MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

INPE-8068-PRE/3884

A STUDY OF THE STATIONKEEPING MANEUVERS FOR THE CBERS SATELLITE

Antonio Fernando Bertachini de Almeida Prado

Paper presented at the German-Brazilian Workshop on Aerospace Technology,
São José dos Campos, Brazil, Sept. 29 to Oct. 03, 1997.

INPE
São José dos Campos
2001

A Study of the Stationkeeping Maneuvers for the CBERS satellite

1. Introduction

The objective of this report is to study the stationkeeping maneuvers that will have to be performed by the CBERS (China-Brazil Earth Resources Satellite) satellite. The CBERS is a remote sensing satellite that is under development by China and Brazil. This satellite is planned to stay in a polar frozen orbit and stationkeeping maneuvers will have to be performed to keep its orbital elements in a specified range. This report will be devoted to study the correction of the semi-major axis, eccentricity and argument of periapse, that are three of the Keplerian elements that will change due to the orbit perturbations.

An algorithm capable of calculating the bi-impulsive transfer orbit with minimum ΔV between any two Keplerian and coplanar orbits is developed and used to calculate the maneuvers expected for the CBERS satellite. Several maneuvers are simulated, both to change the orbital elements individually or in groups. This algorithm is based on the minimum ΔV Lambert's problem shown in Prado (1993) and Prado & Broucke (1994). In the format that it is implemented here it is possible to include constraints in the region of the orbit that an impulse can be applied. The advantage of this approach is that there is no approximation or linearization involved, and, in consequence, the method derived can be used to transfers of any magnitude.

2. Applicable Documents and References

2.1. Applicable Documents

1. Yang Werliang and Valder M. Medeiros "CBERS Satellite Orbit Design." INPE, São José dos Campos, SP, outubro de 1988. [CB-TRT-018]

2. Wilson C.C. da Silva "Multivariable Orbit Correction Dynamics." INPE, São José dos Campos, SP, dezembro de 1995. [CB-ITRT-053]

3. Roberto V.F. Lopes e Kondapalli Rama Rao "CBERS Orbit Control Analysis." INPE, São José dos Campos, SP, dezembro de 1995. [CB-ITRT-054]

2.2. References

Battin, R.H. (1965). *Astronautical Guidance*. McGraw-Hill, New York, NY.

Battin, R.H. (1968). A New Solution for Lambert's Problem. *Proccedings of the XIX International Astronautical Congress*, Oxford, Vol. 2, pp. 131-150,.

Breakwell, J.V., Gillespie, R.W. & Ross, S. (1961). Researches in Interplanetary Transfer. *Journal of American Rocket Society*, Vol. 31, pp. 201-208.

Broucke, R.A. & Prado, A.F.B.A. (1993). Optimal N-Impulse Transfer Between Coplanar Orbits. Paper AAS-93-660. AAS/AIAA Astrodynamics Meeting, Victoria, Canadá.

Prado, A.F.B.A. & Broucke, R.A. (1994). O Problema de Lambert com Mínimo ΔV. Proceedings of the VII Colóquio Brasileiro de Dinâmica Orbital, pp. 16. Santos-SP, Brazil, 21-25 November, 1994 (INPE-5592-PRE/1807).

Gooding, R.H. (1990). A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem. *Celestial Mechanics*, Vol. 48, pp. 145-165.

Herrick, S. (1971). *Astrodynamics*. Van Nostrand Reinhold, London.

Lancaster, E.R. & Blanchard, R.C. (1969). A Unified form of Lambert's Theorem. Technical Note D-5368, NASA, USA.

Lancaster, E.R., Blanchard, R.C. & Devaney, R.A. (1966). A Note on Lambert's Theorem. *Journal of Spacecraft and Rockets*, Vol. 3, pp. 1436-1438.

Lawden, D.F. (1991). Optimal Transfers Between Coplanar Elliptical Orbits. *Journal of Guidance Control and Dynamics*, Vol. 15, No. 3, pp. 788-791.

Prado, A.F.B.A. (1993). Optimal Transfer and Swing-By Orbits in the Two- and Three-Body Problems. Ph.D. Dissertation, University of Texas, Austin, Texas, USA.

Prussing, J.E. (1979). Geometrical Interpretation of the Angles α and β in Lambert's Problem. *Journal of Guidance, Control, and Dynamics,* Vol. 2, pp. 442-443.

Sun, F.T. & Vinh, N.X. (1983). Lambertian Invariance and Application to the Problem of Optimal Fixed-Time Impulsive Orbital Transfer. *Acta Astronautica*, Vol. 10, pp. 319-330.

Taff, L.G. & Randall, P.M.S. (1985). Two Locations, Two Times, and the Element Set. *Celestial Mechanics*, Vol. 37, pp. 149-159.

3. The Minimum ΔV Lambert's Problem

The original Lambert's problem is one of the most important and popular topics in celestial mechanics. Several important authors worked on it, trying to find better ways to solve the numerical difficulties involved (Breakwell *et alii* 1961; Battin, 1965 and 1968; Lancaster *et alii* 1966; Lancaster & Blanchard, 1969; Herrick, 1971; Prussing, 1979; Sun & Vinh, 1983; Taff & Randall, 1985; Gooding, 1990). It can be defined as: "A Keplerian orbit, about a given gravitational center of force is to be found connecting two given points (P_1 and P_2) in a given time Δt."

For this research, we use a problem that is related to the Lambert's problem. The formulation is slightly different from the original one, but it also has many important applications. This new problem is called "Minimum Delta-V Lambert's Problem" and it is formulated as follows: "A Keplerian orbit, about a given gravitational center of force is to be found connecting two given points (P_1 that belongs to an initial orbit and P_2 that belongs to a final orbit), such that the ΔV for the transfer is minimum."

To solve this problem, the analytical expressions for the total increment of the velocity required ΔV (as a function of only one independent variable) and for its first derivative with respect to this variable are obtained. Then, a numerical scheme to get the root of the first derivative and the numeric value of the ΔV at this point is used. More details on this point are available in Prado and Broucke (1994). From this information it is possible to get all the other parameters involved, like the components of the impulses, their locations, etc. This problem is used to search for a minimum two-impulse transfer between two given coplanar orbits in the approach that is used in Lawden (1991), Prado (1993) and Broucke & Prado (1993).

4. Definition of the Orbit Transfer Problem

Suppose that there is a spacecraft in a Keplerian orbit that is called O_0 (the initial orbit). It is desired to transfer this spacecraft to a final Keplerian orbit O_2, that is coplanar with the orbit O_0. To perform this transfer, we start at the point P_1 (r_1, θ_1), where an impulse with magnitude ΔV_1 that has an angle ϕ_1 with the local transverse direction is applied. The transfer orbit crosses the final orbit at the point P_2 (r_2, θ_2), where an impulse with magnitude ΔV_2 making an angle ϕ_2 with the local transverse direction is applied. To get to the basic problem (the "Minimum Delta-V Lambert's Problem"), it is necessary to specify the true anomaly (θ_1) of the departure point in the orbit O_0 (point P_1) and the true anomaly (θ_2) of the point of arrival in the orbit O_2 (point P_2). With these two values given and all the Keplerian elements of both orbits known, it is possible to determine both radius vectors $\vec{r}_1$ and $\vec{r}_2$ at the beginning and at the end of the transfer. Then the problem is to find which transfer orbit connecting these two vectors and using only two impulses is the one that requires the minimum ΔV for the maneuver. This problem is what was defined as the "Minimum ΔV Lambert's Problem". The sketch of the transfer and the variables used are shown in Fig. 1.

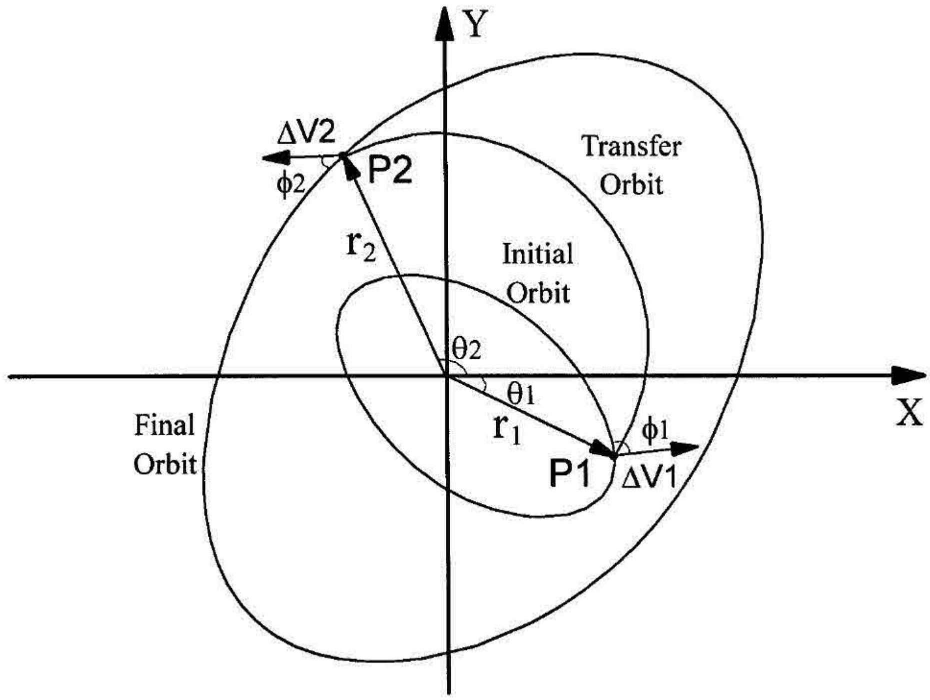


Fig. 1 - Geometry of the Transfer Orbit.

5. Mathematical Development

Using basic equations from the two-body celestial mechanics, it is possible to write an analytical expression for the total ΔV (= $\Delta V_1 + \Delta V_2$) required for this maneuver. To specify each of the three orbits involved in the problem, the elements D, h and k are used. They are defined by the following equations:

$$D = \frac{\mu}{C}; \quad k = e\mathrm{Cos}(\omega); \quad h = e\mathrm{Sin}(\omega) \tag{1}$$

where μ is the gravitational parameter of the central body; C is the angular momentum of the orbit, e is the eccentricity and ω is the argument of the periapse. The subscripts "0" for the initial orbit, "1" for the transfer orbit and "2" for the final orbit are also used. In those variables, the expressions for the radial (subscript r) and transverse (subscript t) components of the two impulses are:

$$\Delta V_{r1} = (D_1 k_1 - D_0 k_0)\mathrm{Sin}(\theta_1) - (D_1 h_1 - D_0 h_0)\mathrm{Cos}(\theta_1) \tag{2}$$

$$\Delta V_{t1} = D_1 - D_0 + (D_1 k_1 - D_0 k_0)\mathrm{Cos}(\theta_1) + (D_1 h_1 - D_0 h_0)\mathrm{Sin}(\theta_1) \tag{3}$$

$$\Delta V_{r2} = (D_2 k_2 - D_1 k_1)\mathrm{Sin}(\theta_2) - (D_2 h_2 - D_1 h_1)\mathrm{Cos}(\theta_2) \tag{4}$$

$$\Delta V_{t2} = D_2 - D_1 + (D_2 k_2 - D_1 k_1)\mathrm{Cos}(\theta_2) + (D_2 h_2 - D_1 h_1)\mathrm{Sin}(\theta_2) \tag{5}$$

The problem now is to find the transfer orbit that minimizes the total ΔV and that satisfies the two following constraints equations, expressing the fact that the orbits intersect:

$$g_1 = D_0^2(1 + k_0\mathrm{Cos}(\theta_1) + h_0\mathrm{Sin}(\theta_1)) - D_1^2(1 + k_1\mathrm{Cos}(\theta_1) + h_1\mathrm{Sin}(\theta_1)) = 0 \tag{6}$$

$$g_2 = D_2^2(1 + k_2\mathrm{Cos}(\theta_2) + h_2\mathrm{Sin}(\theta_2)) - D_1^2(1 + k_1\mathrm{Cos}(\theta_2) + h_1\mathrm{Sin}(\theta_2)) = 0 \tag{7}$$

The problem is reduced to the one of finding the value of D_1 that gives the minimum value for the expression $\Delta V = \sqrt{V_{r1}^2 + V_{t1}^2} + \sqrt{V_{r2}^2 + V_{t2}^2}$.

In the approach used to solve this problem, the constraints (6) and (7) are used to solve this system for two of our variables, making the equation for the ΔV a function of only one independent variable. The system formed by these two equations is symmetric and linear in the variables h_1 and k_1, so the system is solved for these two variables. The results are the equations (8) and (9).

$$k_1 = -\mathrm{Csc}(\theta_1 - \theta_2)\left[\left(\left(\frac{D_0^2}{D_1^2}\right)(1 + k_0\mathrm{Cos}(\theta_1) + h_0\mathrm{Sin}(\theta_1)) - 1\right)\mathrm{Sin}(\theta_2) - \ldots\right.$$
$$\left.\ldots - \left(\left(\frac{D_2^2}{D_1^2}\right)(1 + k_2\mathrm{Cos}(\theta_2) + h_2\mathrm{Sin}(\theta_2)) - 1\right)\mathrm{Sin}(\theta_1)\right] \tag{8}$$

$$h_1 = -\mathrm{Csc}(\theta_1 - \theta_2)\left[\left(\left(\frac{D_2^2}{D_1^2}\right)(1 + k_2\mathrm{Cos}(\theta_2) + h_2\mathrm{Sin}(\theta_2)) - 1\right)\mathrm{Cos}(\theta_1) - \ldots\right.$$
$$\left.\ldots - \left(\left(\frac{D_0^2}{D_1^2}\right)(1 + k_0\mathrm{Cos}(\theta_1) + h_0\mathrm{Sin}(\theta_1)) - 1\right)\mathrm{Cos}(\theta_2)\right] \tag{9}$$

Now that the ΔV is a function of only one variable (D_1), elementary calculus can be used to find its minimum. All that has to be done is to search for the root of the expression $\frac{\partial(\Delta V)}{\partial D_1} = 0$. From the definition of ΔV it is possible to write:

$$\frac{\partial(\Delta V)}{\partial D_1} = 0 = \frac{1}{\Delta V_1}\left[\Delta V_{r1}\frac{\partial(\Delta V_{r1})}{\partial D_1} + \Delta V_{t1}\frac{\partial(\Delta V_{t1})}{\partial D_1}\right] + \frac{1}{\Delta V_2}\left[\Delta V_{r2}\frac{\partial(\Delta V_{r2})}{\partial D_1} + \Delta V_{t2}\frac{\partial(\Delta V_{t2})}{\partial D_1}\right] \quad (10)$$

Now, the chain rule for derivatives is applied to obtain expressions for the quantities $\frac{\partial(\Delta V_{r1})}{\partial D_1}$; $\frac{\partial(\Delta V_{t1})}{\partial D_1}$; $\frac{\partial(\Delta V_{r2})}{\partial D_1}$; $\frac{\partial(\Delta V_{t2})}{\partial D_1}$. A general expression for them is:

$$\frac{\partial(\Delta V_{ij})}{\partial D_1} = \left.\frac{\partial(\Delta V_{ij})}{\partial D_1}\right|_{\text{Direct}} + \frac{\partial(\Delta V_{ij})}{\partial k_1}\frac{\partial k_1}{\partial D_1} + \frac{\partial(\Delta V_{ij})}{\partial h_1}\frac{\partial h_1}{\partial D_1} \quad (11)$$

where i = r,t; j = 1,2 and the word "Direct" stands for the part of the derivative that comes from the explicit dependence of ΔV_{ij} in the variable D_1. The expressions for $\frac{\partial(\Delta V_{ij})}{\partial k_1}$ and $\frac{\partial(\Delta V_{ij})}{\partial h_1}$ can be obtained from the equations (2) to (5) and the expressions for $\frac{\partial k_1}{\partial D_1}$ and $\frac{\partial h_1}{\partial D_1}$ can be obtained from the equations (8) to (9).

With all those equations available, a numerical algorithm can be built to iterate in the variable D_1 to find the unique real root of the equation $\frac{\partial(\Delta V)}{\partial D_1} = 0$. To obtain the value of $\frac{\partial(\Delta V)}{\partial D_1}$ for a given D_1, necessary for the iteration process required, the following steps can be used:

i) Evaluate k_1 and h_1 from equations (8) and (9) for the given D_1;

ii) With D_1, h_1 and k_1 the equations (2) to (5) are used to evaluate ΔV_{r1}, ΔV_{t1}, ΔV_{r2}, ΔV_{t2}, ΔV_1 ($\sqrt{\Delta V_{r1}^2 + \Delta V_{t1}^2}$) and ΔV_2 ($\sqrt{\Delta V_{r2}^2 + \Delta V_{t2}^2}$);

iii) With all those quantities known, it is possible to evaluate $\frac{\partial(\Delta V_{ij})}{\partial k_1}$ and $\frac{\partial(\Delta V_{ij})}{\partial h_1}$ (obtained from equations (2) to (5)) and equation (10) to finally obtain $\frac{\partial(\Delta V)}{\partial D_1}$ for the given D_1.

At this point, it is important to remark that the function $\frac{\partial(\Delta V)}{\partial D_1}$ is very sensitive to small variations in D_1, specially when close to the real root. Its curve is almost a straight line with a slope that goes to infinity when θ_2 - θ_1 goes to 180°. This characteristic is particular for the set of variables used and it is not a physical problem. If another independent variable is used, like the argument of the periapse of the transfer orbit, the curve for the ΔV vs. D_1 has a much less sharp minimum and, in consequence, its derivative has no big jumps.

This behavior makes numerical methods to find the root based on derivatives (like the popular Newton-Raphson) inadequate. In this research, the method of dividing the interval in two parts in each iteration shows to be adequate, although not fast in convergence.

6. Changing the Keplerian Elements Individually

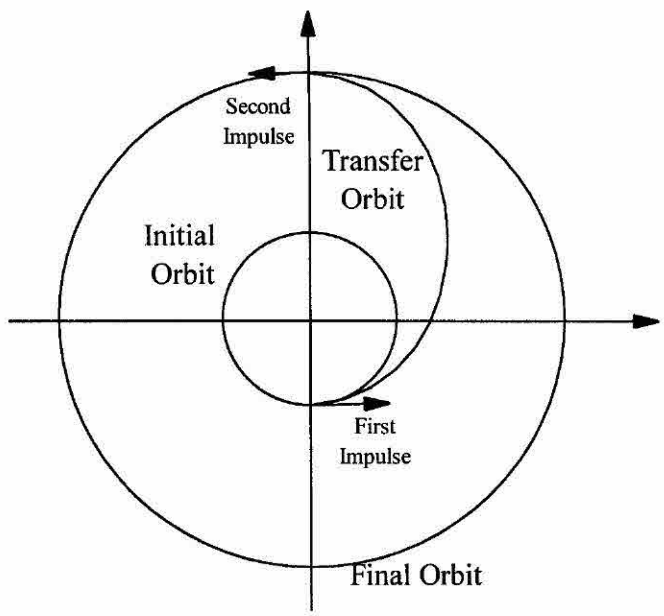
The objective of this section is to get a better understanding of the orbital maneuvers required to change the semi-major axis, eccentricity and argument of periapse, one at a time. Several cases were simulated to have a general idea about the optimal solutions.

1) Change in the semi-major axis

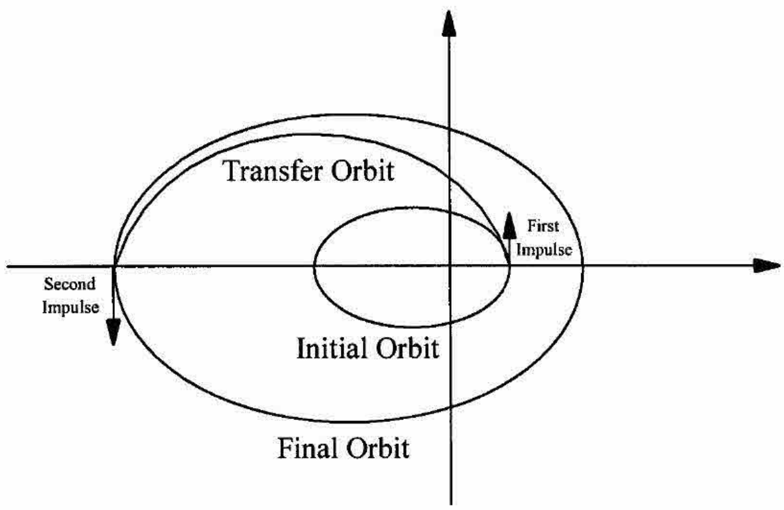
The maneuvers studied in this section has the objective of changing only the semi-major axis, keeping constant all the other orbital elements.

If the two orbits are circular, the best solution is the Hohmann transfer. Transfer where the radius of the final orbit is larger than 11.93876 the initial orbit, that would lead to a bi-elliptical transfer, are not considered in the present study. Transfers with more than two impulses and/or transfers passing by the infinity during the maneuver are also excluded from the present report.

If the orbits are elliptic they are necessarily co-axial and aligned (periapses in the same direction), since the argument of the periapse does not change during the maneuver. In this case, the bi-impulsive transfer that has the minimum value for the ΔV is the transfer that uses the most distant of the apses (the apoapse of the orbit that has the larger semi-major axis). This rule is stated in Marchal (1965) and it was confirmed by the results obtained in the present research. Fig. 2 shows the two situations described.



Hohmann Transfer between two circular orbits.



Transfer between two elliptic orbits.

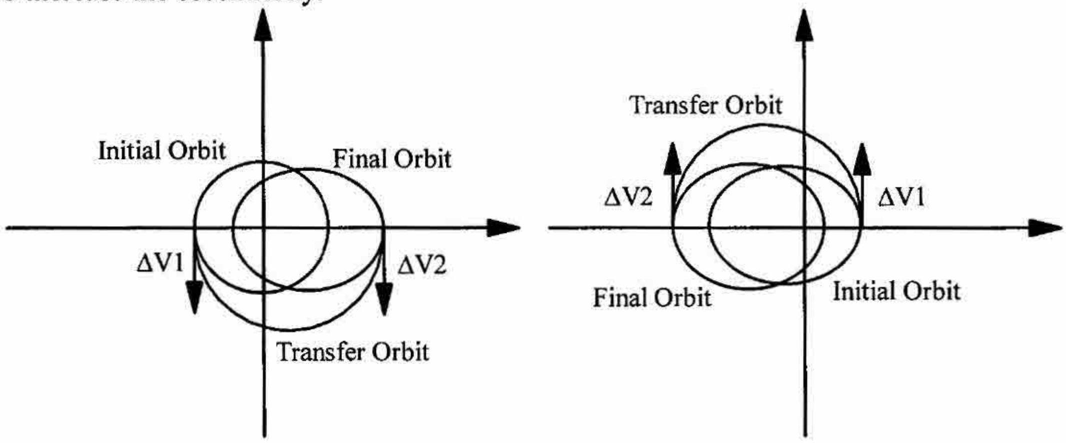
Fig. 2 - Maneuvers to change the semi-major axis.

2) Change in the eccentricity

The maneuvers studied in this section has the objective of changing only the eccentricity, keeping constant all the other orbital elements.

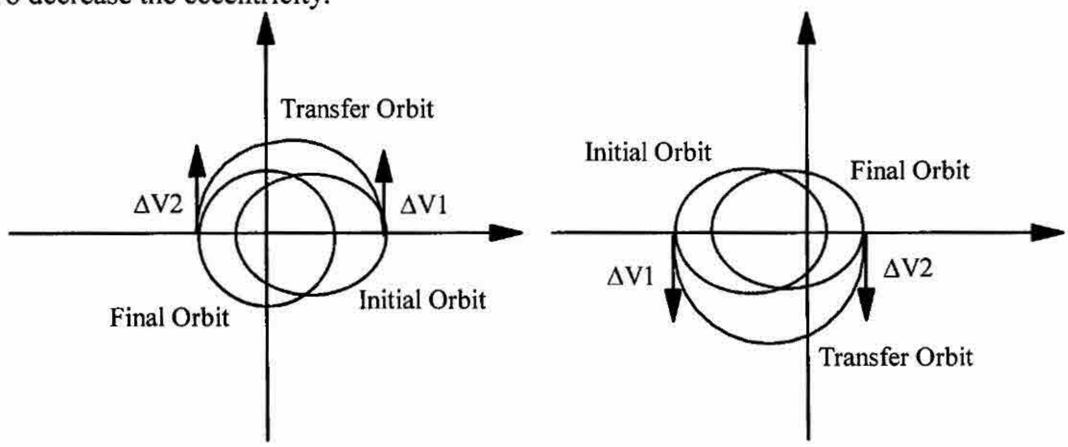
The simulations made in the present research found that the best solution is to apply the two impulses at the apses, always using the most distant of the apses. Fig. 3 shows several examples. Note that to achieve this transfer the two impulses have opposite directions. Those conclusions are valid to increase and to decrease the eccentricity.

To increase the eccentricity:



Circular-Elliptic Transfer Elliptic-Elliptic Transfer

To decrease the eccentricity:



Circular-Elliptic Transfer Elliptic-Elliptic Transfer

Fig. 3 - Maneuvers to change the eccentricity.

3) Change in the argument of the periapse

The maneuvers studied in this section has the objective of changing only the argument of the periapse, keeping constant all the other orbital elements.

The simulations showed that the solution is usually obtained by the application of two impulses with the same magnitude and in directions making angles close to zero (first impulse) and 180° (second impulse) with the transverse direction. The transfer orbit has an argument of periapse that is close to the average between the argument of periapse of the initial and the final orbit. Next, some examples are shown with the numerical data.

i) For the case where $\Delta\omega = 45°$, a= 1.0 and e = 0.2, we have the following solution:

$\theta_1 = 120°$, $\theta_2 = 286°$

Transfer orbit: a = 1.07774, e = 0.18497, $\omega = 22.5°$

$\Delta V_1 = 0.038$ m/s, $\Delta V_2 = 0.038$ m/s, $\Delta V_T = 0.076$ m/s

$\phi_1 = 4.4°$, $\phi_2 = 177.5°$

ii) For the case where $\Delta\omega = 45°$, a= 1.0 and e = 0.4, we have the following solution:

$\theta_1 = 128°$, $\theta_2 = 278°$

Transfer orbit: a = 1.16288, e = 0.36936, $\omega = 22.5°$

$\Delta V_1 = 0.078$ m/s, $\Delta V_2 = 0.078$ m/s, $\Delta V_T = 0.156$ m/s

$\phi_1 = 8.2°$, $\phi_2 = 173.7°$

iii) For the case where $\Delta\omega = 90°$, a= 1.0 and e = 0.2, we have the following solution:

$\theta_1 = 140°$, $\theta_2 = 310°$

Transfer orbit: a = 1.14282, e = 0.14251, $\omega = 45°$

$\Delta V_1 = 0.069$ m/s, $\Delta V_2 = 0.069$ m/s, $\Delta V_T = 0.138$ m/s

$\phi_1 = 2.4°$, $\phi_2 = 177.4°$

iv) For the case where $\Delta\omega = 180°$, a= 1.0 and e = 0.2, we have the following solution:

$\theta_1 = 178°$, $\theta_2 = 360°$

Transfer orbit: a = 1.20000, e = 0.00005, $\omega = 90°$

$\Delta V_1 = 0.096$ m/s, $\Delta V_2 = 0.096$ m/s, $\Delta V_T = 0.192$ m/s

$\phi_1 = 0.0°$, $\phi_2 = 180.0°$

7. The Maneuvers Required by the CBERS Satellite for each Orbital Element

In this section, the maneuvers required by the CBERS satellite are studied for each orbital element. It means that, for each maneuver performed here, two of the three orbital elements that we need to change are kept constant and only the third one is changed. The nominal orbit for the CBERS satellite is:

Semi-major axis (a) = 7148865 m

Eccentricity (e) = 0.0011

Argument of periapse (ω) = 90°

The velocity of the spacecraft, that is required to convert the magnitude of the impulses from the canonical system to m/s, is 7467 m/s.

For the purposes of this first study, it is assumed that the stationkeeping must be performed when at least one of the orbital elements is out of the range that is specified by the following constraints:

Error in semi-major axis = 200 m

Error in eccentricity = 0.0001

Error in argument of periapse = 5°

1) Change in semi-major axis

For this maneuver it is assumed that the semi-major axis decayed 200 m while the eccentricity and argument of periapse remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 0.9999720235$

$e_i = 0.0011$

$\omega_i = 90°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 106°$, $\theta_2 = 286°$

Transfer orbit: $a = 1.00000$, $e = 0.00111$, $\omega = 90.2°$

$\Delta V_1 = 0.0522$ m/s, $\Delta V_2 = 0.0522$ m/s, $\Delta V_T = 0.1044$ m/s

$\phi_1 = 0.0°$, $\phi_2 = 0.0°$

2) Change in eccentricity

For this maneuver it is assumed that the eccentricity was reduced by 0.0001 while the semi-major-axis and argument of periapse remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 1.0$

$e_i = 0.0010$

$\omega_i = 90°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 90°$, $\theta_2 = 270°$

Transfer orbit: $a = 1.00005$, $e = 0.00105$, $\omega = 90°$

$\Delta V_1 = 0.1865$ m/s, $\Delta V_2 = 0.1868$ m/s, $\Delta V_T = 0.3733$ m/s

$\phi_1 = 0.1°$, $\phi_2 = 180°$

3) Change in argument of periapse to 95°

For this maneuver it is assumed that the argument of periapse was increased by 5° while the eccentricity and semi-major axis remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 1.0$

$e_i = 0.0011$

$\omega_i = 95°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 2.0°$, $\theta_2 = 182.0°$

Transfer orbit: $a = 1.000048$, $e = 0.0011$, $\omega = 92.5°$

$\Delta V_1 = 0.1791$ m/s, $\Delta V_2 = 0.1791$ m/s, $\Delta V_T = 0.3582$ m/s

$\phi_1 = 1.1°$, $\phi_2 = 179.1°$

4) Change in argument of periapse to 85°

For this maneuver it is assumed that the argument of periapse was decreased by 5° with the eccentricity and semi-major axis remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 1.0$

$e_i = 0.0011$

$\omega_i = 85°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 178.0°$, $\theta_2 = 358.0°$

Transfer orbit: a = 1.000048, e = 0.0011, $\omega = 87.5°$

$\Delta V_1 = 0.1791$ m/s, $\Delta V_2 = 0.1791$ m/s, $\Delta V_T = 0.3582$ m/s

$\phi_1 = 0.9°$, $\phi_2 = 179.1°$

8. The Maneuvers Required by the CBERS Satellite for each Two of the Orbital Elements

In this section, the maneuvers required by the CBERS satellite are studied for each two of the orbital elements. It means that, for each maneuver performed here, one of the three orbital elements that we need to change remains constant and the two other are changed.

1) Change in semi-major axis and eccentricity

For this maneuver it is assumed that the semi-major axis decayed 200 m, the eccentricity was reduced by 0.0001 and argument of periapse remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 0.9999720235$

$e_i = 0.0010$

$\omega_i = 90°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 90°$, $\theta_2 = 270°$

Transfer orbit: a = 1.000036, e = 0.00106, $\omega = 90°$

$\Delta V_1 = 0.2387$ m/s, $\Delta V_2 = 0.1346$ m/s, $\Delta V_T = 0.3733$ m/s

$\phi_1 = 0.1°$, $\phi_2 = 180°$

2) Change in semi-major axis and argument of periapse

For this maneuver it is assumed that the semi-major axis decayed 200 m, the argument of periapse was reduced by 5° and the eccentricity remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 0.9999720235$

$e_i = 0.0011$

$\omega_i = 85°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 178°$, $\theta_2 = 358°$

Transfer orbit: $a = 1.000034$, $e = 0.0011$, $\omega = 88.23°$

$\Delta V_1 = 0.2314$ m/s, $\Delta V_2 = 0.1269$ m/s, $\Delta V_T = 0.3583$ m/s

$\phi_1 = 0.9°$, $\phi_2 = 179.1°$

3) Change in argument of periapse and eccentricity

For this maneuver it is assumed that the argument of periapse was reduced by 5°, the eccentricity was reduced by 0.0001 and semi-major axis remained in the nominal value. So, the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 1.0$

$e_i = 0.0010$

$\omega_i = 85°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 130°$, $\theta_2 = 310°$

Transfer orbit: a = 1.000068, e = 0.00105, $\omega = 87.62°$

$\Delta V_1 = 0.2528$ m/s, $\Delta V_2 = 0.2531$ m/s, $\Delta V_T = 0.5059$ m/s

$\phi_1 = 0.2°$, $\phi_2 = 180°$

9. The Maneuver Required by the CBERS Satellite

In this section, the maneuvers required by the CBERS satellite are studied for all the orbital elements at the same time. It means that all the three orbital elements are supposed to be in the limit allowed by the requirements and all of them will be changed by a single bi-impulsive maneuver. Two situations are simulated: the one where the argument of periapse decreased by 5° and the one where the argument of periapse increased by 5°. So, for the first situation the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 0.9999720235$

$e_i = 0.0010$

$\omega_i = 85°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 130°$, $\theta_2 = 310°$

Transfer orbit: a = 1.000054, e = 0.00106, $\omega = 88.13°$

$\Delta V_1 = 0.3050$ m/s, $\Delta V_2 = 0.2010$ m/s, $\Delta V_T = 0.5060$ m/s

$\phi_1 = 0.1°$, $\phi_2 = 179.9°$

For the second situation the initial (subscript "i") and final (subscript "f") conditions, in canonical units, are:

$a_i = 0.9999720235$

$e_i = 0.0010$

$\omega_i = 95°$

$a_f = 1.0$

$e_f = 0.0011$

$\omega_f = 90°$

The solution obtained is:

$\theta_1 = 50°$, $\theta_2 = 230°$

Transfer orbit: $a = 1.000064$, $e = 0.00106$, $\omega = 91.87°$

$\Delta V_1 = 0.3050$ m/s, $\Delta V_2 = 0.2010$ m/s, $\Delta V_T = 0.5060$ m/s

$\phi_1 = 0.0°$, $\phi_2 = 180.0°$

Table 1 shows the results obtained from the above simulations.

Table 1 - Fuel consumed for all the maneuvers simulated

Maneuver	ϕ_1 (deg)	ϕ_2 (deg)	ΔV_T (m/s)
Semi-major axis	0.0	0.0	0.1044
Eccentricity	0.1	180.0	0.3733
Argument of Periapse (from 95°)	1.1	179.1	0.3582
Argument of Periapse (from 85°)	0.9	179.1	0.3582
Semi-major axis and Eccentricity	0.1	180.0	0.3733
Semi-major axis and Arg. Per. (85°)	0.9	179.1	0.3583
Eccentricity and Arg. Per. (85°)	0.2	180.0	0.5059
Total (from 95°)	0.1	179.9	0.5060
Total (from 85°)	0.0	180.0	0.5060

10. Conclusions

There are many conclusions that we can reach from the data shown in Table 1. First of all, we can see that all the impulses applied are very close to tangential. The angles ϕ_1 and ϕ_2 are always close to 0 or 180 degrees. This is a very interesting characteristic, because it validates the use of methods of optimal transfers that are restricted to tangential impulses. Next, we can see that the fuel consumed is smaller for the correction in semi-major axis (0.1044 m/s) then for the corrections in eccentricity (0.3733 m/s) or in argument of periapse (0.3582 m/s). The two possible situations for the drift in the argument of periapse (positive until $\omega = 95°$ or negative until $\omega = 85°$) does not change anything in the fuel consumed in the maneuver to correct this element individually or in the maneuver that correct all the elements at the same time. From the analyses of the maneuvers to correct two or three elements at a time, we can see that the correction in the semi-major axis can be obtained with a zero cost, because the total ΔV for the combined maneuver and the maneuver that do not include the semi-major axis are the same. In the case of the maneuvers in eccentricity and argument of periapse, the combined maneuvers requires 0.5059 m/s, showing a savings of 0.2256 m/s over the total for the two maneuvers realized individually (0.7315 m/s).

Another characteristic that is visible from the data is that the correction in semi-major axis uses two impulses in the positive direction (direction of the motion of the satellite), while the corrections in eccentricity and argument of periapse both use two impulses in opposite directions (the first one is positive and the second one is negative). In the combined maneuvers the correction in eccentricity and argument of periapse dominates the transfer and the impulses are in opposite directions.