AIP | Physics of Fluids

The single-particle and collective descriptions of the anomalous Doppler resonance and the role of ion dynamics

R. O. Dendy, C. N. Lashmore-Davies, and A. Montes

Citation: Physics of Fluids **29**, 4040 (1986); doi: 10.1063/1.865746
View online: http://dx.doi.org/10.1063/1.865746
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof1/29/12?ver=pdfcov
Published by the AIP Publishing

The single-particle and collective descriptions of the anomalous Doppler resonance and the role of ion dynamics

R. O. Dendy, C. N. Lashmore-Davies, and A. Montes[a)]
Culham Laboratory (Euratom/UKAEA Fusion Association), Abingdon, Oxfordshire OX14 3DB, United Kingdom

(Received 29 January 1986; accepted 5 August 1986)

The connections between three aspects—single-particle, beam, and continuous velocity distribution—of the anomalous Doppler effect are investigated. The key quantity is the power $\mathrm{Re}(\mathbf{j}\cdot\mathbf{E}^*)$ dissipated by electrostatic waves interacting with a non-Maxwellian electron velocity distribution. Its spatial components describe energy flows that are the counterparts in classical electrodynamics to the quantum properties of the single-particle anomalous Doppler effect. A complete plasma physics treatment requires the inclusion of ion dynamics. Examination of the large-k region of wavenumber space—in contrast to the well-known small-k region—shows that ion Landau damping is important in stabilizing plasmas for which the electron velocity distribution considered alone is destabilizing. Finally, by choosing simple models for the superthermal electron velocity distribution, general features of the instability, and those specific to particular tail models, are identified analytically and numerically.

I. INTRODUCTION

The anomalous Doppler effect[1,2] is a fundamental relaxation mechanism believed to underlie certain observed instabilities of superthermal tails in the distribution of electron velocities parallel to the magnetic field in tokamak plasmas. It is a mechanism by which the kinetic energy of electron motion parallel to the magnetic field can be transformed simultaneously into the kinetic energy of electron gyration perpendicular to the magnetic field and into wave energy. The parallel phase velocity of the excited wave is less than the parallel velocity of the electrons that drive the instability. These characteristics make the anomalous Doppler effect a probable explanation[3,4] of the relaxation oscillations seen in the soft x-ray signal from Ohmic plasmas[5] and in association with lower hybrid current drive.[6,7] In this application, the divergence of fast-electron motion from the parallel direction, as the anomalous Doppler instability proceeds, is responsible for the shift in the predominant direction of electron bremsstrahlung, which follows the fast-electron motion.

The general characteristics of the anomalous Doppler effect, which we have just outlined, are widely known. However, despite many interesting studies of the instability, and particularly its quasilinear development, the theoretical description has remained incomplete. First, although concepts arising from a single-particle description of the phenomenon have been employed in contexts where a collective description is appropriate, the justification for this has not been clear. Here, we generalize Nezlin's work[2] on the single-particle and beam aspects of the anomalous Doppler effect to the case where the fast electrons have a continuous distribution of velocities. This case is of most relevance to tokamak plasmas. Consideration of the different spatial components of the field energy dissipation $\mathrm{Re}(\mathbf{j}\cdot\mathbf{E}^*)$ is fundamental to this generalization. It enables us to identify the flow of particle kinetic energy in the directions parallel and perpendicular to the magnetic field, and the energy flow to the excited wave. These energy flows can be related to the positive- and negative-energy wave aspects of the beam description, and to the single-particle description.

Second, the conditions that must be satisfied by the tail and plasma parameters if the anomalous Doppler instability is to occur, or be suppressed, require clarification. It has not always been clear what properties are intrinsic to the instability, and what properties are specific to the model chosen for the superthermal distribution from the wide range considered. In this area, some of the most general results have been given by Muschietti, Vaclavik, and Appert.[8] Here, we first extend these general formulas by including the previously neglected but significant role of ion dynamics. We then concentrate on two simple and general tail models that require very few parameters: first, a flat tail, which requires three parameters (fraction μ of electrons in the tail, maximum velocity v_M, and characteristic perpendicular velocity $v_{T\perp}$); and second, a simple monotonically decreasing tail, which requires four parameters (fraction μ of electrons in the tail, characteristic drift velocity v_D, parallel velocity spread $v_{T\parallel}$, and characteristic perpendicular velocity $v_{T\perp}$). Our analysis shows that in the first case, the linear instability is independent of $v_{T\perp}$. It follows that tails which can be represented in this manner cannot be linearly stabilized against the anomalous Doppler effect by increasing their perpendicular temperature, for example, by electron cyclotron resonance heating. Conversely, where such stabilization has been observed experimentally,[6] our result indicates that the tail is not a simple flat one with a Maxwellian distribution of perpendicular velocities that is characterized by a single parameter $v_{T\perp}$, independent of parallel velocity. For the second class of tail, despite the simplicity of its analytical representation, it is not possible to arrive at an exact criterion for instability. Nevertheless, we show that, in this case, the perpendicular tail temperature can act as a stabilizing influence. There are analytical arguments for concluding that this wide class of tail is in general stable against the anomalous

[a)] Permanent address: Instituto de Pesquisas Espaciais, C.P. 515, 12 200 S. José dos Campos, São Paolo, Brazil.

 0031-9171/86/124040-07$01.90

Doppler effect, and this conclusion is supported numerically.

II. GENERAL PROPERTIES OF THE ANOMALOUS DOPPLER RESONANCE

Let us turn first to the single-particle description of the anomalous Doppler effect. Consider a system with total relativistic energy E and internal energy U, traveling with velocity βc in a medium with refractive index n. When it emits a photon at an angle θ to its direction of motion, the energy $\Delta E = (h/2\pi)\omega > 0$ supplied to the photon is related to the internal energy change ΔU of the system by[1]

$$\Delta E = (1-\beta^2)^{1/2}\Delta U/(1-n\beta\cos\theta). \quad (1)$$

For motion with $n\beta < 1$, Eq. (1) gives the Doppler frequency shift with $\Delta U = (h/2\pi)\omega_0$, where ω_0 is the frequency in the rest frame of the system. For superluminal motion with $n\beta > 1$, a positive value for ΔE remains possible within the cone $|\theta| < \theta_0 \equiv \cos^{-1}(1/n\beta)$ provided that ΔU is negative. In the case of a superluminal electron in a magnetized medium, the negative ΔU corresponds to an increase in the energy of perpendicular gyration. This is the single-particle anomalous Doppler effect, in which the parallel kinetic energy lost by the electron exceeds that given to the photon (wave), and the balance is transferred to the electron gyromotion.

Consider next a cold beam of electrons in a magnetized plasma. The beam will support waves additional to the bulk plasma waves. These can be negative-energy waves, whose excitation involves a loss in parallel kinetic energy and a smaller gain in perpendicular kinetic energy. When such a wave resonates with a bulk plasma wave, both will grow. Nezlin[2] has pointed out the correspondence between this effect and the single-particle anomalous Doppler effect. There is a net flow from the parallel beam kinetic energy into the perpendicular component of collective motion associated with the beam wave, and into the bulk wave. The diagnostic potential of this process has also been noted.[9]

A collective description of the anomalous Doppler effect arising from the interaction of resonant electrons with electrostatic waves in a magnetized plasma follows from the dielectric response function.[10] We shall show below that ion dynamics play a significant role in wave damping under conditions where the wavenumber is sufficiently large that the ions can be regarded as unmagnetized. For this reason, we include the corresponding ion contribution in the expression for the full dielectric response function:

$$\epsilon = 1 - \frac{\omega_p^2}{k^2}\sum_{n-\infty}^{\infty}\int_{v_\parallel=-\infty}^{\infty}\int_{v_\perp=0}^{\infty}\frac{2\pi v_\perp\,dv_\perp\,dv_\parallel}{n\Omega + k_\parallel v_\parallel - \omega}\times\left(\frac{n\Omega}{v_\perp}\frac{\partial f}{\partial v_\perp} + k_\parallel\frac{\partial f}{\partial v_\parallel}\right)J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right) - \frac{\omega_{pi}^2}{k}\int_{-\infty}^{\infty}\frac{dv_i}{kv_i-\omega}\frac{df_i}{dv_i}. \quad (2)$$

Here $v_\perp$ and $v_\parallel$ characterize the electron velocity perpendicular and parallel to the magnetic field direction, $f(v_\perp, v_\parallel)$ is the electron velocity distribution function, J_n denotes the Bessel function of order n, and ω_p and Ω are, respectively, the electron plasma and cyclotron frequencies; $k_\perp$ and $k_\parallel$ are the components of the wave vector $\mathbf{k}$ perpendicular and parallel to the magnetic field direction, $f_i(v_i)$ is the ion velocity distribution function, and ω_{pi} is the ion plasma frequency. For ions of charge Z in a neutral plasma, $\omega_{pi}^2 = Z(m_e/m_i)\omega_p^2$, where m_e and m_i are the electron and ion masses. The real part $\epsilon_{\rm real}$ of ϵ is in general dominated by cold electron plasma terms independent of the exact form of $f(v_\perp, v_\parallel)$. For $(\omega_p/\Omega)^2 \lesssim 0.2$, the electrostatic waves that are roots of $\epsilon_{\rm real} = 0$ are well described by $\omega = \omega_p k_\parallel/k$ in the frequency range of interest. The imaginary part $\epsilon_{\rm im}$ of ϵ describes wave-particle resonance, which leads to electrostatic wave growth or damping at a rate given by

$$\gamma = -(\omega_p k_\parallel/k)(\epsilon_{\rm im}/2). \quad (3)$$

By Eq. (2), a negative contribution from the electrons to $\epsilon_{\rm im}$ can arise from the $n = -1$ term at the parallel resonant velocity $v_{\rm AD} \equiv (\omega + \Omega)/k_\parallel$. This is the anomalous Doppler resonance.

For a wave of given $(\omega, k_\perp, k_\parallel)$, Eq. (3) leads to wave growth if the Landau damping terms associated with $\partial f/\partial v_\parallel$ at $v_\parallel = \omega/k_\parallel$ and df_i/dv_i at $v_i = \omega/k$ in Eq. (2) are so weak that they can be overcome by the anomalous Doppler resonant term involving $\partial f/\partial v_\perp$ at $v_\parallel = v_{\rm AD}$. The latter is proportional to the magnitude, rather than the parallel gradient, of the parallel component of the electron distribution function at the superthermal parallel velocity $v_\parallel = v_{\rm AD}$. Let v_B denote the thermal velocity associated with the isotropic bulk Maxwellian distribution. Electron Landau damping is weak first for $v_\parallel \gtrsim 2.5\, v_B$, beyond the main body of the bulk, and also for $v_\parallel \ll v_B$, deep in the bulk distribution. The first possibility for wave growth has been examined in the literature[3,4,8,11–16]; the second does not appear to have been studied previously.

Let us consider an isotropic bulk Maxwellian electron distribution, together with a small tail whose distribution in $v_\parallel$ is not yet specified, but which has a fixed perpendicular thermal velocity $v_{T\perp}$:

$$f(v_\perp, v_\parallel) = \frac{(1-\mu)}{\pi^{3/2}v_B^3}e^{-v_\perp^2/v_B^2}e^{-v_\parallel^2/v_B^2} + \frac{\mu}{\pi v_{T\perp}^2}e^{-v_\perp^2/v_{T\perp}^2}F(v_\parallel). \quad (4)$$

In general, $\mu \ll 1$. We also specify a thermal ion distribution

$$f_i(v_i) = (1/\pi^{1/2}v_{Ti})e^{-v_i^2/v_{Ti}^2}, \quad (5)$$

where v_{Ti} is the ion thermal velocity. We shall assume the bulk electron and ion temperatures to be equal, so that $v_{Ti} = (m_e/m_i)^{1/2}v_B$.

Substituting Eqs. (4) and (5) in Eqs. (2) and (3), we obtain

$$\frac{\gamma}{\omega_p} = \frac{\gamma_{\rm AD}}{\omega_p} - \left(\frac{\gamma_{\rm LB}}{\omega_p} + \frac{\gamma_{\rm LT}}{\omega_p} + \frac{\gamma_{\rm LI}}{\omega_p}\right), \quad (6)$$

$$\frac{\gamma_{\rm AD}}{\omega_p} = \mu\pi^{1/2}\left(\frac{\omega_p}{kv_B}\right)^2\Lambda_1(\beta_T)\left[\frac{\Omega}{kv_B}\left(\frac{v_B}{v_{T\perp}}\right)^2\pi^{1/2}v_B F(v_\parallel) + \frac{k_\parallel}{k}\frac{\pi^{1/2}v_B^2}{2}\frac{dF}{dv_\parallel}\right]_{v_\parallel = v_{\rm AD}}, \quad (7)$$

$$\frac{\gamma_{\rm LB}}{\omega_p} = (1-\mu)\pi^{1/2}\left(\frac{\omega_p}{kv_B}\right)^3\Lambda_0(\beta_B)\frac{k_\parallel}{k}e^{-(\omega_p/kv_B)^2}, \quad (8)$$

$$\frac{\gamma_{\rm LT}}{\omega_p} = \mu\pi^{1/2}\left(\frac{\omega_p}{kv_B}\right)^2 \Lambda_0(\beta_T)\,\frac{k_\parallel}{k} \times\left(-\frac{\pi^{1/2}v_B^2}{2}\frac{dF}{dv_\parallel}\right)_{v_\parallel=\omega_p/k}, \tag{9}$$

$$\frac{\gamma_{\rm LI}}{\omega_p} = \pi^{1/2}\left(\frac{\omega_p}{kv_B}\right)^3 Z\left(\frac{m_i}{m_e}\right)^{1/2} \times (k_\parallel/k)^2\, e^{-[(\omega_p/kv_B)^2(m_i/m_e)(k_\parallel/k)^2]}. \tag{10}$$

Here $\gamma_{\rm AD}$ is the anomalous Doppler driving term, $\gamma_{\rm LB}$ and $\gamma_{\rm LT}$ describe electron Landau damping in the bulk and tail distributions, respectively, $\gamma_{\rm LI}$ describes damping on the ions, $\beta_B = k_\perp^2 v_B^2/2\Omega^2$, $\beta_T = (v_{T\perp}^2/v_B^2)\beta_B$, and $\Lambda_n(\beta) = e^{-\beta}I_n(\beta)$, where $I_n(\beta)$ is the modified Bessel function of order n. The representation in Eqs. (7)–(10) indicates those features of the instability that are universal, rather than specific to particular choices of parallel electron tail distribution $F(v_\parallel)$.

The dielectric response function ϵ in Eq. (2) is given in terms of the dielectric tensor ϵ_{ij} by $\epsilon = k_i k_j \epsilon_{ij}/k^2$. In the region of instability it will be shown that the ion contribution to ϵ_{ij} can be neglected. For the electrostatic waves excited by the anomalous Doppler instability, we have $E_i = |\mathbf{E}|k_i/k$. Let us take $\mathbf{k} = (k_\perp, 0, k_\parallel)$. In this case, the rates of energy dissipation by the electrostatic field on the electron motion perpendicular and parallel to the magnetic field are given by

$$P_x = {\rm Re}(j_x E_x^*) = \omega\frac{|\mathbf{E}|^2}{4\pi}\left(\frac{k_\perp^2}{k^2}\epsilon_{xx}^{\rm im} + \frac{k_\perp k_\parallel}{k^2}\epsilon_{xz}^{\rm im}\right), \tag{11}$$

$$P_z = {\rm Re}(j_z E_z^*) = \omega\frac{|\mathbf{E}|^2}{4\pi}\left(\frac{k_\perp k_\parallel}{k^2}\epsilon_{xz}^{\rm im} + \frac{k_\parallel^2}{k^2}\epsilon_{zz}^{\rm im}\right). \tag{12}$$

From the definition of ϵ_{ij} (see Ref. 17), Eqs. (11) and (12) give

$$P_x = (|\mathbf{E}|^2/4\pi)2\gamma_{\rm AD}\Omega/\omega, \tag{13}$$

$$P_z = (|\mathbf{E}|^2/4\pi)[2\gamma_{\rm LB} + 2\gamma_{\rm LT} - (1+\Omega/\omega)2\gamma_{\rm AD}], \tag{14}$$

where we have used Eqs. (7)–(10). Here $|\mathbf{E}|^2/4\pi$ is the electrostatic field energy density of the wave, and the factors of 2 arise from the quadratic dependence of power on field amplitude. Equation (13) describes the field energy dissipated by the wave in increasing the perpendicular kinetic energy of the electrons undergoing the anomalous Doppler resonance. The first two terms of Eq. (14) describe the field energy dissipated by Landau damping on the electrons, which increases their parallel kinetic energy. The final term in Eq. (14) describes the parallel kinetic energy given up by the anomalous Dopper resonant electrons. The net flow of parallel kinetic energy from these electrons to the field and to perpendicular kinetic energy occurs in the ratio $1{:}\Omega/\omega$. This reflects the original concept of Kadomtsev and Pogutse,[18] who treated $(h/2\pi)\Omega$ as the energy quantum of perpendicular gyromotion, and $(h/2\pi)\omega$ as the quantum of wave energy, both of which are drawn from the electron parallel kinetic energy. If the energy transfer to the field is sufficient to overcome the Landau damping losses, γ is positive and wave growth occurs: thus, $-(P_x + P_z) = 2\gamma|\mathbf{E}|^2/4\pi$ as expected. A larger energy transfer occurs from the parallel to the perpendicular component of electron motion. In both these respects, Eqs. (13) and (14) demonstrate explicitly how closely the collective anomalous Doppler effect follows the single-particle effect described at the beginning of Sec. II. The essential difference lies in the existence of a threshold, since Landau damping of the wave by the bulk distribution and by the ions, where appropriate, must be overcome in the collective case. This also differentiates the instability of an extended tail from that of a beam in a cold plasma. We note also that the contribution to P_z from the anomalous Doppler term in Eq. (14) can be negative, even when the relative magnitudes of $\gamma_{\rm LB}$, $\gamma_{\rm LT}$, and $\gamma_{\rm AD}$ are such as to give overall wave damping. It follows that anomalous Doppler resonant electrons may reduce the absorption of driven waves that are undergoing damping, while they increase their perpendicular energy at the expense of their parallel energy.

III. THE EFFECT OF THE TAIL STRUCTURE ON INSTABILITY

Let us return to Eqs. (6)–(10). These describe the general properties of the various contributions to the growth rate for the anomalous Doppler instability, and appear also in the spatial components of ${\rm Re}(\mathbf{j}\cdot\mathbf{E}^*)$ in Eqs. (13) and (14). No specific choice of the parallel electron velocity distribution $F(v_\parallel)$ has yet been made. Our aim in this section is to use very simple and general choices of $F(v_\parallel)$, which are specified by a minimal number of parameters, to identify general properties of the instability. We note first that, subject to other constraints, Eqs. (6)–(10) indicate that instability is favored by small values of $k_\parallel/k$. Second, for $\beta \ll 1$, $\Lambda_0(\beta) \simeq 1$ and $\Lambda_1(\beta) \simeq \beta/2$; however, for $\beta \gtrsim 2$, $\Lambda_0(\beta) \simeq \Lambda_1(\beta) \simeq 0.2$. There are thus two candidate regimes for instability.

(1) The first is $k_\perp$ small such that $\beta_B, \beta_T \ll 1$. In this case, ω_p/kv_B can be sufficiently large that the exponential term in Eq. (10) renders ion Landau damping negligible. The stability of the wave is determined by the electron velocity distribution alone. Electron Landau damping can be weak because the parallel velocity lies beyond the bulk thermal electron distribution. Owing to the factor $\Lambda_1(\beta_T) \simeq \beta_T$, the ratio of $\gamma_{\rm AD}$ to $\gamma_{\rm LB}$ is independent of the perpendicular temperature of the tail, and decreases as Ω/ω_p increases at a given density.

(2) The second candidate regime is $k_\perp$ large such that $\beta_B, \beta_T \gtrsim 1$. In this case, the parallel phase velocity ω_p/k can be much less than v_B, and lie in a region where electron Landau damping is again weak. Since $\Lambda_1(\beta_T) \simeq \Lambda_0(\beta_B)$, the ratio of $\gamma_{\rm AD}$ to $\gamma_{\rm LB}$ increases with Ω/ω_p, and decreases as the perpendicular temperature of the tail is increased. In this low phase velocity regime, Landau damping on resonant ions becomes a significant phenomenon.

Consider by way of illustration a flat tail that extends as far as a maximum velocity v_M:

$$F(v_\parallel) = \begin{cases} 1/\pi^{1/2}v_M, & 0 < v_\parallel < v_M, \\ 0, & \text{otherwise}. \end{cases} \tag{15}$$

For instability in region (1), combining Eq. (15) with Eqs. (5)–(9), we obtain

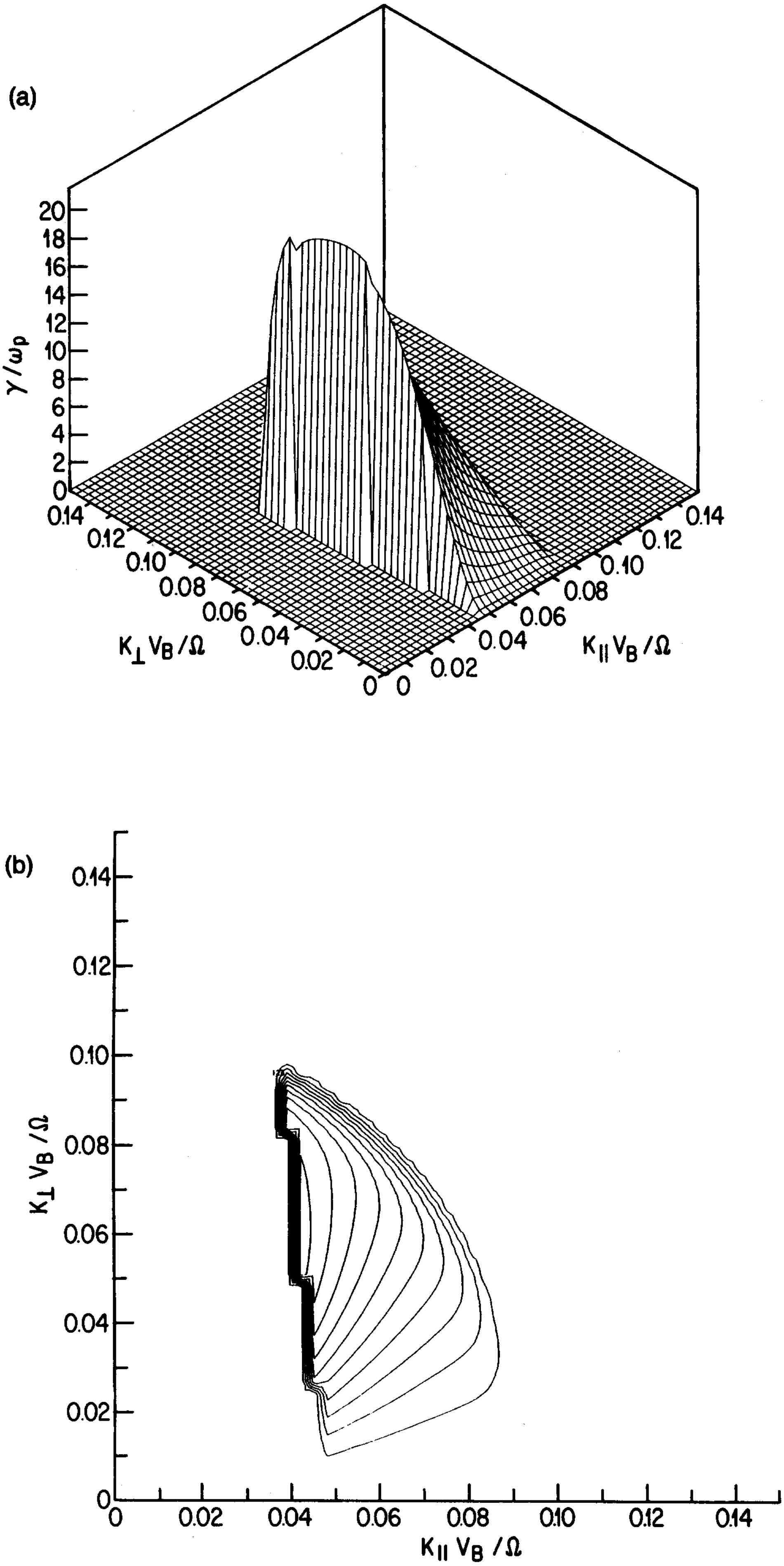

FIG. 1. Growth rate $\gamma/\omega_p \times 10^6$ (a) as a function of $k_\perp v_B/\Omega$ (left axis) and $k_\parallel v_B/\Omega$ (right axis) for a flat tail with $\mu = 10^{-3}$, $v_{max} = 30 v_B$, $v_{T\perp} = v_B$, and $\omega_p/\Omega = 0.4$; (b) corresponding contour plot.

$$\frac{\gamma}{\omega_p} = \pi^{1/2}\left(\frac{\omega_p}{kv_B}\right)^3 \Lambda_0(\beta_B)\left[\mu\left(\frac{v_B}{v_{T\perp}}\right)^2\left(\frac{v_B}{v_M}\right)\left(\frac{\Omega}{\omega_p}\right) \times \frac{\Lambda_1(\beta_T)}{\Lambda_0(\beta_B)} - (1-\mu)\,\frac{k_\parallel}{k}\,e^{-(\omega_p/kv_B)^2}\right], \tag{16}$$

provided that $0 < v_{\rm AD} < v_M$, so that

$$k_\parallel > \Omega/(v_M - \omega_p/k). \tag{17}$$

In Fig. 1(a), the growth rate γ/ω_p given by Eq. (16) is plotted as a function of the dimensionless wavenumber coordinate $(k_\perp v_B/\Omega,\, k_\parallel v_B/\Omega)$ for a distribution function as in Eq. (15) with $\mu = 0.001$, $v_M = 30v_B$, $v_{T\perp} = v_B$, and $\omega_p/\Omega = 0.4$. The corresponding contour plot is shown in Fig. 1(b). The dependence of growth rate on tail and plasma parameters has been examined numerically. For a tail fraction $\mu = 0.001$ (0.1% of electrons in the extended tail) we find the following.

(i) For $\omega_p/\Omega = 0.4$, instability occurs only when $v_M \geqslant 15\, v_B$. The growth rate rises to $\gamma = 2\times 10^{-5}\omega_p$ when $v_M = 30\, v_B$, and thereafter is insensitive to v_M.

(ii) For $v_M = 20\, v_B$, $\gamma = 1.6\times 10^{-5}\,\omega_p$ when $\omega_p/\Omega = 0.4$, but γ falls to zero when $\omega_p/\Omega < 0.29$.

(iii) The growth rate is independent of $v_{T\perp}$.

We note from Eq. (2) that the ratio of the anomalous Doppler parallel resonant velocity $v_{\rm AD}$ to the parallel velocity $v_{\rm L}$ at which electron Landau damping occurs is given by

$$v_{\rm AD}/v_{\rm L} = 1 + \Omega/\omega. \tag{18}$$

Thus, point (i) illustrates the fact that for instability to occur, the tail must extend sufficiently far for electrons to exist at the value of $v_{\rm AD}$ given by Eq. (18) when $v_{\rm L}$ is a few times v_B, so that Landau damping is weak. Points (ii) and (iii) quantify and confirm remarks made above. For the tail parameters considered, it is possible to suppress the instability by a relatively small increase in magnetic field strength at constant density. Result (iii) holds only for a Maxwellian distribution of perpendicular velocities in the tail. If there were a plateau in the $v_\perp$ distribution in the anomalous Doppler resonant region, it is clear from Eq. (2) that it would not be possible to obtain expressions in terms of $\Lambda_n(\beta_T)$, and Eq. (16) would no longer apply.

For region (2) we find that no instability occurs. While there can exist a region of wavenumber space—typically at $k_\parallel v_B/\Omega \simeq 0.3$, $k_\perp v_B/\Omega \simeq 3$—where the anomalous Doppler resonance can overcome the effects of electron Landau damping, this effect is always overcome by ion Landau damping. From Eqs. (8) and (10), we have

$$\frac{\gamma_{\rm LI}}{\gamma_{\rm LB}} = Z\left(\frac{m_i}{m_e}\right)^{1/2}\frac{k_\parallel}{k}\,\frac{1}{\Lambda_0(\beta_B)} \times \exp\left(-\left\{\left(\frac{\omega_p}{kv_B}\right)^2\left[\left(\frac{m_i}{m_e}\right)\left(\frac{k_\parallel}{k}\right)^2 - 1\right]\right\}\right). \tag{19}$$

In the region of wavenumber space indicated, $\gamma_{\rm LI}$ exceeds $\gamma_{\rm LB}$ by a factor of order 20 for the case of hydrogen ions, and this is sufficient to keep γ given by Eq. (6) negative, even though $\gamma_{\rm AD} > \gamma_{\rm LB}$.

Now let us examine further the extent to which the properties of the instability depend on the choice of tail representation. A wide range of monotonically decreasing superthermal tails in the electron velocity distribution can be represented by Eq. (4) when

$$F(v_\parallel) = (1/\pi^{1/2} v_{T\parallel})\,e^{-(v_\parallel - v_D)^2/v_{T\parallel}^2}. \tag{20}$$

The parameters $(\mu, v_D, v_{T\parallel})$ can be chosen so that the tail structure has a slow, plateau-like decline [Fig. 2(a)], or a much steeper falloff [Fig. 2(b)]. In the parameter range of

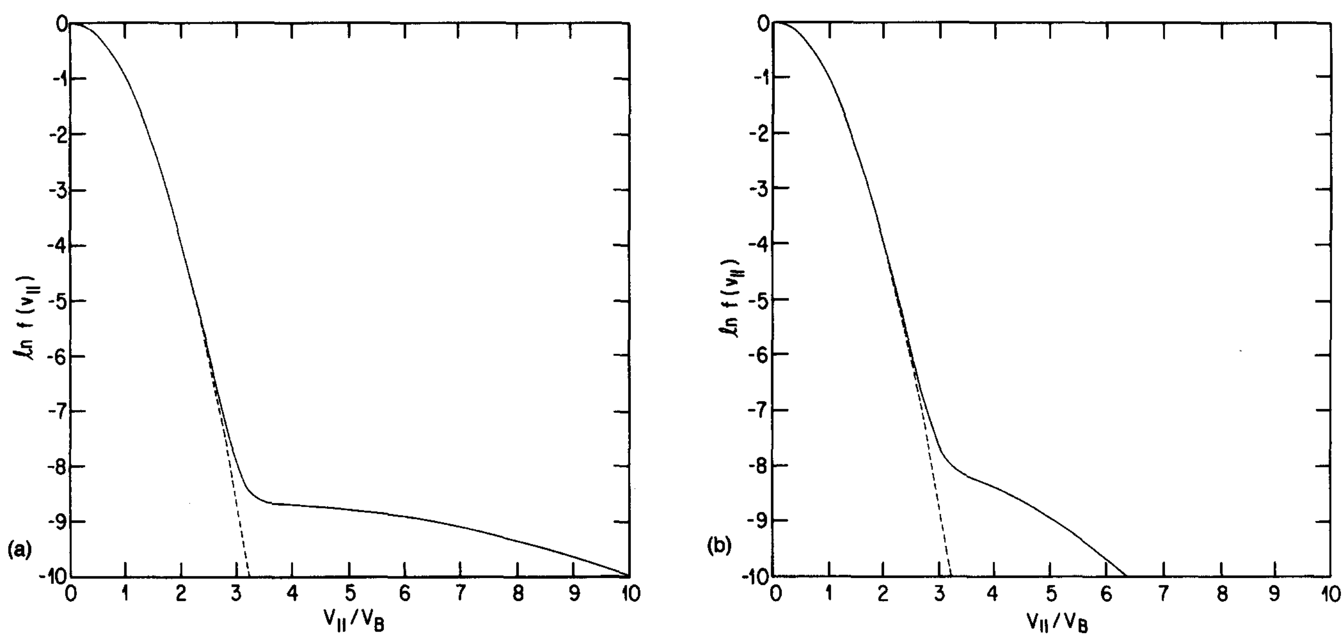

FIG. 2. Use of a drifted Maxwellian component to represent (a) a plateau-like monotonically decreasing tail, with $\mu = 0.001$, $v_D = 3v_B$, $v_{T\parallel} = 6v_B$, $v_{T\perp} = v_B$; (b) a more steeply monotonically decreasing tail, with $\mu = 0.001$, $v_D = 2v_B$, $v_{T\parallel} = 3v_B$, $v_{T\perp} = v_B$.

interest, the value of v_D is kept below $4v_B$ in order to avoid describing tails that have a bump rather than monotonic decrease. We have also taken steps in our code to preclude spurious effects arising from a positive γ_{LT} given by Eq. (9) near $v_\parallel = 0$, where γ_{LB} is small. Physically, these constraints ensure that the electron distribution function is monotonically decreasing in $v_\parallel$ throughout its range. Combining Eqs. (7) and (20), we obtain

$$\frac{\gamma_{AD}}{\omega_p} = \mu\pi^{1/2}\left(\frac{\omega_p}{kv_{T\perp}}\right)^2 \Lambda_1(\beta_T) e^{-(v_{AD}-v_D)^2/v_{T\parallel}^2} \frac{1}{kv_{T\parallel}} \times\left[\Omega - k_\parallel\left(\frac{v_{T\perp}}{v_{T\parallel}}\right)^2 (v_{AD}-v_D)\right]. \tag{21}$$

We note that in general, $v_{AD} - v_D$ can be of either sign, so that the effect of the perpendicular tail temperature can be either stabilizing or destabilizing. Because of the large number of independent parameters, it is not possible to devise an exact condition for instability from Eqs. (6), (8), (20), and (21). However, a number of conclusions can be drawn. As we noted above, for the tail to be monotonically decreasing, we require $v_D \leqslant 4v_B$. Second, it is clear both in general and from our discussion of the stability of the flat tail distribution given by Eq. (15) that for instability, $v_L = \omega/k_\parallel = \alpha v_B$, where α is a few times unity. Landau damping can be sufficiently weak to permit instability only if this condition is satisfied. These two constraints lead to the conclusion that

$$v_{AD} - v_D \simeq \Omega/k_\parallel, \tag{22}$$

for the modes whose instability is potentially greatest. Combining Eqs. (21) and (22), we obtain

$$\frac{\gamma_{AD}}{\omega_p} \simeq \mu\pi^{1/2}\left(\frac{\omega_p}{kv_{T\perp}}\right)^2 \Lambda_1(\beta_T) e^{-(\Omega/k_\parallel v_{T\parallel})^2} \times\frac{\Omega}{kv_{T\parallel}}\left(1 - \frac{v_{T\perp}^2}{v_{T\parallel}^2}\right). \tag{23}$$

It follows that for monotonically decreasing tails, no instability can occur if $v_{T\perp}^2 \geqslant v_{T\parallel}^2$. This demonstrates explicitly the stabilizing effect of increasing the perpendicular tail temperature. This effect has its origin in the term $k_\parallel\, \partial f/\partial v_\parallel$ in Eq. (2), evaluated at $v_\parallel = v_{AD}$. This term is zero for the flat tail of Eq. (15), where in contrast the growth rate was found to be independent of $v_{T\perp}$.

Now let us consider the magnitude of the exponential term in Eq. (23). It is clear that significant growth requires $k_\parallel \lesssim \Omega/v_{T\parallel}$. We have already established that $v_L = \omega/k_\parallel = \omega_p/k = \alpha v_B$, where α is a few times unity, is a necessary though not sufficient condition for instability. Also, necessarily $k_\parallel < k$, so that

$$\left(\frac{\Omega}{k_\parallel v_{T\parallel}}\right)^2 > \left(\frac{\Omega}{kv_{T\parallel}}\right)^2 = \alpha^2\left(\frac{\Omega}{\omega_p}\right)^2\left(\frac{v_B}{v_{T\parallel}}\right)^2. \tag{24}$$

Here, although $v_B/v_{T\parallel} \ll 1$, both Ω/ω_p and α exceed unity. It follows that the negative exponential term in Eq. (23) may give rise to a number that is very small, even for the mode for which γ_{AD} is greatest. These considerations suggest that for most sets of parameters (μ, v_D, $v_{T\parallel}$, $v_{T\perp}$), monotonically decreasing tails of the type described by Eqs. (4) and (20) are stable against the anomalous Doppler effect, and this result is supported numerically.

IV. CONCLUSIONS

In this paper, we have established the connection between the three aspects—single-particle, beam, and continuous velocity distribution—of the anomalous Doppler effect. The key step was the calculation of the spatial components of the field energy $\mathrm{Re}(\mathbf{j}\cdot\mathbf{E}^*)$ dissipated under anomalous Doppler resonance conditions. For a general continuous distribution $F(v_\parallel)$ of electron velocities parallel to the magnetic field, the flows of energy from parallel to perpendicular particle kinetic energy and into the wave have been identified. We have shown how these aspects of a classical electrodynamic approach to the anomalous Doppler effect for a continuous distribution of velocities can reproduce the characteristics of the single-particle anomalous Doppler effect, which is based on a quantum treatment. Next, in order to give a complete treatment of the anomalous Doppler effect in the context of plasma physics, we have included the role of ion dynamics in addition to electron dynamics. We have shown how ion Landau damping is responsible for stabilizing electrostatic waves in a region of wavenumber space ($k_\perp v_B/\Omega \simeq 3$, $k_\parallel/k_\perp \simeq 0.1$) that would otherwise be unstable against the anomalous Doppler effect. Finally, we have turned to specific choices of $F(v_\parallel)$. Our aim was to identify characteristics of the instability that arise independently of the precise choice of tail model. We therefore chose very simple models of tail distributions, which possess a minimal number of parameters. As a first example, we considered a flat tail with a Maxwellian distribution of perpendicular velocities, characterized by a $v_{T\perp}$ independent of $v_\parallel$. The linear instability of this distribution against the anomalous Doppler effect was found to be independent of $v_{T\perp}$. Our second example was a broad class of monotonically decreasing tails. Even in this simple case, it was not possible to devise an exact criterion for instability. However, the stabilizing role of perpendicular tail temperature was demonstrated. In addition, we gave analytical arguments that in general indicated stability against the anomalous Doppler effect for this class of distribution, and this conclusion was borne out numerically.

ACKNOWLEDGMENT

We wish to thank Dr. V. Fuchs and Dr. M. Lontano for helpful discussions

One of the authors (A.M.) wishes to thank the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico (CNPq), Brazil for partial financial support.

[1]V. L. Ginzburg, Sov. Phys. Usp. **2**, 874 (1960).
[2]M. V. Nezlin, Sov. Phys. Usp. **19**, 946 (1976).
[3]V. V. Parail and O. P. Pogutse, Sov. J. Plasma Phys. **2**, 126 (1976).
[4]V. V. Parail and O. P. Pogutse, Nucl. Fusion **18**, 303 (1978).
[5]H. Knoepfel and D. A. Spong, Nucl. Fusion **19**, 785 (1979).
[6]S. C. Luckhardt, G. Bekefi, P. I. Bonoli, K.-I. Chen, B. Coppi, R. C. Eng-

lade, A. S. Fisher, K. Hackett, S. F. Knowlton, M. J. Mayberry, F. S. McDermott, M. Porkolab, J. S. Levine, M. E. Read, and V. L. Granatstein, *Proceedings of the Third Varenna–Grenoble International Symposium on Heating in Toroidal Plasmas* (Commission of the European Communities, Brussels, 1982), p. 529.

[7]S. von Goeler, J. Stevens, C. Karney, S. Bernabei, M. Bitter, T. K. Chu, P. Efthimion, K. Hill, W. Hooke, F. Jobes, E. Mazzucato, E. Meservey, R. Motley, P. Roney, N. Sauthoff, S. Sesnic, G. Taylor, F. Tenney, and E. Valeo, *Proceedings of the Fifth APS Topical Conference on RF Plasma Heating* (University of Wisconsin, Madison, 1983), p. 96.

[8]L. Muschietti, J. Vaclavik, and K. Appert, Plasma Phys. **24**, 987 (1982).

[9]R. O. Dendy and C. N. Lashmore-Davies, Plasma Phys. **26**, 1347 (1984).

[10]S. Ichimaru, *Basic Principles of Plasma Physics* (Benjamin, Reading, MA, 1973).

[11]K. Molvig, M. S. Tekula, and A. Bers, Phys. Rev. Lett. **38**, 1404 (1977).

[12]C. S. Liu, Y. C. Mok, K. Papadopoulos, F. Engelmann, and M. Bornatici, Phys. Rev. Lett. **39**, 701 (1977).

[13]L. Muschietti, K. Appert, and J. Vaclavik, Phys. Fluids **24**, 151 (1981).

[14]C. S. Liu., V. S. Chan, D. K. Bhadra, and R. W. Harvey, Phys. Rev. Lett. **48**, 1479 (1982).

[15]E. Moghaddam-Taaheri, L.Vlahos, H. L. Rowland, and K. Papadopoulos, Phys. Fluids **28**, 3356 (1985).

[16]S. C. Luckhardt, *Proceedings of the Sixth AIP Topical Conference on Radiofrequency Plasma Heating* (American Institute of Physics, New York, 1985), p. 158.

[17]N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw–Hill, London, 1973), p. 405.

[18]B. B. Kadomtsev and O. P. Pogutse, Sov. Phys. JETP **26**, 1146 (1968).