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16.Summary/Notes

A comparison of FABNE and Burg spectra for artificial samples showed that for single sinusoids or for peaks widely separated in frequency, the FABNE method was distinctly superior. However, for samples having peaks in a wide frequency band, or having close-by peaks, both methods showed almost similar results, not always satisfactory.

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COMPARISON OF MAXIMUM ENTROPY SPECTRAL ANALYSIS (MESA) AND LEAST-SQUARES LINEAR PREDICTION (LSLP) METHODS FOR SOME ARTIFICIAL SAMPLES

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ABSTRACT

A comparison of the spectra obtained by the Least-Squares Linear Prediction (LSLP) algorithm developed by Barrodale and Erickson (termed by them as FABNE) and the BURG algorithm of Maximum Entropy Spectral Analysis (MESA) showed that FABNE was distinctly superior. However, for samples having peaks in a wide frequency band, or having close-by peaks, both methods showed almost similar results, not always satisfactory.

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1. Introduction

Spectral analysis is a very useful technique for studying geophysical problems. In earlier days, the only methods available were those of Fourier analysis or the method of Blackman and Tukey (1959) based on auto-correlation function. Recently, Burg (1967,1968) introduced the Maximum Entropy Spectral Analysis (MESA), which gives good resolution even for periods comparable to the data length. Ulrych and Bishop (1975) have given a critical appraisal of the Burg's algorithm. Several workers noticed and reported some inherent shortcomings of Burg's algorithm. Thus, Chen and Stegan (1974) showed that, for truncated sinusoids, the spectral maxima showed frequency shifts sometimes as large as 20%, depending upon the initial phase and the length of the sample. Also, under certain conditions, the Burg spectra display line-splitting in the presence of low noise, and, as the noise is increased, the multiple peaks coalesce into a single peak shifted substantially away from the correct value (Fougere et al, 1976; Fougere, 1977). These defects can be rectified by the elaborate computer program given by Fougere (1977). Another difficulty is about the selection of the appropriate length of the Prediction Error Filter (LPEF). Whereas low LPEF is generally inadequate to resolve all the peaks, high LPEF, while resolving all peaks, produces instability in the spectra and gives spurious peaks. For determining the optimum LPEF, Ulrych and Bishop (1975) suggested the use of the Akaike's (1969) final prediction Error (FPE) criterion. And if this failed, an LPEF of about 50% of the data length was suggested tobe generally adequate. Gutowski et al. (1978) suggested the use of partial correlation coefficient. Berryman (1978) suggested an empirical solution LPEF = 2 N/(ln2N)(where N=No.of data points).Our experience (Kane 1977, 1979) indicated that, for samples containing peaks in a wide range of frequency, LPEF of about 50% of data length was adequate to resolve frequencies exceeding the fifth harmonic, while for lower harmonics, LPEF even as high as 90% was sometimes needed, with the danger of peak-splitting ever present.

The limitations of the Burg algorithm are caused by its imposition of a Toeplitz structure on the matrix of the system of equations which yield the AR (auto regressive) parameters. This procedure is responsible for the computational efficiency of the Burg's algorithm; but there is enough empirical evidence to show that the resulting spectra so obtained are inferior (in accuracy of frequency determination) to those obtained by using Least-Square (LS) solutions to the AR model (see, for example, Ulrych and Clayton, 1976).

Estimation of AR parameters by LS methods has previously been unpopular because of the large computational effort involved, besides other reasons. However, Barrodale and Erickson (1980 a and b) have recently developed an algorithm for solving the Least-Squares Linear Prediction (LSLP) problem directly (without forcing a Toeplitz structure on the AR model) which is claimed to be computationally efficient and numerically stable. From the example presented in that publication, their new algorithm seems superior to the Burg algorithm specially for very low frequencies.

In Geophysics, one often encounters data having periodicities in a wide band of frequency. In this communication, we present a comparative study of the results of analysis by this FABNE algorithm (as the above authors have designated it) and the usual Burg algorithm for which we used the program given by Anderson (1974). We examine the spectra of artificial samples for a wide range of values of LPEF, including the one which is indicated by the Akaike's FPE criterion.

2. Artificial Samples

First, we produced an artificial sample of 101 data points, by superposing six sinusoids of periods T = 5,10,20,40,80,160. All the input data (101 data points) were calculated to six significant digits. Fig 1 shows the FABNE spectra (full lines) and the Burg spectra (dashed lines with a shifted zero level) for various LPEF. The ordinate scale is logarithmic with constant steps of (log T). The step chosen was Δ (log T) = 0.005, so that the step accuracy is uniformaly 1.2% for all values of T (e.g. T=8.0 ± 0.1 and T=80.0 ± 1.0). All the six sinusoids have the same amplitude (unity) and a small Gaussian noise (amplitude 0.001) is added.

The vertical dashed lines in Fig 1 indicate the positions where peaks (T=5,10,20,40,80,160) are expected. The following may be noted:

(1) The FPE criterion showed an optimum LPEF=18. The top curves in Fig 1 represent spectra for LPEF=18. In both the methods (FABNE as well as BURG), T=5,10,20 are resolved very well. T=40 is indicated with about 10% error. Higher periods (T=80, 160) are not resolved.

(2) The next two curves, for LPEF=40, show good resolution for T=5,10,20,40, and a rough resolution (error about 15%) for T=80. The highest period (T=160) is not resolved.

(3) The next two curves for LPEF=60 show good resolution for T=5,10,20,40. The period T=80 is indicated within an error of about 6%, and T=160 with an error of aobut 15-25%. In all cases, frequency shifts in the FABNE spectra are smaller than those in BURG spectra. Thus, FABNE method is superior, but only slightly.

(4) For LPEF=65, the FABNE spectra (not shown here) showed results similar to LPEF=60. For still higher LPEF, the FABNE program failed, even in double precision. For BURG spectra only, the results for LPEF=80 are shown at the bottom of Fig 1. No improvement over LPEF=60 is seen.

(5) The sample used in Fig 1 is produced as:

Gaussian noise (Amplitude 0.001) $+ \sum_{n=1}^{6} \mathrm{Sin}\,[2\pi(t/T_n) + \phi]$

with $T_1 = 5$, $T_2 = 10$, $T_3 = 20$, $T_4 = 40$, $T_5 = 80$ and $T_6 = 160$, with $t = 0$ to 100. Initially, the phase ϕ was shosen as 60^0 and the results shown in Fig 1 refer to $\phi = 60^0$. Later, we repeated the analysis for $\phi = 0$ and $\phi = 120^0$. The results were similar. T = 160 showed shifts as large as 30%, more so in Burg spectra.

(6) Even though the input signals had the same amplitude (unity), the peaks in Fig 1 are not of the same height. According to Lacoss (1971), in the Burg algorithm, the heights of the output peaks are expected to be proportional to the square of the power of the signal, and the area under the peak is expected to be proportional to the power of the signal. In Fig 1, the Burg spectral peaks seem to be more similar in heights than the BABNE spectral peaks. However, this may be misleading in view of the fact that our abscissa scale is not linear in frequency. We repeated the whole analysis by using the conventional method of Δf increasing in constant steps. In Fig 2, we show the corresponding FABNE spectra only for LPEF=18,30,40,50,60. Two steplengths were chosen viz. $\Delta f = 0.0001$ for the low frequency range $f = 0.00$ to 0.03 (shown in the left half of Fig 2) and $\Delta f = 0.001$ for the high frequency range $f = 0.03$ to 0.25 (shown in the right half of Fig 2). The results of Fig 2 are similar to those of Fig 1. Thus, the logarithmic scale chosen for T in Fig 1 does not produce any complication or distortion and has the advantage of showing the whole spectrum in a reasonable size of the graph.

In Fig 2, the peaks at T=5 and T=10 are very sharp indeed, reducing by several db, even with one Δf step on either side. Thus, the area under the curve is essentially determined by the height of the peak itself. Table 1 gives the ratios of the actual heights of the various peaks at T=10,20,40,80,160 with respect to the height of the peak at T=5.

TABLE 1

RATIOS OF THE ACTUAL HEIGHTS OF THE PEAKS AT T=10,20,40,80,160 WITH RESPECT TO THE HEIGHT OF THE PEAK AT T=5

LPEF	$\frac{T_{10}}{T_5}$	$\frac{T_{20}}{T_5}$	$\frac{T_{40}}{T_5}$	$\frac{T_{80}}{T_5}$	$\frac{T_{160}}{T_5}$
18	6.3	0.1	0.2	X	X
30	1.8	0.9	1.0	X	X
40	0.1	0.1	2.5	0.2	X
50	0.2	0.9	2.0	1.1	3.8
60	0.1	1.0	0.1	0.1	2.6

Thus, only for LPEF near 30, the ratios are near unity. On this criterion only, LPEF=30 could be considered as the most appropriate one. However, it fails to resolve peaks at T=80 and 160 as seen in Fig 2. This problem of relationship between input power signal and height of the peak needs detailed scrutiny.

To conclude from this mixed sample as also from a study of similar other samples (T values other than the present series 5,10, 20,40,80,160), the FABNE program does not seem to give results very much better than the Burg program. Where the FABNE program fails (LPEF about 65% of the data length), the Burg program still works, and at higher and higher LPEF, it generally yields better resolution for larger periods. However, the peak-splitting problem as also the problem of peak-shifting are very much there for high LPEF in the

Burg program and can be taken care of only by the elaborate extra computation proposed by Fougere (1977).

These results are surprising and somewhat discouraging in view of the fact that Barrodale and Erickson (1980a) have presented the results for at least one artificial sample, where the FABNE spectra showed resolution at lower LPEF as compared to Burg spectra. We reexamine here their sample. The sample test signal was of two sinusoids of frequencies 0.20 and 0.03, sampled 10 times per second, and 75 such data points were used for analysis. In terms of periods T, 75 data points each of 0.1 sec. are equivalent to a data length of 7.5 seconds. Thus, in a set of a fundamental period T=7.5, frequencies f=0.20 and 0.03, i.e. periods T=5.0 sec. and T=33.3 sec., were searched. Fig 3 shows the results. The FABNE spectra are obviously more accurate and detect the peaks correctly even at low LPEF.

Fig 4 shows spectra for a simple sinusoid of T=160, for a phase $\phi = 60^{0}$. FABNE spectra are more accurate and remained so even when ϕ was changed to $\phi = 0$ and $\phi = 120^{0}$.

Finally, we examine samples with triplets of amplitudes unity using the expression

$$\sum_{n=1}^{3} \mathrm{Sin}\left[2\pi \, (t/Tn) + \phi \right],$$

using 3 nearby values of the period T_n. Initially, the phase ϕ was set to zero and 101 data points with an accuracy of eight significant digits were produced (t=0 to 100) and a Gaussian noise of amplitude 0.01 was added. Fig 5 shows the results for five different sets of T_n triplets. The abscissa scale is common for all and is logarithmic in T with Δ (log T) steps of 0.005. Fig 5(a) shows the results for the triplet $T_1 = 4.5$, $T_2 = 5.0$ and $T_3 = 5.5$. For LPEF = 20 or more, the triplet is resolved, by both the methods. Thus, FABNE has no particular advantage over Burg method for this triplet. In Fig 5(b) for the triplet $T_1 = 9.0$, $T_2 = 10.0$, $T_3 = 11.0$, for LPEF = 20, the Burg spectra show only a doublet while FABNE spectra show a triplet. Thus, for this sample, FABNE method gives resolution at lower LPEF. In Fig 5(c) for the triplet T_1 = 18.0, T_2 = 20.0, T_3 = 22.0, at LPEF = 30, both methods show only a doublet. But at LPEF = 40, FABNE spectra show the triplet with better accuracy than the Burg spectra. In Fig 5(d) for the triplet T_1 = 36.0, T_2 = 40.0, T_3 = 44.0, up to LPEF = 40, only doublets are indicated. However, at LPEF = 65, the FABNE spectra still show only a doublet; but the Burg spectra show a triplet. At higher LPEF, FABNE program fails; but at LPEF = 90, the Burg spectra (Fig 5(d), bottom) show clear resolution of the triplet. Thus, for this sample, Burg spectra are superior to FABNE spectra. In Fig 5(e) for the triplet T_1 = 72.0, T_2 = 80.0, T_3 = 88.0, up to LPEF = 65, both methods show only a doublet. For higher LPEF, FABNE program fails. For LPEF = 90, the Burg spectra show some indication of a triplet (Fig 5(e), bottom) but the peaks are shifted from 72,80,88 to 76,88,107. Thus, Burg spectra, though superior to FABNE spectra for this sample, are not fully satisfactory even at very high LPEF.

For resolution of nearby peaks at frequencies, say f_1 and f_2, the data length necessary is roughly $1/(f_1 - f_2)$. In case of the Burg algorithm, because of an inherent smoothing procedure, this requirement is halved to $1/2\ (f_1 - f_2)$ (Ulrych and Bishop, 1975). The resolution seen in Fig 5(a),(b),(c), is as per expectation but for T = 40 ± 4 in Fig 5(d), the success of the Burg method in resolving the triplet was unexpected and shows the superiority of the Burg method over FABNE method in this case.

3. Conclusions

A comparison of the FABNE and Burg spectra for several artificial samples comprising of sinusoids revealed the following:

1) For single sinusoids or for peaks separated wide apart in frequency, the FABNE method gives much better frequency accuracy as compared to the Burg spectra, specially for periods comparable to or exceeding the fundamental period (data length). Peak splitting and peak-shifting are common in Burg spectra at

high LPEF and are dependent on the initial phase. The FABNE program fails at high LPEF (exceeding about 60% of data length); but high LPEF are generally not needed and accurate frequency determination is possible at lower LPEF, irrespective of the initial phase.

2) For resolution of nearby peaks in the Burg algorithm the criterion is that a data length of $1/2\ (f_1 - f_2)$ is needed to resolve two peaks at frequencies f_1 and f_2. This criterion is obeyed in the FABNE method too; but Burg spectra seem to be superior in this respect as lesser data length seems to be needed for similar resolution, though higher LPEF need to be used with the ever-present danger of peak-splitting and peak-shifting.

3) In samples with peaks in a wide range of frequecy, the FABNE method is only slightly better than the Burg method. For high periods comparable to or exceeding the data length, the Burg method shows frequency shifts as high as 25% while FABNE method may show shifts of about 15%.

Contemporarily but probably independently, Marple (1980) has reported an algorithm similar to that of Barrodale and Erickson (1980 a,b). We used it for the above samples and found that for some samples, results from both these algorithms were similar. However, this needs furthur scrutiny.

Acknowledgements

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CAPTIONS FOR FIGURES

Fig.1 - Burg spectra (dashed lines) and FABNE spectra (full lines) for 101 data points of an artificial sample of sinusoids with periods $T = 5,10,20,40,80,160$ all of the same amplitude and initial phase $\phi = 60^0$, with a (0.1%) Gaussian noise added. The abscissa scale is in constant steps of log T viz. $\Delta \log T = 0.005$ giving a step accuracy of about 1.2% for all T. LPEF = 18,40,60, for both spectra and LPEF = 80 for Burg spectra only.

Fig.2 - FABNE spectra for the artificial sample containing periods $T = 5,10,20,40,80,160$, all of same amplitudes with initial phase 60^0 and a 0.1% Gaussian noise. The abscissa scale is in constant steps of frequency, with $\Delta f = 0.0001$ for the low frequency region $f = 0.00$ to 0.03 (left half) and $\Delta f = 0.001$ for the high frequency region $f = 0.03$ to 0.25 (right half).

Fig.3 - Burg spectra (full lines) and FABNE spectra (dashed lines) for two sinusoids of frequencies 0.2 and 0.03 i.e., periods $T = 5.0$ and $T = 33.3$ (indicated by vertical dashed lines) for LPEF = 10 only for FABNE spectra and for LPEF = 10 and 30 for Burg spectra. Data length was 75 points and frequency step was $\Delta f = 0.001$. Initial phase ϕ was zero.

Fig.4 - FABNE spectra for LPEF = 6,20,30 (upper half,dashed lines) and Burg spectra for LPEF = 20,40,60,90 (lower half, full lines) for 101 data points of a single sinusoid of period $T = 160$, for an initial phase $\phi = 60^0$. Vertical line indicates the position where the peak is expected.

Fig.5 - Burg spectra (full lines) and FABNE spectra (dashed lines) for samples containing 3 nearby peaks at periods of (A) $T = 4.5, 5.0, 5.5$, (B) $T = 9, 10, 11$, (C) $T = 18, 20, 22$, (D) $= 36, 40, 44$ and (E) $= 72, 80, 88$ in data samples of 101 points, all sinusoids of the same amplitude. Noise of amplitude 1% was added.

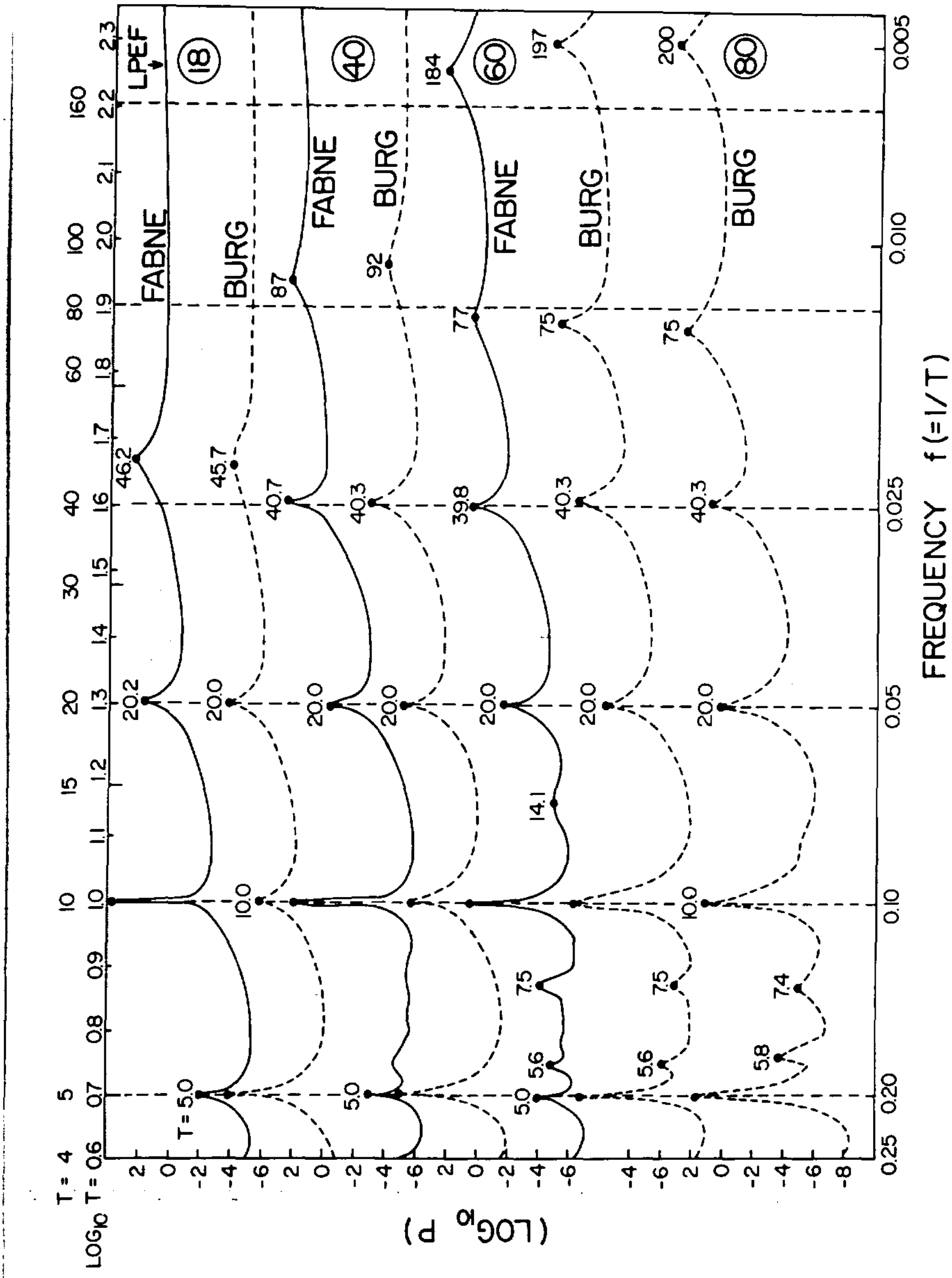

FABNE
BURG
LPEF
18
40
60
80
FREQUENCY f(=1/T)
(LOG_{10} P)
T = 4
LOG_{10} T = 0.6
T = 5.0

Fig. 1

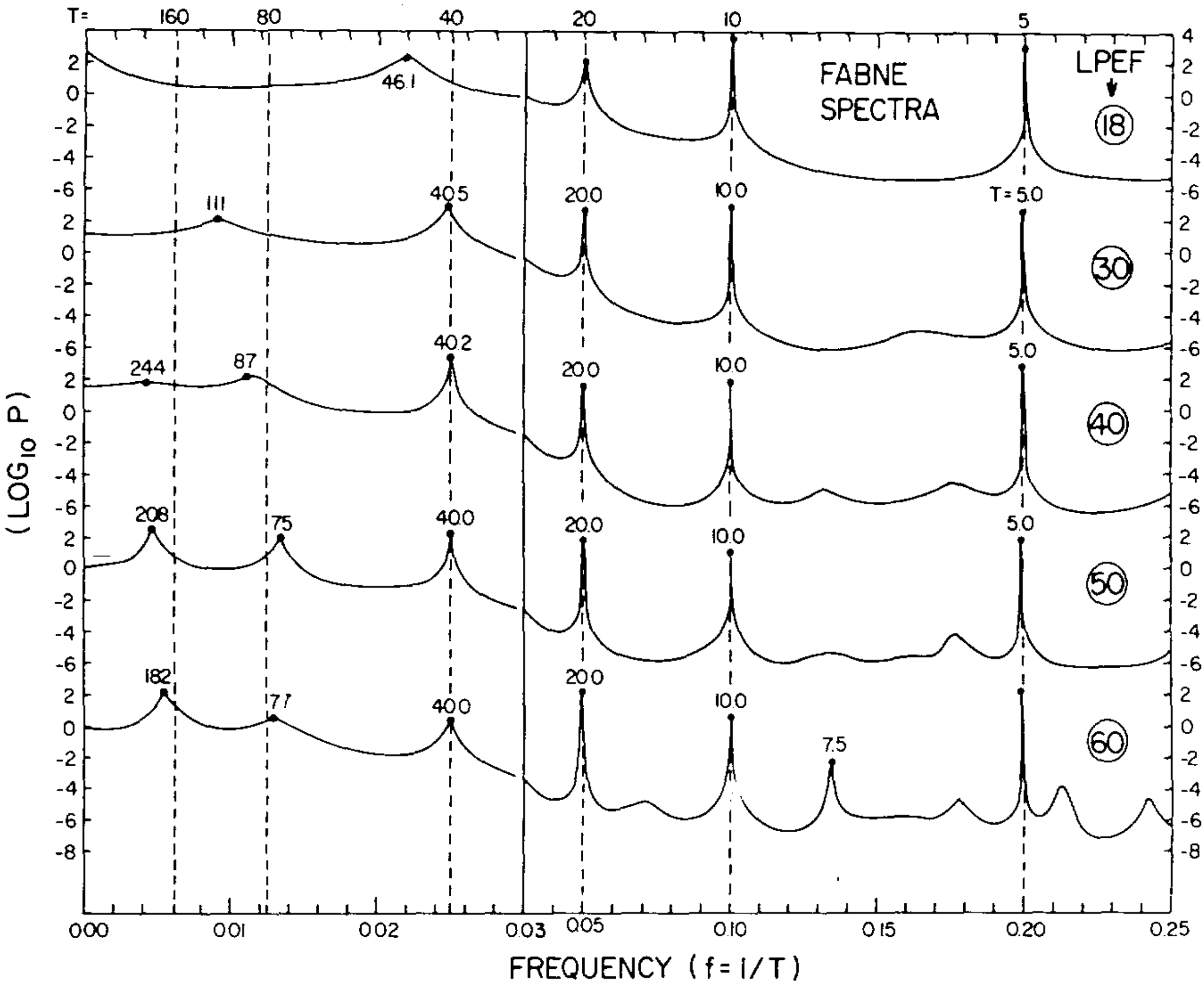

T=
160
80
40
20
10
5
FABNE
SPECTRA
LPEF
18
30
40
50
60
46.1
111
40.5
20.0
10.0
T=5.0
244
87
40.2
5.0
208
75
40.0
182
77
7.5
(LOG10 P)
FREQUENCY (f=1/T)

Fig. 2

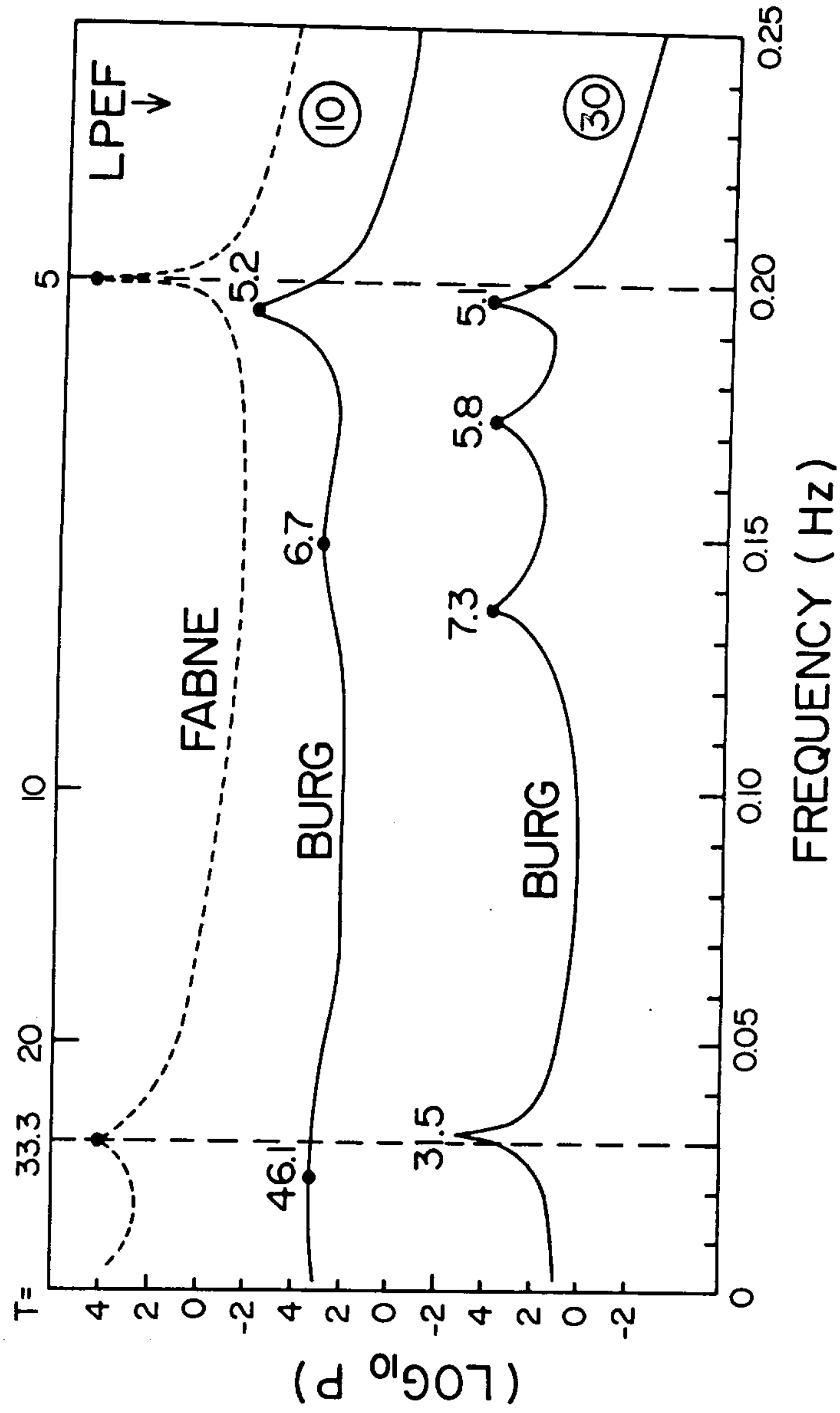
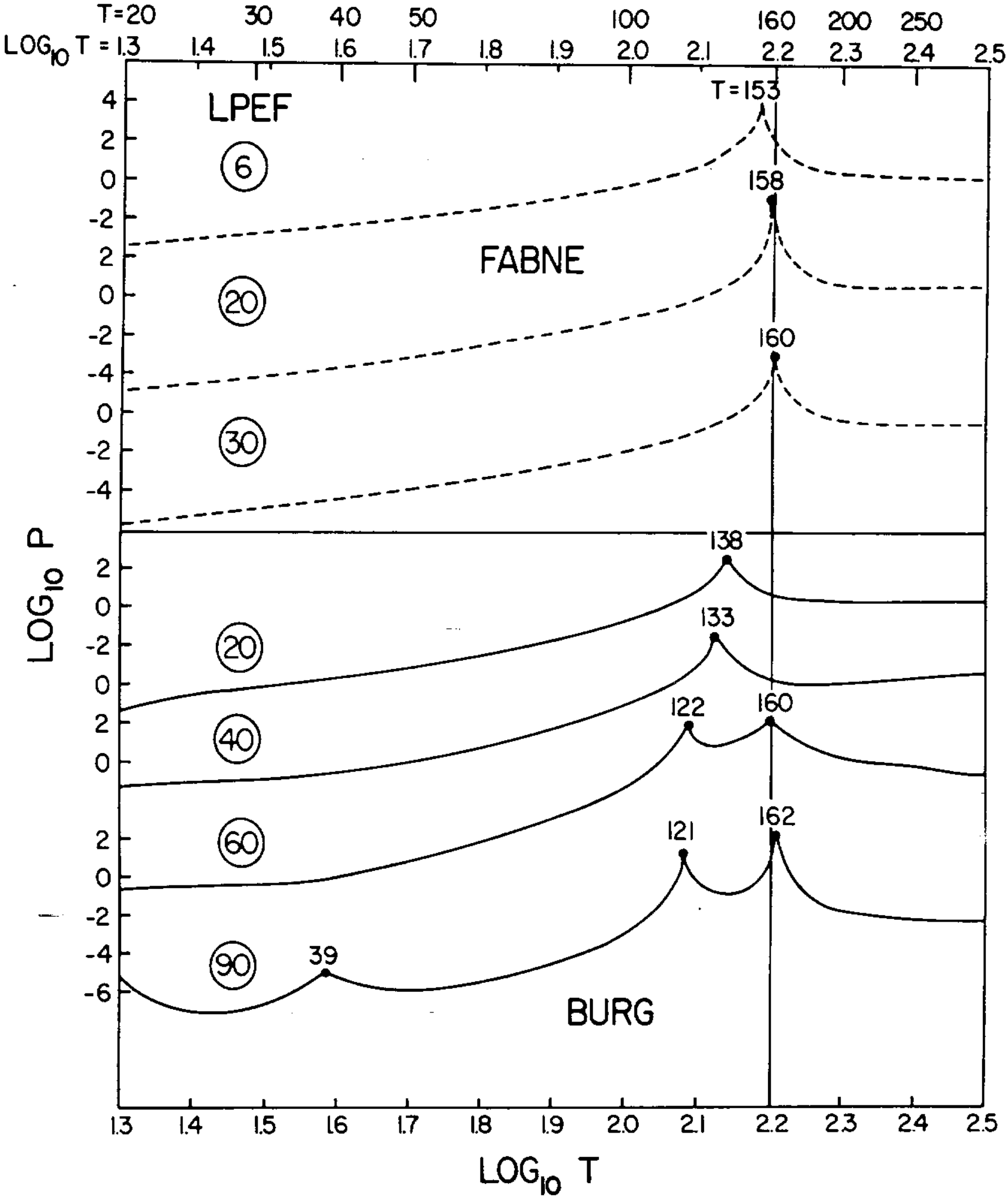

T=
5
10
20
33.3
LPEF
FABNE
BURG
BURG
10
30
5.2
6.7
46.1
5.1
5.8
7.3
31.5
4
2
0
-2
(LOG10 P)
0
0.05
0.10
0.15
0.20
0.25
FREQUENCY (Hz)

Fig. 3



T=20
30
40
50
100
160
200
250
LOG10 T = 1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
LPEF
6
T=153
158
FABNE
20
160
30
LOG10 P
138
20
133
40
122
160
60
121
162
90
39
BURG
LOG10 T

Fig. 4

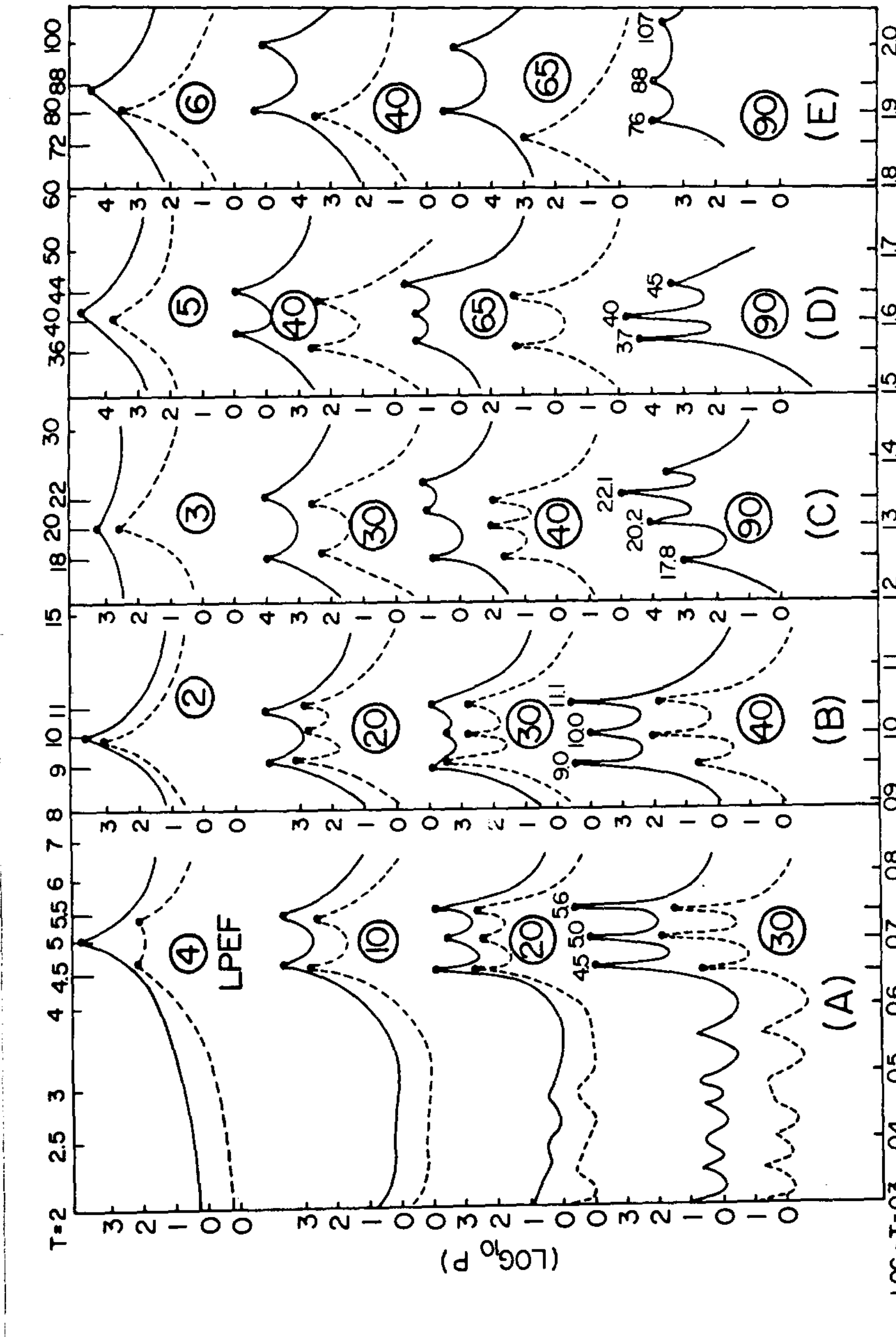

(LOG10 P)
LPEF
(A)
(B)
(C)
(D)
(E)
T=2
LOG10 T=0.3
f=0.500
0.200
0.100
0.050
0.025
0.010

Fig. 5

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