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DETERMINATION OF CONDUCTIVE THERMAL RESISTANCE OF PLATES WITH NONUNIFORM INTERNAL HEAT GENERATION AND UNDER VARIOUS BOUNDARY CONDITIONS

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### ABSTRACT

The conductive resistance of thin rectangular plates with internal heat generation in two regions, with one end subject to an internal flux is determined using the finite element method. The dependance of the thermal resistance on various parameters is discussed. A comparison is made with the one dimensional resistance.

## INTRODUCTION

The motivation for this work arose while trying to make a thermal analysis of the first Brazilian satellite. The thermal analysis of satellite is done by dividing the satellite into many isothermal nodes and the knowledge of the internodal conductive resistance or conductive coupling coefficients is essential for the thermal design and analysis. The calculation of these coupling coefficients is often difficult because of geometrical complexities and uncertainties about

the contact resistance. Often in satellite thermal control design, for lack of information, a one-dimensional approximation is made.

Oliveira and Forslund (1974) considered a convectively cooled plate with constant heat flux and no internal heat generation. Schneider et al. (1980) extended the same to include the case of nonuniform heat flux, but again with no internal heat generation. Venkataraman et al. (1981) extended the problem to include internal heat generation. Three different internal heat generation cases were considered and thermal resistance was calculated for the various cases. The objective of this work is to find the thermal resistance between two heat generating boxes (which may be the heat dissipation by electronic components) mounted on a thin rectangular plate as shown in Figure 1. One of the sides is subjected to an external heat flux, the adjacent side is at a constant temperature and all other sides are maintained at zero temperature. If the other sides are at a non-zero constant temperature, the problem can be reduced to the present one by using superposition.

#### FORMULATION

The geometry is shown in Figure 1. We consider a thin rectangular plate of length a and width b, with the coordinate axes as shown. The plate is subjected to a constant internal heat generation  $g_1$  per unit volume confined to the region  $x_1 \le x \le x_2$ ,  $y_1 \le y \le y_2$  and constant heat generation of  $g_2$  per unit volume confined to  $x_3 \le x \le x_4$ ,  $y_3 \le y \le y_4$ . The bottom side is subjected to constant heat flux  $q_0$ . The side x=0 is maintained at a constant temperature. The problem is to find the thermal resistance between the two heat generating boxes. The governing differential equation for the temperature  $\theta(x,y)$  is the Poisson equation given by

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = -\frac{g(x,y)}{k}$$
 (1)

where k is the plate thermal conductivity, g(x,y) is the internal heat generation per unit volume:

$$\theta(0,y) = \theta_0 \tag{1-a}$$

$$\theta(\mathbf{x},\mathbf{b}) = 0 \tag{1-b}$$

$$\theta(\mathbf{a},\mathbf{y}) = 0 \tag{1-c}$$

$$-k\left(\frac{\partial\theta}{\partial x}\right)_{y=0} = q_0 \tag{1-d}$$

## THE SOLUTION

The finite element method (Chung, 1978) has been used because it offers flexibility in choosing various types of internal generation and boundary conditions, and there is better control an error propagation.

The functional corresponding to the Euler's Equation (1) is (Heuser, 1973)

$$\mathbf{I} = \mathbf{k} \, \mathbf{l} \int_{\mathbf{S}} \left[ \frac{1}{2} \left( \frac{\partial \theta}{\partial \mathbf{x}} \right)^2 + \frac{1}{2} \left( \frac{\partial \theta}{\partial \mathbf{y}} \right)^2 \right] d\mathbf{s} - \mathbf{l} \int_{\mathbf{S}} \mathbf{g}(\mathbf{x}, \mathbf{y}) \, \theta d\mathbf{s} + \mathbf{l} \int_{\mathbf{F}} \mathbf{q}(\mathbf{x}) \, \theta d\mathbf{F}$$
 (2)

Here q(x) is the external heat flux and  $\ell$  is the plate thickness. The first two integrals are over the plate area and the last one is on the plate boundary. The problem has been solved starting from this function using the method of finite elements based on the method of Ritz. A triangular element with six nodes and quadratic interpolation have been used. The interpolation function was selected such that the second derivatives of the temperature with respect to x and y are constant. The condition of the temperature continuity along the boundaries of the triangle was imposed, with no restriction on the normal derivatives. One could presumably obtain better results by using the Hermite interpolation polynomials with continuous first derivatives across the interelement boundaries.

The solution of the problem in terms of the shape functions N. are

$$\theta(\mathbf{x},\mathbf{y}) = \sum_{j=1}^{6} N_{j} (\mathbf{x},\mathbf{y})\theta_{j}$$
(3)

The shape functions in terms of the area coordinates  $\xi_1$ ,  $\xi_2$  and  $\xi_3$  are

$$N_1 = \xi_1(2\xi_1 - 1),$$
  $N_2 = \xi_2(2\xi_2 - 1),$   $N_3 = \xi_3(2\xi_3 - 1)$  (4)  
 $N_4 = 4\xi_1\xi_2,$   $N_5 = 4\xi_2\xi_3,$   $N_6 = 4\xi_3\xi_1.$ 

By dividing the domain and the boundaries into finite subdonains, we can write the function as

$$I = \sum_{e=1}^{n} I^{e}$$
 (5)

where the superscript e corresponds to the element and n the number of elements. Substituting Equations (2) and (3) in Equation (5) transforms the problem from function space to  $\mathbb{R}^N$  space, where the variables are  $\theta_1, \theta_2, \ldots, \theta_N$ . The problem of finding the function that makes the functional stationary is reduced to the problem of obtaining the points of  $\mathbb{R}^N$  that will make the function a minimum. Thus:

$$\frac{\partial \mathbf{I}}{\partial \theta_{\mathbf{k}}} \delta \theta_{\mathbf{k}} = 0, \qquad \mathbf{k} = 1, \dots, \mathbf{N}$$
 (6)

from which we obtain a system of n equations

$$\frac{\partial I}{\partial \theta_k} = 0 \qquad k = 1, \dots, N \tag{7}$$

Solving this equation, we obtain the solution in each element (subdomain), in the form

$$\theta^{E}(x,y) = \sum_{i=1}^{6} N_{i}(\xi_{1},\xi_{2},\xi_{3})\theta_{j}^{e}$$
 (8)

where  $\theta_{j}^{e}$  are the values of  $\theta_{k}$  at the nodal element e.

Substituting from Equation (5) into Equation (6) we get

$$\frac{\partial}{\partial \theta_{k}} = \sum_{e} I^{e} = 0 \quad \text{ou} \quad \sum_{e} \frac{\partial I^{e}}{\partial \theta_{i}} = 0$$
 (9)

where  $\partial I^e/\partial \theta_i$  for each i corresponds to the nodes of the elements and forms the matrix of element e.

 $\theta_i^e$  (i=1,...,6) being the nodal parameter of element e, we have, for the problem considered, six expressions of the type

$$\frac{\partial I^{e}}{\partial \theta_{i}^{e}} \qquad i=1,\dots,6$$
 (10)

Substituting Equation (2) in Equation (10)

Using Equation (8), the first term of Equation (11) becomes

$$\frac{\partial \theta}{\partial \mathbf{x}} = \frac{\partial \theta}{\partial \mathbf{u}} = \frac{6}{3} \left( \frac{\partial \mathbf{N}_{\mathbf{j}}}{\partial \mathbf{x}} \right) = \frac{6}{3} \left( \frac{\partial \mathbf{N}_{\mathbf{j}}}{\partial \mathbf{x}} \right) = \frac{6}{3} \left( \frac{3}{2} \frac{\partial \mathbf{N}_{\mathbf{j}}}{\partial \mathbf{x}} - \frac{\partial \xi_{\mathbf{j}}}{\partial \mathbf{x}} \right) \frac{\partial \mathbf{N}_{\mathbf{j}}}{\partial \xi_{\mathbf{m}}} - \frac{\partial \xi_{\mathbf{m}}}{\partial \mathbf{x}} \right) \theta_{\mathbf{j}}$$
(12)

and similarly for y. From the relation between the cartesian and natural coordinates

$$\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}$$
(13)

which can be writeen as

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$$
(14)

we obtain

$$\frac{\partial \xi_{\ell}}{\partial x} = \frac{g_{\ell 2}}{\Delta} , \qquad \frac{\partial \xi_{m}}{\partial x} = \frac{g_{m^{2}}}{\Delta}$$

$$\frac{\partial \xi_{\ell}}{\partial y} = \frac{g_{\ell 3}}{\Delta} , \qquad \frac{\partial \xi_{m}}{\partial y} = \frac{g_{m^{3}}}{\Delta}$$
(15)

where  $\Delta$  is the determinant of the matrix of Equation (13) .

Using Equations (10), (11) and (15) we get

$$\frac{\partial I^{e}}{\partial \theta_{i}} = \frac{k\ell}{\Delta^{2}} \int_{j=1}^{6} \left[ \sum_{\ell} \sum_{m} (g_{\ell 2} g_{m2} + g_{\ell 3} g_{m3}) \int_{S}^{\frac{\partial N_{i}}{\partial \xi_{\ell}}} \frac{\partial N_{j}}{\partial \xi_{m}} \right] \theta_{j} - \ell \int_{S} g(x,y) N_{i} ds + \ell \int_{F} q(x) N_{i} dF \qquad i=1,2,...,6$$
(16)

The above six equation can be written in matrix notation as

$$[K^e] \{\theta^e\} + \{F^e\} = 0$$
 (17)

where

$$K_{ij}^{e} = \frac{k \ell}{\Delta^{2}} \sum_{\ell m} \left( g_{\ell 2} g_{m2} + g_{\ell 3} g_{m3} \right) \int_{S}^{\frac{\partial N_{i}}{\partial \xi_{\ell}}} \frac{\partial N_{j}}{\partial \xi_{m}} ds$$
 (18)

$$\mathbf{F}_{i}^{e} = \mathbf{F}_{q_{i}}^{e} + \mathbf{F}_{g_{i}}^{e} \tag{19}$$

$$\mathbf{f}_{\mathbf{i}}^{\mathbf{e}} = + i \int_{\mathbf{F}}^{\mathbf{q}(\mathbf{x})} \mathbf{N}_{\mathbf{i}} d\mathbf{F}$$
 (20)

$$\mathbf{F}_{\mathbf{g}_{\mathbf{i}}}^{\mathbf{e}} = - l \int_{\mathbf{S}} \mathbf{g}(\mathbf{x}, \mathbf{y}) \, \mathbf{N}_{\mathbf{i}} \, d\mathbf{S}$$
 (21)

The value of the matrix elements are shown in appendix A.

Following an analogous approach for the remaining nodes and using Equation (7), we have the following final system of matrix equations.

where the matrices K, 0 and F correspond to the entire plate.

#### NONDIMENSIONALIZATION

To have more generality, the variables are nondimensionalized as follows.

Non dimensional lengths 
$$\bar{x} = \frac{x}{a}$$
,  $\bar{y} = \frac{y}{a}$ ,  $\epsilon = \frac{b}{a}$ ,  $\bar{x}_1 = \frac{x_1}{a}$ ,  $\bar{x}_2 = \frac{y_2}{a}$ , ... etc.

Non dimensional temperature  $\theta * = \frac{k \ell \theta}{Q + Q_V}$ 

where

Q = qoal is the total external heat flow

$$Q_{V} = i \int_{\mathbf{x}_{1}}^{\mathbf{y}_{2}} \int_{\mathbf{y}_{1}}^{\mathbf{y}_{2}} dx dy + i \int_{\mathbf{x}_{2}}^{\mathbf{y}_{2}} \int_{\mathbf{x}_{3}}^{\mathbf{y}_{3}} dx dy$$

is the total internal heat generation.

Ratio of internal heat generation to external heat flux,  $G_1 = ag_1/q_0$ ,  $G_2 = ag_2/q_0$ .

Boundary condition parameter  $C = k\theta_0/aq_0$ .

The matrix {F} can be written as

$$\{F\} = \{F_{g_2}\} + \{F_{g_2}\} + \{F_q\}$$
 (23)

where F and F are source matries due to  $g_1$  and  $g_2$ , and  $F_q$  is the matrix due to the heat flux, for the entire plate.

Substituting in Equation (22), we have the nondimensional form of the matrix equation

$$[K^*]\{\theta^*\} + G\{F_q^*\} + G_1G\{F_{q_1}^*\} + G_2G\{F_{q_2}^*\} = 0$$
 (24)

where

$$G = \frac{1}{1 + G_1(\bar{x}_2 - \bar{x}_1)(\bar{y}_2 - \bar{y}_1) + G_2(\bar{x}_4 - \bar{x}_3)(\bar{y}_4 - \bar{y}_3)}$$
(25)

and the asterisks represent the nondimensionalized matries.

# NONDIMENSIONAL THERMAL RESISTANCE

The nondimensional thermal resistance R is defined as

$$R = \frac{\overline{\theta}_1^* - \overline{\theta}_2^*}{|\vec{Q}_{12}|} \tag{26}$$

where  $\overline{\theta}_1^*$  and  $\overline{\theta}_2^*$  are the average temperatures of the zones of heat generation  $g_1$  and  $g_2$ , and  $Q_{12}$  is the nondimensional heat flow from region 1 to region 2.

$$Q_{12} = \left[ \left\{ \int_{\overline{y}_{A}}^{\overline{y}_{B}} \frac{\partial \theta^{*}}{\partial \overline{x}} d\overline{y} \right\}^{2} + \left\{ \int_{\overline{x}_{A}}^{\overline{x}_{B}} \frac{\partial \theta^{*}}{\partial \overline{y}} d\overline{x} \right\}^{2} \right]^{\frac{1}{2}}$$

$$\left[ \left\{ \int_{\overline{y}_{A}}^{\overline{y}_{D}} \frac{\partial \theta^{*}}{\partial \overline{x}} d\overline{y} \right\}^{2} + \left\{ \int_{\overline{x}_{A}}^{\overline{x}_{D}} \frac{\partial \theta^{*}}{\partial \overline{y}} d\overline{x} \right\}^{2} \right]^{\frac{1}{2}}$$
(28)

where  $(\bar{x}_A, \bar{y}_A)$ ,  $(\bar{x}_B, \bar{y}_B)$ ,  $(\bar{x}_C, \bar{y}_C)$ ,  $(\bar{x}_D, \bar{y}_D)$  are the coordinates of the limiting lines of heat flux. These limiting lines are obtained graphically as shown in Figure 2 for a typical case. The isotherms are normalized between 0 and 100 and are plotted by a Calcomp plotter. The limiting flux lines (perpendicular to the isotherms) between the two regions are then drawn and thus the coordinates of the points A, B, C and D, on the boundaries of these heat generating regions are determined.

#### RESULTS AND DISCUSSIONS

Equation (22) was solved by the method of Crout and Banachiewicz (Crandall, 1956) by dividing the plate into 400 elements. For confirmation purposes the problem was also solved by the finite difference method and the comparison has been very good. The nondimensional thermal resistance has been calculated as a function of the parameters C,  $G_1$ ,  $G_2$ ,  $\varepsilon$ , size and position of the heat generating regions. For simplicity, the size of these regions, the value of  $\epsilon$  and their relative position in the y direction are fixed, and analysis is made by varying  $G_1$ ,  $G_2$ ,  $\epsilon$  and their relative position in x direction. For comparison purposes the nondimensional one dimensional resistance  $R_{1D}$  between the two regions is also calculated. Figures 3, 4 and 5 show the behabiour of R and the percetage difference between R,D and R against  $G_1/G_2$  for various values of C with  $\varepsilon$  and  $\bar{x}_3 - \bar{x}_2$  fixed. Figures 6 and 7 show the variation of these resistances and the percentage difference against  $\bar{x}_3 - \bar{x}_2$  with  $G_1/G_2$  fixed but for different values of C. Figure 2 shows isotherms for a typical case. For small values of  $G_1/G_2$  the resistance is considerably larger than the one dimensional resistance, but rapidly decreases to a constant value whichis less than the one dimensional resistance. The resistance tends to increase with C, because an increase in C implies a higher temperature gradient along the plate. The resistance increases as the regions of heat generation are moved apart, steeply incresting to infinity at some separation distance. The one dimensional resistance as expected increases lineary with the separation distance.

In conclusion, the nondimensional resistance can considerably differ from the nondimensional one dimensional resistance depending upor plate geometry and the boundary conditions.

## APPENDIX A

From Equation (18) we have for the elements of the matrix [Ke]

$$K_{11}^{e} = \frac{1}{2\Lambda} (g_{12}^{2} + g_{13}^{2})$$

$$K_{12}^{e} = K_{21}^{e} = -\frac{1}{6\Delta} (g_{12}g_{22} + g_{13}g_{23})$$

$$K_{13}^e = K_{31}^e = -\frac{1}{6\Delta} (g_{12}g_{32} + g_{13}g_{33})$$

$$K_{14}^{e} = K_{41}^{e} = \frac{2}{3\Delta} (g_{12}g_{22} + g_{13}g_{23})$$

$$K_{15}^e = K_{51}^e = 0$$

$$K_{16}^{e} = K_{61}^{e} = \frac{2}{3\Delta} (g_{12}g_{32} + g_{13}g_{33})$$

$$K_{22}^e = \frac{1}{2\Lambda} (g_{22}^2 + g_{23}^2)$$

$$K_{23}^e = K_{32}^e = -\frac{1}{6\Delta} (g_{22}g_{32} + g_{23}g_{33})$$

$$K_{24}^{e} = K_{42}^{e} = -\frac{2}{3\Delta} (g_{12}g_{22} + g_{13}g_{23})$$

$$K_{25}^e = K_{52}^e = -\frac{2}{3\Delta} (g_{22}g_{32} + g_{23}g_{33})$$

$$K_{26}^{e} = K_{62}^{e} = 0$$

$$K_{33}^e = \frac{1}{2\Delta} (g_{32}^2 + g_{33}^2)$$

$$K_{34}^e = K_{43}^e = 0$$

$$K_{35}^e = K_{53}^e = \frac{2}{3\Lambda} (g_{22}g_{32} + g_{23}g_{33})$$

$$K_{36}^{e} = K_{63}^{e} = \frac{2}{3\Delta} (g_{12}g_{32} + g_{13}g_{33})$$

$$K_{44}^{e} = \frac{4}{3\Delta} (g_{12}^{2} + g_{13}^{2} + g_{12}g_{22} + g_{13}g_{23} + g_{22}^{2} + g_{23}^{2})$$

$$K_{45}^{e} = K_{54}^{e} = -8 K_{13}$$

$$K_{46}^{e} = K_{64}^{e} = -8 K_{23}$$

$$K_{55}^{e} = \frac{4}{3\Delta} (g_{22}^{2} + g_{23}^{2} + g_{22}g_{32} + g_{23}g_{33} + g_{32}^{2} + g_{33}^{2})$$

$$K_{56}^{e} = -8 K_{12}$$

Considering that g(x,y) is defined in two regions, we have the following elements for the matrix  $\{F^e\}$ 

 $K_{66}^{e} = \frac{4}{3\Lambda} (g_{12}^{2} + g_{13}^{2} + g_{12}g_{32} + g_{13}g_{33} + g_{32}^{2} + g_{33}^{2})$ 

$$F_{g_1}^e = 0$$
,  $F_{g_2}^e = 0$ ,  $F_{g_3}^e = 0$ ,  $F_{g_4}^e = F_{g_5}^e = F_{g_6}^e = -\frac{g_1^\Delta}{6}$ 

where  $g_i = g_1$  for region 1 and  $g_i = g_2$  for region 2.

From Equation (20) we obtain the matrix elements of  $\{F_q^e\}$  as

$$F_{q_1}^e = \frac{q_0^L}{6}$$
,  $F_{q_2}^e = \frac{q_0^L}{6}$ ,  $F_{q_3}^e = 0$ ,  $F_{q_4}^e = \frac{2q_0^L}{3}$ ,

$$\mathbf{F}_{\mathbf{q}_5}^{\mathbf{e}} = \mathbf{F}_{\mathbf{q}_6}^{\mathbf{e}} = \mathbf{0}.$$

where L is the characteristic dimension of the element perpendicular to the direction of the specified flux  $q_0$ .

## LIST OF SYMBOLS

9 : Plate temperature.

 $\theta_0$ : Temperature of one side of plate.

 $\theta_1, \dots, \theta_N$ : Temperature parameter in  $R^N$  space.

0.07,02 : Nondimensional temperature.

e : Plate geometric parameter.

 $\xi_1, \xi_2, \xi_3$ : Area coordinates.

a,b : Plate dimensions.

C : Boundary condition parameter.

g(x,y) : Internal heat generation.

g1,g2 : Internal heat generation in the two regions.

811,812,...,866 : Matrix elements of Eq. (14).

G<sub>1</sub>,G<sub>2</sub> : Nondimensional heat generation parameter.

i,j,k : Suffixes.

 $K_{ij}^e, F_{qi}^e, F_{gi}^e$ : Elements of the generalized matrices  $K^e, F_q^e$  and  $F_g^e$ .

2 : Plate thickness.

N : Total number of nodes.

n : Total of number of elements.

 $N_1, \dots, N_5$ : Shape functions.

q(x) : External heat flux.

constant external heat flux.

Q : Total external heat flux.

 $\mathbf{Q_v}$ : Total internal heat generation.

R : Nondimensional thermal resistance.

R : Nondimensional one dimensional thermal resistance.

x,y : Rectangular coordinates.

x.y : Nondimensional coordinates.

 $(\bar{x}_1,\bar{y}_1),(\bar{x}_2,\bar{y}_2)$ : Nondimensional coordinates of region 1.

 $(\bar{x}_3,\bar{y}_3),(\bar{x}_4,\bar{y}_4)$ : Nondimensional coordinates of region 2.

 $(\bar{x}_A, \bar{y}_A)$ ....: Coordinates of limiting heat flux lines.

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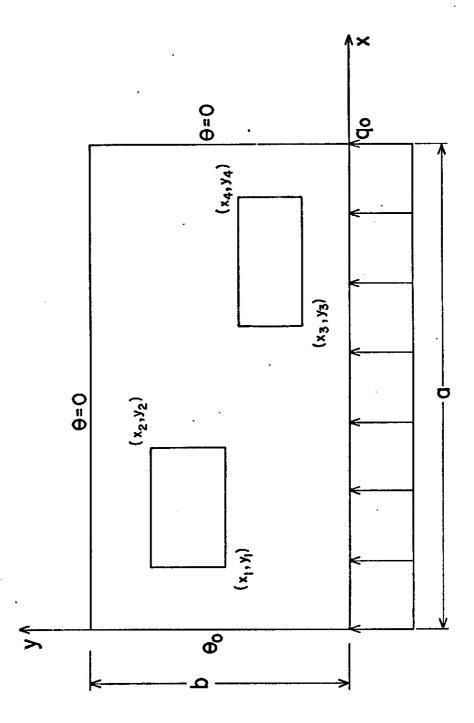


FIGURE 1 - THE GEOMETRY

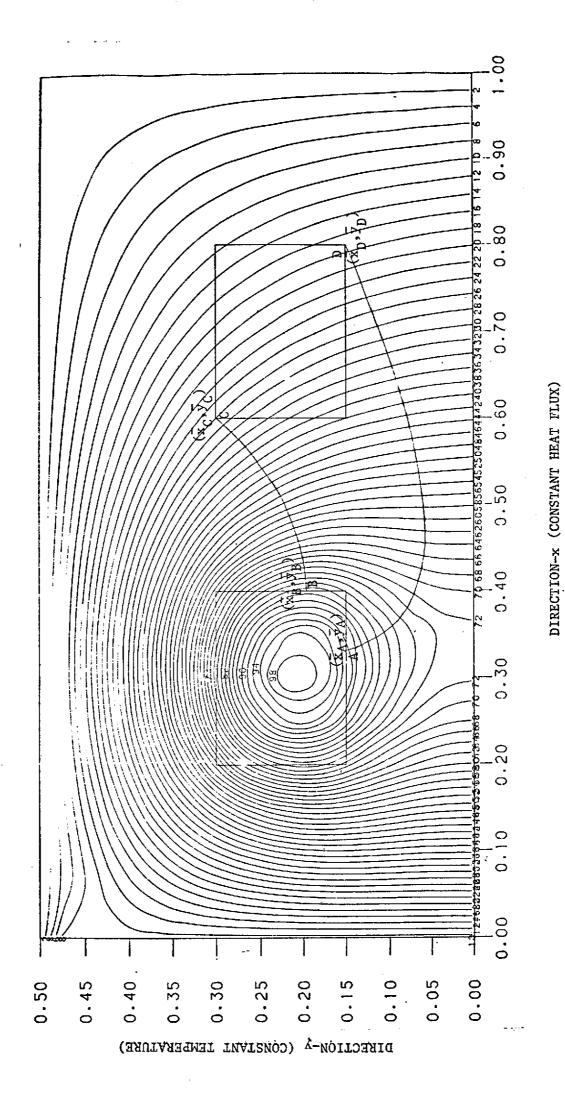
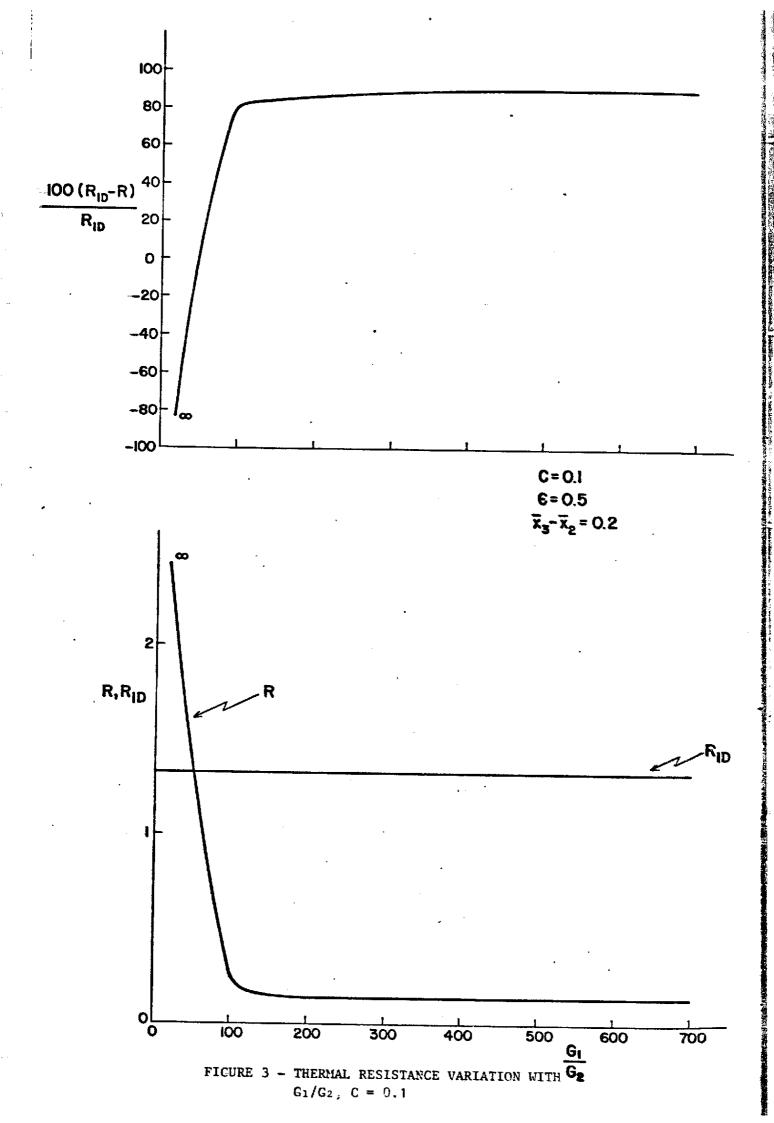
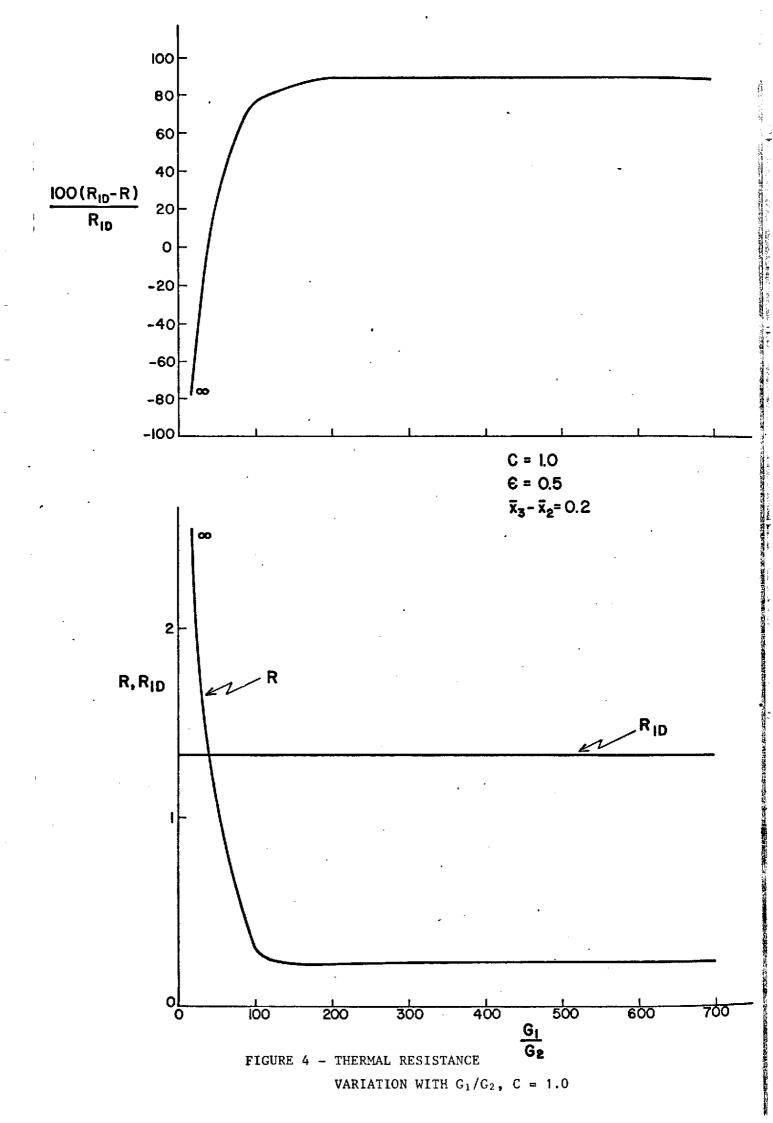
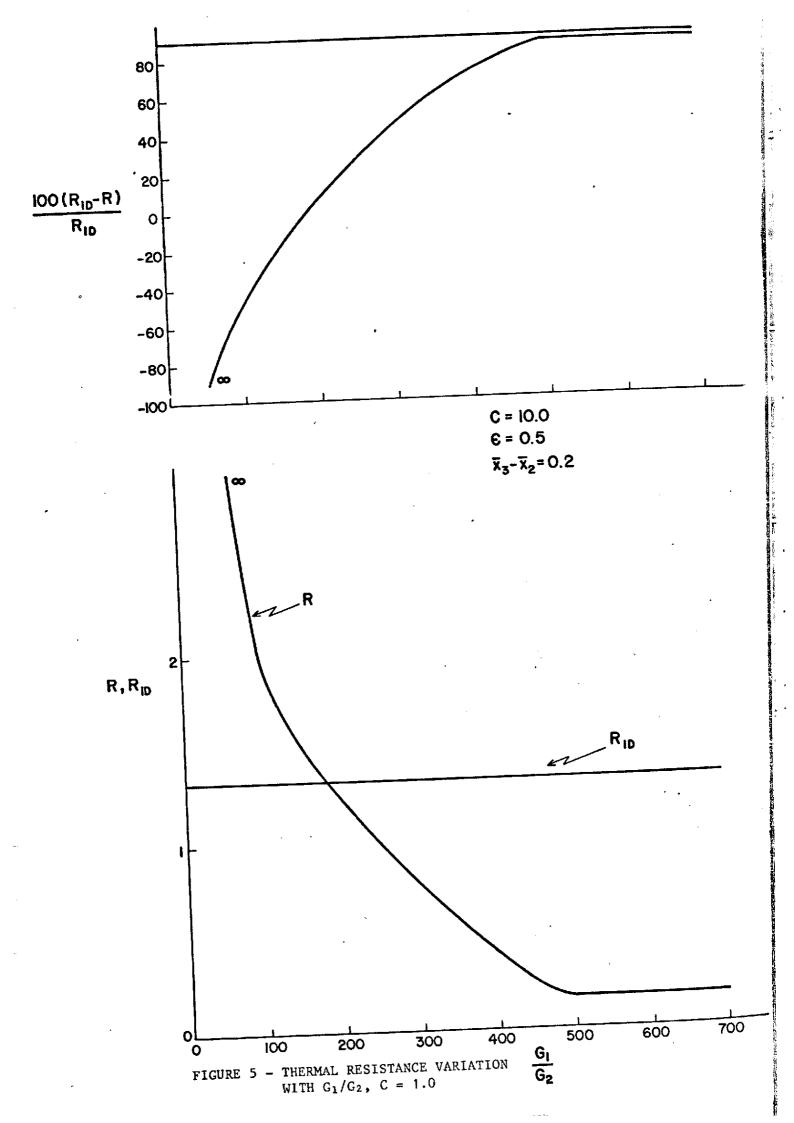
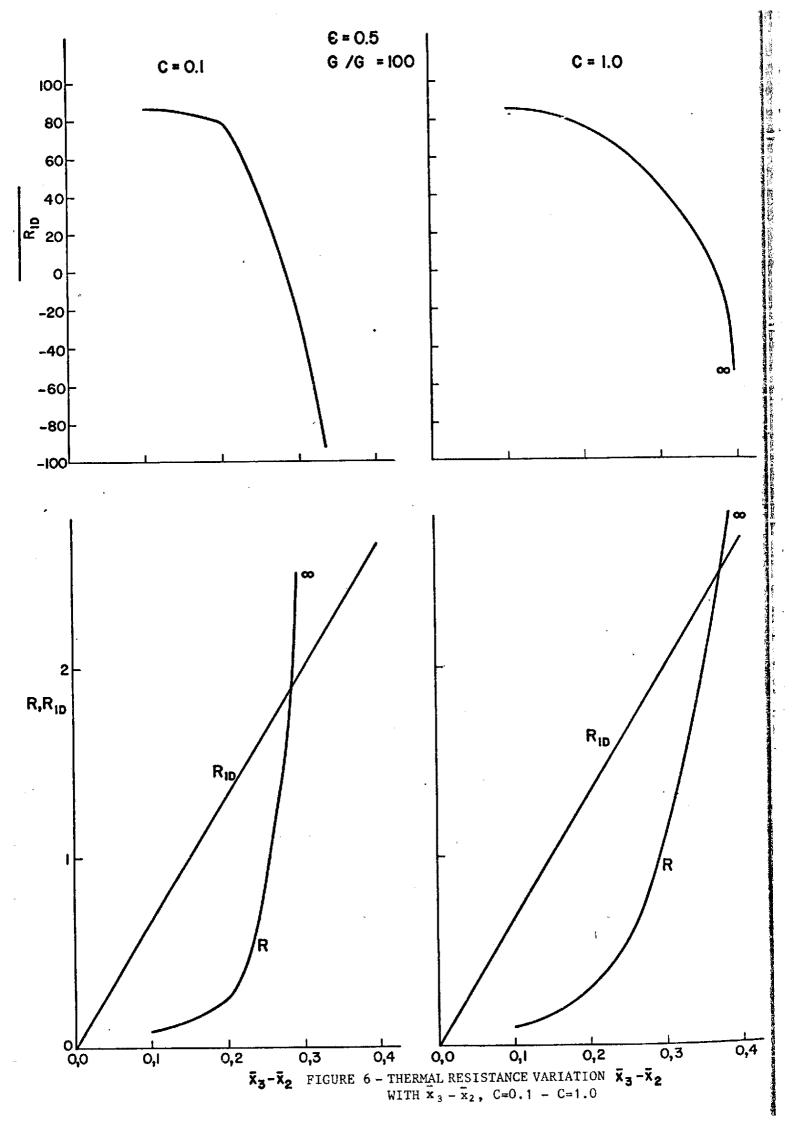


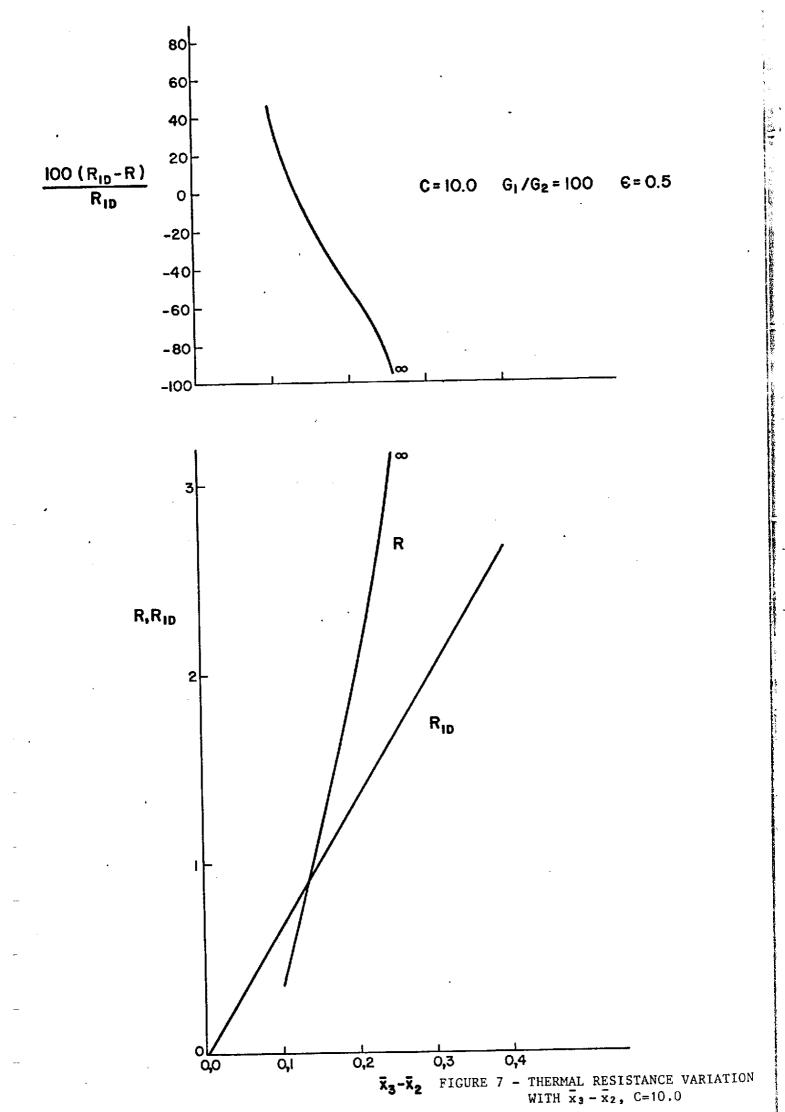
FIGURE 2 - THE DISTRIBUTION OF NORMALIZED TEMPERATURE













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SANTA FE, Setiembre 13 de 1982

Doctor Nelson de Jesús Parada INPE Caixa Postal 515 12200 - Sao José dos Campos BRASIL

Por la presente acuso recibo del manuscrito final del trabajo: "DETERMINATION OF CONDUCTIVE THERMAL RESISTANCE OF PLATES WITH NONUNIFORM INTERNAL HEAT GENERATION AND UNDER VARIOUS BOUNDARY CONDITIONS". O.B.Oliveira Filho, S.A.Tavares y N.S.Venkataraman.

el cual ha de ser publicado en las "Actas" del I Congreso Latinoamericano de Transferencia de Calor y Materia.

Agradeciéndole su envío hago propicia la oportunidad para saludarlo muy atentamente.

OR- JUAN PERTO BRESSAN

JAB:eg/ez



26 July, 1982.

Dr. Juan Alberto Bressan Secretario Ejecutivo Comite Argentino de Transferencia de Calor y Materia Casilla de Correo nº 91 3000, Santa Fé República Argentina Ref.: 30.650.200.006/82

Dear Dr. Bressan:

I thank you for informing that our papers, 1) Determination of Conductive Thermal Resistance of Plates With Non-uniform Internal Heat Generation and Under Various Boundary Conditions and 2) Wall Temperature Distribution in a Cylindrical Thermo-vacuum Chamber, have been accepted for presentation in the congress.

I am sending again the abstracts of these two papers prepared as per your format.

Thanking you once again.

Sincerely yours

Nellore S. Venkataraman

Mothertaramen

NSV/abg

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DETERMINATION OF CONDUCTIVE THERMAL RESISTANCE OF PLATES WITH NON-UNIFORM INTERNAL HEAT GENERATION AND UNDER VARIOUS BOUNDARY CONDITIONS

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### ABSTRACT

The determination of the equivalent thermal resistance is important in thermal analysis and has many engineering applications. For example, in satellite thermal analysis the structure is divided into a large number of isothermal nodes and the knowledge of internodal thermal resistance is necessary for thermal analysis as well as thermal simulation. Often many of these nodes may include sources of heat dissipation like electronic components and this can be approximated as internal heat generation. This work determines the thermal conductive resistance of rectangular plates with non-uniform internal heat generation with one end subject to an external heat flux and with temperature specified on the other ends. The internal heat generation is considered in two regions, which represent thermal dissipation by two electronic boxes on the mounting plate. We are interested in determining the thermal resistance between these boxes. The governing differential equation for the temperature distribuition is solved numerically by finite difference and finite element methods and the results are compared. The nondimensional thermal resistance is a function of the plate geometry, the ratio of total internal heat generation to total heat flux, their relative positions and a parameter which relates the temperature boundary condition to the heat flux boundary condition. The influence of these parameters is discussed. A comparison is also made with the one dimensional resistance which is sometimes used for quick estimates.