 0$ leads to the solution of the fluid equations in the stationary state [5].

Much simplification can be introduced in the description of the plasma equilibrium of this fully ionized plasma composed of several components if one considers the essentialy inertialess character of the electrons. In the limit $m_e \to 0$, the entropy production rate becomes

$$T\sigma = T(\vec{q} + 5kT\vec{j}_c/2e) \cdot \nabla T^{-1} + \vec{j}_c \cdot [\vec{E}' + (m_e/e)(\nabla \mu_e)_T] \geq 0,$$

where $\vec{j}_c = -e\vec{\Gamma}_e/m_e$. Using Spitzer's notation the constitutive relations for a weakly magnetized plasma are written

$$\vec{j}_c = \eta^{-1}[\vec{E}' + (m_e/e)(\nabla \mu_e)_T] + \alpha \nabla T,$$

$$\vec{q} = -\beta[\vec{E}' + (m_e/e)(\nabla \mu_e)_T] - \kappa \nabla T,$$

where η is the resistivity, κ the thermal conductivity, α the Seebeck coefficient and β, according to Onsager's relations, is given by $\beta = \alpha T + 5kT/2e\eta$.

In the relation $(\nabla\mu_e)_T = (\nabla\mu_e)_{p,T} + \nabla p/m_e n$, the gradient of the pressure p can be expressed in terms of the mechanical and electromagnetic forces by the condition of mechanical equilibrium, which, for a quasi-neutral plasma ($\rho \approx 0$), becomes

$$\nabla p = \rho_m \nabla(|\vec{\Omega}\times\vec{r}|^2/2) + \vec{j}_c \times (\vec{B}_0 + \vec{B}).$$

The explicit contribution of the Hall electric field $\vec{j}_c \times (\vec{B}_0 + \vec{B})/en$ in the expression of σ vanishes, that is, this term corresponds to a nondissipative transport effect. However, at mechanical equilibrium the influence of the Hall field in the entropy production rate is implicitly present in the ∇p term. Thus to determine $\vec{j}_c$ one has to solve the equation

$$\vec{j}_c - \vec{j}_c \times (\vec{B}_0 + \vec{B})/en\eta = \vec{E}^*/\eta + \alpha\nabla T,$$

where $\vec{E}^*$ is a generalized electric field defined by

$$\vec{E}^* = \vec{E}' + (\rho_m/en)\nabla(|\vec{\Omega}\times\vec{r}|^2/2) + (kT/e)n_e^{-1}\nabla n_e.$$

In the low-beta approximation one can neglect the magnetic induction generated by the plasma ($|\vec{B}| \ll |\vec{B}_0|$). Introducing the unit vector $\hat{b}_0 = \vec{B}_0/|\vec{B}_0|$ one obtains

$$\vec{j}_c = (\vec{E}^*/\eta + \alpha\nabla T)\cdot\hat{b}_0\hat{b}_0 + [1+(B_0/en\eta)^2]^{-1}(|\vec{B}_0|/en\eta)(\vec{E}^*/\eta + \alpha\nabla T)\times\hat{b}_0 -$$
$$- [1+(B_0/en\eta)^2]^{-1}[(\vec{E}^*/\eta + \alpha\nabla T)\times\hat{b}_0]\times\hat{b}_0.$$

Assuming that only electric conduction takes place but no temperature gradient exists the expression for σ becomes (with $\vec{E}^*_\parallel = \vec{E}^*\cdot\hat{b}_0$)

$$T\sigma = \vec{j}_c\cdot\vec{E}^* = |\vec{E}^*_\parallel|^2/\eta + [1+(B_0/en\eta)^2]^{-1}|\vec{E}^*_\perp|^2/\eta \geq 0.$$

In this case the variational principle assumes the form of Steenbeck's minimum principle [7].

$$\delta\int_V (|\vec{E}^*_\parallel|^2 + [1+(B_0/en\eta)^2]^{-1}|\vec{E}^*_\perp|^2)d^3r = 0.$$

The assumption of constant transport coefficient is justifiable in this case, because the resistivity is basically a function of the temperature T. The variational principle can be used to compute the electrostatic potential distribution in the plasma column consistent with an assumed particle density distribution. Application of this scheme to a cylindrical discharge is the purpose of the next section. To conclude this section consider the virial theorem in the inertial frame. Within the rigid-rotor approximation ($\vec{u} = \vec{\Omega}\times\vec{r}$)

and assuming negligible viscosity effects ($\overleftrightarrow{\pi} \simeq 0$) in a low-beta plasma ($\vec{\beta} \simeq 0$) this theorem can be written in the form

$$\int_V d^3r \langle \rho_m |\vec{\Omega}\times\vec{r}|^2 + 3p + \varepsilon_0 E^2/2 + \vec{r}\cdot(\vec{J}\times\vec{B}_0)\rangle = \oint_S d^2\vec{r}\cdot\langle\vec{r}(p+\varepsilon_0 E^2/2) - \vec{r}\cdot\varepsilon_0\vec{E}\vec{E}\rangle,$$

where the brackets indicate average values (steady-state values) over a large time interval. The virial theorem relates the values of the angular rotation frequency and the voltage drop in the plasma column. The voltage drop, or the electrostatic potential distribution for that matter, is computed using the variational principle, consistently with a prescribed density profile. Finally, the value of Ω and the calculated electrostatic potential profile can be used to compute the separation factors for each ion species.

3 Application to the plasma centrifuge

In this section the variational principle derived in the previous section is utilized to compute the profiles of the self-consistent radial electric field and azimuthal current density in the quasi-equilibrium steady-state of the plasma centrifuge. Only a brief outline of the method is presented with application to a simple case. Consider a cylindrical rotating plasma in a uniform magnetic field in the axial direction. All equilibrium quantities depend on r only. For this type of equilibrium, Steenbeck's minimum principle becomes

$$\delta\int_0^{r_0}\left[1+\left(\frac{B_0}{en\eta}\right)^2\right]^{-1}\left(-\frac{d\phi'}{dr}+\frac{\rho_m}{en}\Omega^2 r+\frac{kT}{en_e}\frac{dn_e}{dr}\right)^2 r\,dr = 0.$$

The boundary conditions on ϕ' are $\phi'(0) = 0$, $\phi'(r_0) = \phi_0' = \phi_0 - \Omega B_0 r_0^2/2$. Now assume a fully ionized plasma formed by electrons of density n_e and a dilute binary mixture of ions so that $n_2 \ll n_1$. Assuming, furthermore, that $m_2 \sim m_1$, one has (in the limit $m_e \to 0$) $\rho_m \simeq n_1 m_1$. By the condition of quasi-neutrality, $\rho = (n_1 Z_1 + n_2 Z_2 - n_e)e \simeq 0$, it follows that $n_e \simeq n_1 Z_1$ and $n = n_1 + n_2 + n_e \simeq n_1(1+Z_1)$. Substitution in the minimum principle gives

$$\delta\int_0^{r_0}\left[1+\left(\frac{Z_1 B_0}{(1+Z_1)en_e\eta}\right)^2\right]^{-1}\left[\frac{d}{dr}\left(-\phi'+\frac{m_1\Omega^2 r^2}{2(1+Z_1)e}+\frac{kT}{e}\ln\hat{n}_e\right)\right]^2 r\,dr = 0,$$

where $\hat{n}_e$ is the electron density normalized to some reference value. Now $Z_1B_0/(1+Z_1)en_e\eta \sim \Omega_e\tau_{ei}$, where $\Omega_e = eB_0/m_e$ and τ_{ei} is the electron-ion collision time. For typical parameters in the plasma centrifuge [6] this is a large number, that is, the electrons are strongly magnetized. This is inconsistent with the simplified form that was assumed for the constitutive relations and the present calculation is strictly valid only in the weakly magnetized plasma regime. However, since it is intented as an illustrative example of the method, the strongly magnetized plasma approximation will be adopted in the following. One expects the conclusions to be approximately valid if the proper value of the perpendicular resistivity is used. With the assumption $\Omega_e\tau_{ei}>>1$, one obtains (with $x = r/r_0$)

$$\delta\int_0^1 \hat{n}_e^2\left[\frac{d}{dx}\left(-\frac{e\phi'}{kT}+\frac{m_1\Omega^2r_0^2}{2(1+Z_1)kT}x^2+\ell n\hat{n}_e\right)\right]^2 x\,dx = 0.$$

According to the Rayleigh-Ritz method, the electrostatic potential can be approximated by the polynomial expansion

$$e\phi'/kT = (e\phi_0'/kT)\left[x^2+\sum_{i=1}^{n}c_ix^{i+1}(x-1)\right]$$

and the variational procedure leads to a set of linear equations for c_i ($i = 1, 2, \ldots, n$). If the electron density has a Gaussian profile such that

$$\hat{n}_e = \exp\left[-\left(\frac{m_1\Omega^2r_0^2}{2(1+Z_1)kT}-\frac{e\phi_0'}{kT}\right)\frac{r^2}{r_0^2}\right] = \exp\left(\frac{r^2}{a^2}\right)$$

the equations are homogeneous, $c_1 = c_2 = \ldots = c_n = 0$ and the electrostatic potential profile is exactly parabolic $\phi' = \phi_0'r^2/r_0^2$. In this case the conduction current density and the entropy production rate vanish and one has a situation of thermodynamic equilibrium with electrostatic confinement. In general, this is not the case and the electrostatic density profile is modified by the current. Anyhow, the calculated profiles can be substituted in the virial theorem leading to an equation which relates the angular rotation frequency and the plasma parameters. For a rotating cylindrical discharge

$$\int_0^{r_0}[\rho_m\Omega^2r^2+3p+\varepsilon_0E_r^2/2+\rho\Omega B_0r^2+(j_c)_\theta B_0r]rdr = r_0^2(p-\varepsilon_0E_r^2/2)_{r=r_0}.$$

For a very low-beta plasma one may neglect pressure effects. Hence

$$\Omega \approx \left[\int_0^{r_0}(-\rho)r^3dr\right]^{-1}\left[(r^2\varepsilon_0E_r^2/2B_0)_{r_0}+\int_0^{r_0}(\varepsilon_0E_r^2/2B_0)rdr+\int_0^{r_0}(j_c)_\theta r^2dr\right].$$

As an example, consider the simple case of electrostatic confinement ($j_c = 0$) with $E_r \approx -2\phi_0 r/r_0^2$ and $\rho \approx -4\varepsilon_0\phi_0/r_0^2$. It follows that

$$\Omega \approx \frac{\phi_0}{2r_0^2B_0}\left[1+r_0^4\left(\int_0^{r_0}r^3dr\right)^{-1}\right] = \frac{5\phi_0}{2r_0^2B_0}.$$

One verifies that electrostatic confinement is possible if $\Omega \approx 5\phi_0/2r_0^2B_0$ ($E_r' \approx -E_r/4$) and $a \approx 2r_0(e\phi_0/kT)^{-1/2}$. For this equilibrium the angular rotation frequency decreases for large values of the external induction. This simple case illustrates the usefulness of the variational method in achieving solutions to the problem of quasi-equilibrium in the plasma centrifuge. In general, one has to consider the effects of finite diamagnetic currents and plasma pressure. Furthermore, to describe correctly the plasma centrifuge problem the minimum principle has to be extended to include flow terms in the axial direction and the strongly magnetized plasma constitutive relations must be used. One expects to deal with this last aspect defining a local potential [11] appropriate to the centrifuge problem. Full application of the method will be presented elsewhere.

4 References

[1] Alfvén, H.: Rev. Mod. Phys. 32 (1960) 710-713

[2] Bonnevier, B.: Plasma Phys. 13 (1971) 763-774

[3] Del Bosco, E.; Dallaqua, R.S.; Ludwig, G.O.: Rev. Bras. Apl. Vac. 5 (1985) 160-167

[4] Del Bosco, E.; Dallaqua, R.S.; Ludwig, G.O.; Bittencourt, J.A.: Phys. Lett. (submitted)

[5] De Groot, S.R.; Mazur, P.: Non-Equilibrium Thermodynamics (1st. edn.) Amsterdam: North-Holland Publishing Co. 1962

[6] Geva, M.; Krishnan, M.; Hirshfield, J.L.: J. Appl. Phys. 56 (1984) 1398-1413

[7] Hoyaux, M.F.: Arc Physics. New York: Springer-Verlag 1968

[8] James, B.W.; Simpson, S.W.: Plasma Phys. 18 (1976) 289-300

[9] Krishnan, M.; Geva, M.; Hirshfield, J.L.: Phys. Rev. Lett. 46 (1981) 36-38

[10] Walsh, C.J.; Brand, G.F.; James, B.W.: Phys. Lett. 67A (1978) 33-34

[11] Woods, L.C.: The Thermodynamics of Fluid Systems. Oxford: Clarendon Press 1975

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