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14. Abstract/Notes

In this paper, it is presented a derivation of a reduced set of equations for the electron motion, based upon Lorentz equation, where the applicability conditions and approximations employed are clearly indicated. As an example of practical interest, scaling relations are discussed in the analysis of cavities appropriate for high efficiency operation.

15. Remarks

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DERIVATION OF GYROTRON'S REDUCED EQUATIONS AND ITS APPLICATION TO THE ANALYSIS OF RESONANT CAVITIES

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The realization of high efficiency, high power gyrotrons (>1MW at frequencies above 100GHz) demands high current beams and low Q resonant cavities operating in high order modes. The requirement of extended-space systems, to increase the gyrotron power handling capability, sets strong restrictions to the stable single mode operation of the device. As the cavity cross-section increases, the mode spectrum becomes particularly severe. Furthermore, the normal electromagnetic structure of a low Q cavity is modified by the presence of intense beams. These points can only be dealt with by means of self-consistent models and multi-mode nonlinear treatments. These theories require a set of equations to describe the electron dynamics.

In this paper, it is presented a derivation of a reduced set of equations for the electron motion, based upon Lorentz equation, where the applicability conditions and approximations employed are clearly indicated. As an example of practical interest, scaling relations are discussed in the analysis of cavities appropriate for high efficiency operation.

The electron motion in a weakly irregular waveguide immersed in a magnetostatic field $\hat{z}B_0$, operating near the cutoff of a purely circular TE mode, can be described by [1]

$$\dot{r}=V_r\ ,\quad \dot{\theta}=V_\theta/r\ ,\quad \dot{z}=V_z \tag{1}$$
$$\dot{V}_i=a_i-\beta_i T\ ,\quad i=r,\theta,z$$

where

$$a_r=-(e/\gamma m)[E_r+(B_0+B_z)V_\theta-B_\theta V_z]+V_\theta^2/r$$
$$a_\theta=-(e/\gamma m)[E_\theta-(B_0+B_z)V_r+B_r V_z]-V_r V_\theta/r \tag{2}$$
$$a_z=-(e/\gamma m)[B_\theta V_r-B_r V_\theta]$$
$$T=\beta_r a_r+\beta_\theta a_\theta+\beta_z a_z$$

with $\beta_i=V_i/c$ $(i=r,\theta,z)$, the relativistic factor $\gamma=(1-\beta_r^2-\beta_\theta^2-\beta_z^2)^{-1/2}$ and the cylindrical coordinate system is depicted in Fig. 1. The weakly irregular waveguide acts as an open resonator where, near cutoff conditions, the effects of the RF magnetic field can be neglected. Assuming thereafter $B_r=B_\theta=0$, the electron axial momentum is conserved ($\gamma\beta_\parallel$=const.) and eqs. (1) give

$$dV_\perp/dt=-(e/\gamma m)[E_r\cos\varepsilon+E_\theta\sin\varepsilon](1-V_\perp^2) \tag{3a}$$
$$d\varepsilon/dt=-(e/\gamma m)V_\perp^{-1}[-E_r\sin\varepsilon+E_\theta\cos\varepsilon]+eB_0/\gamma m-(V_\perp/r)\sin\varepsilon \tag{3b}$$

where, according to Fig. 1, $V_\perp^2=V_r^2+V_\theta^2$ and $\tan\varepsilon=V_\theta/V_r$. Note that under the condition $\gamma\beta_\parallel$=const., the electron energy is related to the transverse velocity $V_\perp$ by

$$\gamma^2/\gamma_0^2=(1-\beta_{\perp 0}^2)/(1-\beta_\perp^2) \tag{4}$$

where the subscript "0" refers to the initial conditions. These equations are sufficient to describe the mechanism of energy transfer between the electrical field and the electrons, inasmuch as the electron energy constitutes the only quantity of interest. The solution of eqs. (3) gives the value of $V_\perp$ at the cavity outlet and relation (4) gives the corresponding electron energy.

Introducing the following normalization scheme [1]: velocity: $\bar{V}=V/c=\beta$; time: $\bar{t}=t\omega_0$; length: $\bar{L}=L.k_0$; electric field: $\bar{E}=E/(mc^2k_0/e)$; magnetic induction: $\bar{B}=B/(mck_0/e)$; voltage: $\bar{V}=VC_{om}/(mc^2/e)$, where $\omega_0=ck_0$ is the wave angular frequency and $C_{om}=[\sqrt{\pi}x_{om}J_0(x_{om})]^{-1}$ is a normalization constant (x_{om} is the mth nontrivial root of $J_0'(x)=0$); using the geometrical relations (Fig. 1)

$$\sin\delta=(R_0/R)\sin\theta$$
$$\cos\delta=(R_0/R)\cos\theta+r/R$$

and adopting a slow time scale formulation, eqs. (3) can be rewritten as

$$d\beta_\perp/dt=(\bar{E}_{\phi 0}/2\gamma)R_0R_1^{-1}k_{om}(z)J_1(k_{om}R)(1-\beta_\perp^2)(V_R\sin\psi-V_I\cos\psi) \tag{5a}$$
$$d\psi/dt=(\bar{E}_{\phi 0}/2\gamma)R_0R^{-1}k_{om}(z)J_1(k_{om}R)\beta_\perp^{-1}(V_R\cos\psi+V_\perp\sin\psi)+(\omega_{ce}/\omega_0-1) \tag{5b}$$

where, for sake of clarity, the bar was omitted except for the dimensionless field amplitude $\bar{E}_{\phi 0}$; ω_{ce} is the electron cyclotron frequency; $k_{om}(z)=x_{om}/R_w(z)$ is the transverse wavenumber, with $R_w(z)$ the waveguide radius; V_R and V_I are respectively the real and imaginary parts of the complex eigenfuction which characterizes the RF longitudinal field profile in the resonant cavity, subject to radiation boundary conditions at both ends [1]; and $\psi=\varepsilon+\theta-\omega t$ is the new slow time scale phase variable. Using Gegenbauer's addition theorem for the Bessel functions and keeping only the first term (fundamental resonance) in the expansion for $J_1(x)/x$, one has

$$J_1(k_\perp R)\simeq(R/R_0)J_1(k_\perp R_0) \tag{6}$$

where a weakly relativistic beam ($k_\perp r \sim (\gamma/\gamma_0)\beta_\perp << 1$) was assumed. With these approximations, eqs. (5) become, after the transformation $z = \omega z/\beta_{\parallel 0} c$ is carried out,

$$d\beta_\perp/dz = (\bar{E}_{\phi 0}/2\gamma_0)F(z)[V_R \sin\psi - V_I \cos\psi](1-\beta_\perp^2) \qquad (7a)$$

$$d\psi/dz = (\bar{E}_{\phi 0}/2\gamma_0 V_\perp)F(z)[V_R \cos\psi + V_I \sin\psi] + \omega_{ce0}/\omega - \gamma/\gamma_0 \qquad (7b)$$

where $\omega_{ce0} = (\gamma/\gamma_0)\omega_{ce}$ and $F(z) = k_{om}(z)J_1(k_{om}R_0)$.

Eqs. (7) can be reduced further if the condition $\beta_{\perp 0}^2/2 << 1$ is fulfilled and the beam is considered to be weakly relativistic. Under these conditions and using the basic conservation relation (4), the single particle efficiency $\eta_s = (\gamma_0-\gamma)/(\gamma_0-1)$ can be written as

$$\eta_s \simeq (2/\beta_{\perp 0}^2)(1-\gamma/\gamma_0)(\alpha^2/1+\alpha^2) = \eta_{\perp,s}(\alpha^2/1+\alpha^2) \qquad (8)$$

where $\eta_{\perp,s}$ is the perpendicular efficiency for a single electron and $\alpha = \beta_{\perp 0}/\beta_{\parallel 0}$. Hence, eqs. (7) become

$$d\eta_{\perp,s}/d\zeta = -2\bar{E}F(\zeta)[V_R \sin\psi - V_I \cos\psi](1-\eta_{\perp,s})^{1/2} \qquad (9a)$$

$$d\psi/d\zeta = \bar{E}F(\zeta)[V_R \cos\psi + V_I \sin\psi](1-\eta_{\perp,s})^{-1/2} - \Delta + \eta_{\perp,s} \qquad (9b)$$

where $\bar{E} \equiv \bar{E}_{\phi 0}/(\gamma_0 \beta_{\perp 0}^3)$, $\zeta = (\beta_{\perp 0}^2/2)\omega z/\beta_{\parallel 0} c$ is a normalized axial coordinate and $\Delta \equiv (2/\beta_{\perp 0}^2)(1-\omega_{c0}/\omega)$ is the magnetic detuning parameter [2]. In order to calculate the perpendicular interaction efficiency $\eta_\perp = \langle \eta_{\perp,s} \rangle_{\theta_0}$, where the brackets denote an ensemble average over initial phase, eqs. (9) are integrated for a system consisting of several electrons uniformly distributed over the initial gyration circle. As apparent from eqs. (9a,b) the perpendicular efficiency depends upon three variables, namely $\eta_\perp = \eta_\perp(\bar{E}, \Delta, \zeta_{out})$, with $\zeta_{out} = (\beta_{\perp 0}^2/2)\omega L/\beta_{\parallel 0} c$ where L defines the interaction length. Optimizing $\eta_\perp$ with respect to the parameters $\bar{E}$ and Δ, results a scaling relation $\eta_\perp^{MAX.} = \eta_\perp^{MAX.}(s)$ for $s = 2\zeta_{out} = (L/R_c)x_{om}\alpha\beta_{\perp 0}$ where R_c is the critical cross section radius $R_c = x_{om}c/\omega_0$. The quantity s seems to be quite useful for design purposes. It explicitly embodies the beam parameters (via α and $\beta_{\perp 0}$), the cavity geometry (L and R_c) and the operating mode (x_{om}), characterizing thus the optimized gyrotron interaction.

As an application of the reduced eqs. (9a,b) in cavity analysis, Fig. 2 displays the real and imaginary parts of the complex eigenfuction V(z) associated with a special cavity type of continuous profile [1]. The results in Fig. 3 are obtained by taking $0.9 \leq \alpha \leq 2.5$ whereas the beam voltage values are within the range 20-90kV. The data seems to follow a smooth curve and a perpendicular efficiency as high as 0.87 for an ensemble of 16 electrons, occurs at s=80.0. It should be noted, however, that the soft-self excitation regime corresponds to the interval $s \leq 42$. Regarding the applicability of these equations to a system with a tapered magnetic field [2], some questions arise from the fact that the electron axial momentum is not conserved due to the presence of a radial magnetic field component.

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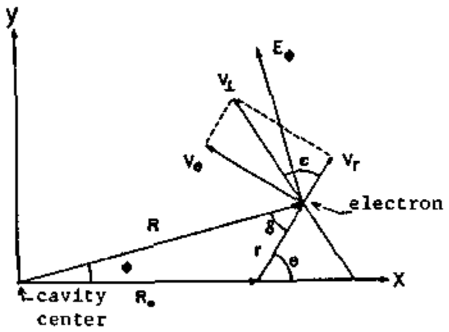

Fig.1. Definition of geometrical parameters for the gyrotron interaction.

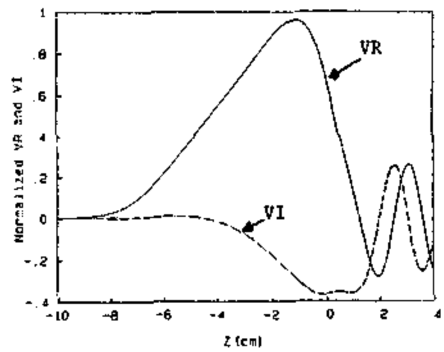

Fig. 2. Real and imaginary parts of the complex eigenfunction V(z) associated with a highly efficient 35GHz cavity [1] operating in the TE_{021} mode.

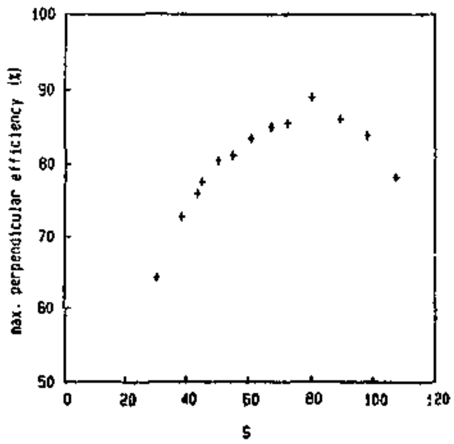

Fig. 3. Scaling relation corresponding to the longitudinal complex field profile shown in Fig. 2.

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