I

1. Classification *INPE-COM.3/RPI* *C.D.U.: 621.372.8*	2. Period	4. Distribution Criterion internal [X] external [ ]
3. Key Words (selected by the author) *CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION* *TRANSITION IMPEDANCE* *TRANSITION PROJECT*		
5. Report No. *INPE-1995-RPI/039*	6. Date *February, 1981*	7. Revised by *Carlos W. Santana*
8. Title and Sub-title *CHARACTERISTIC IMPEDANCE OF A CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION WHICH MAINTAINS CONSTANT CUTOFF WAVELENGTH*		9. Authorized by *Nelson de Jesus Parada* *Director*
10. Sector *DEE/DET*	Code	11. No. of Copies *03*
12. Authorship *Pawel Rozenfeld* 13. Signature of first author		14. No. of Pages *14* 15. Price

16. Summary/Notes

The characteristic impedance of a circular to rectangular waveguide transition which maintains constant cutoff wavelength is computed by the finite-difference method and is presented in a graphical form. It is shown, also, that the curve which defines such a transition does not agree with the one computed by the transverse resonance method [4].

17. Remarks *Submitted for publication in the IEEE Transactions on Microwave Theory and Techniques.*

CHARACTERISTIC IMPEDANCE OF A CIRCULAR TO RECTANGULAR WAVEGUIDE TRANSITION WHICH MAINTAINS CONSTANT CUTOFF WAVELENGTH

Pawel Rozenfeld

Instituto de Pesquisas Espacaiais - INPE
Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq
12200 - São José dos Campos, SP, Brasil

ABSTRACT

The characteristic impedance of a circular to rectangular waveguide transition which maintains constant cutoff wavelength is computed by the finite-difference method and is presented in a graphical form. It is shown, also, that the curve which defines such a transition does not agree with the one computed by the transverse resonance method |4|.

I - INTRODUCTION

A circular to rectangular waveguide transition is a very useful device for construction of a mode transducer, [1], [2], [3]. While in [2] and [3] the transition is made by deforming conveniently only one dimension of a circle, in [1] this is done by deforming simultaneously both of the dimensions. The idea of such a deformation was firstly exposed in [4], and a curve which relates the cross-sectional dimensions of such a transition was presented.

The basis of this idea is to truncate conveniently the cross-section of a circular homogeneous waveguide in a manner that the final cross-section is a rectangular one, and the cutoff wavelength of the fundamental TE_{11} mode of a circular through transition to the fundamental TE_{10} of a rectangular waveguide is the same. Figure 1 shows a typical cross-section of such a transition.

Using the transverse resonance method, Pyle [4] computed a curve of (d_1/r) versus (d_2/r) which defines the cross-section of that transition. The computed results obtained did not pass through the origin as it should. It was assumed that this was caused by numerical errors, and the curve was made to pass throught the origin by multiplying it by a convenient correction factor. This factor was thought to be a bit arbitrary.

Also, [4] did not present the characteristic impedance of such a transition. This parameter is very important in the design of mode transducers [1], [2].

In this work, the curve for truncation of a circular waveguide in order to get such a transition as well as its characteristic impedance computed by the finite-difference method are presented.

II - THEORY

Define d as the largest dimension of a figure and k_c as the cutoff wave-number of a waveguide whose cross-section is this figure. For the fundamental TE_{11} mode of a circular waveguide, it can be shown that

$$(k_c d)_{circ} = 2\, p'_{11} \qquad (1)$$

where $p'_{11} = 1.84118$4 is the first zero of the derivative of the Bessel function of the first kind and first order.

For the fundamental TE_{10} mode of a rectangular waveguide, we get

$$(k_c d)_{rect} = \Pi \qquad (2)$$

where d is either the diameter of the circular waveguide or the largest side (a) of the rectangular waveguide.

For a transition originating from a circular waveguide in which both dimensions are truncated and which maintains the cutoff frequency, $(k_c d)_{trans}$ can be cast in form

$$(k_c d)_{trans} = 2\, p'_{11}(1 - \frac{d_2}{r}) \qquad (3)$$

In this case, the largest dimension d is the dimension a of Figure 1. Notice that (3) is reduced to (1) for $d_2 = 0$.

By equating (2) and (3), it is obtained the value of the largest dimension (a) of the rectangular waveguide which maintains the same cutoff wavelength as the transition. This gives the value of

$$\frac{d_{2\ max}}{r} = 0.1468553.$$

From geometrical considerations, it is found that

$$\frac{d_{1\ max}}{r} = 0.4783255.$$

The dimensions d_1 and d_2 for the transition cannot be found analytically and will be computed numerically.

Following [5], the mode characteristic impedance of a waveguide for TE modes is defined as

$$Z_o = \frac{2P}{II^*}$$

where P is the total power transmitted by the waveguide and I is an associated axial current. The normalized characteristic impedance at infinite frequency is give by

$$z_o = \frac{Z_{o(\text{frequency} = \infty)}}{\eta} \qquad (4)$$

where η is the characteristic impedance of free space. The characteristic impedance at any finite frequency can be found by dividing (4) by

$$\sqrt{1 - (\frac{f_c}{f})^2}$$

where f_c is the cutoff frequency of the guide, and f is its working frequency.

It can be shown that for a circular waveguide the normalized characteristic impedance is

$$z_0 = \frac{\Pi(p'^2_{11} - 1)}{8} \tag{5}$$

while for a rectangular waveguide

$$z_0 = \frac{\Pi^2}{8} \frac{b}{a} \tag{6}$$

In the case of the rectangular waveguide treated in this work

$$z_0 = \frac{\Pi^2}{8} \frac{(1 - \frac{d_{1\ max}}{r})}{(1 - \frac{d_{2\ max}}{r})} \tag{6'}$$

From (6) it is seen that the characteristic impedance of a rectangular waveguide depends on its cross-sectional dimensions, while from (5) it is seen that it does not for a circular waveguide. This fact leads us to suspect that for a transition which maintains the cutoff wavelength of a circular waveguide constant, there is no linear relatioship between the cross-sectional dimensions of the transition and its characteristic impedance.

III - RESULTS

A computer program using the finite-difference method and giving $(k_c d)$ and z_0, for a waveguide of arbitrary cross-section, was run for several transition cross-sections. For fixed values of (d_2/r), different values of (d_1/r) were assumed and the values of $(k_c d)$ and (z_0) were computed. The values of (d_1/r) and (d_2/r), which produced the lowest error as compared to the theoretical $(k_c d)_{trans}$ given by (3), were selected. With these values of (d_1/r) and (d_2/r), Figure 2 was drawn.

For all plotted points of the figure, the error in $(k_c d)_{trans}$ was less than 0.1%. Pyle's [4] curve is also shown on Figure 2.

It is seen that there is a good agreement between Pyle's [4] and our curves for very high or very low values of (d_1/r) and (d_2/r). In the intermediate region, the difference becomes quite significant. This discrepancy could be explained by rather arbitrary correction factor adopted by Pyle in [4].

In Figure 3 three characteristic impedance curves are represented: (I) computed by the finite-difference program, (II) and (III) computed by applying the results of this work and Pyle's one [4] to equation (6), respectively.

The characteristic impedance data obtained from the finite-difference computer program showed an error of 3.6% when compared to the known theoretical results for circular and rectangular waveguides. Therefore all the computed values for the characteristic impedances were scaled down by this factor and are presented as curve (I) on Figure 3.

Two other curves presented in this figure are based on the assumption of [1], which assumes that expression (6) is valid for the transition. From Figure 3, it can be seen that this is indeed true for (d_1/r) in the range 0.4783255-0.44. For values of (d_1/r) lower than that it is not so, and the error in the value of characteristic impedance is maximum for the circular waveguide as it should be.

By assuming the proportionality of the impedance of the truncated circle to the ratio b/a, as is done in [1], we are, actually, approximating a truncated circle by a circumscript rectangle of dimensions $b \times a$. This is poor approximation, especially for the cases in which d_1/r and d_2/r are small when the figure is more of a circle than a rectangle. In this cases, the field lines in the waveguide are better described by the Bessel functions than by the trigonometric functions. This fact shows up in the expression of the characteristic impedance.

IV - CONCLUSIONS

A circular to rectangular waveguide transition which maintains the cutoff wavelength of the fundamental TE_{11} mode of the circular waveguide is computed by the finite-difference method.

The curve which relates the dimension of the truncation is presented and compared with the one presented in [4]. The results of this work are less than 0.1% of the theoretical ones as far as cutoff wavelength is concerned. The precision of the results of [4] is not known.

The normalized characteristic impedance of such a transition is also computed and compared with the one used in [1]. It is shown that the simplified assumption of [1], as far as the normalized characteristic impedance is concerned, is valid only for large truncation of a circular waveguide. This corresponds to a rectangular waveguide with slight rounding of the corners. For the other region, where the truncation of the circular waveguide is not large, the simplified assumption of [1] is in error. This fact could be accountable for the measured reflection coeficient of the mode transducer described in [1] being twice the theoretical one.

REFERENCES

[1] - Bathker, D.A. "A Stepped Mode Transducer Using Homogeneous Waveguides", *IEEE Transactions Microwave Theory Tech* (Corresp.) vol. MTT-15, pp. 128-130, Feb. 1967.

[2] - S. Stuchly and A. Kraszewski, "Wide-Band Rectangular to Circular Waveguide Mode and Impedance Transformer", *IEEE Transactions Microwave Theory Tech* (Corresp.), vol. MTT-13, pp.379-380, May 1965.

[3] - G. R. Valenzuela, "The Cutoff Wavelengths of Composite Waveguides", *IEEE Transactions Microwave Theory Tech* (Corresp.), vol. MTT-9, pp. 363-364, July 1961.

[4] - Pyle, J. R., *A Circular to Rectangular Waveguide Transition Maintaining a Constant Cutoff Wavelength.* Australian Defence Scientific Service Weapons Research Establishment, Salisbury, Tec. Note Pad 94, September 1964.

[5] - J. B. Davies and C.A. Muilwyk "Numerical Solution of Uniform Hollow Waveguides with Boundaries of Arbitrary Shape", Proc. IEE - 113, pp. 277-284, Feb. 1966.

LIST OF FIGURES

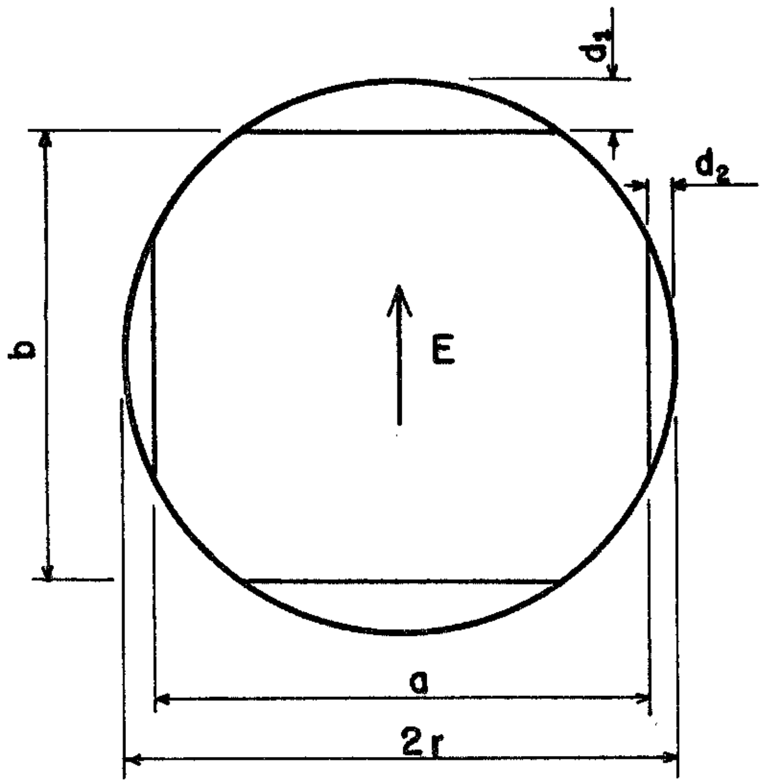

Figure 1

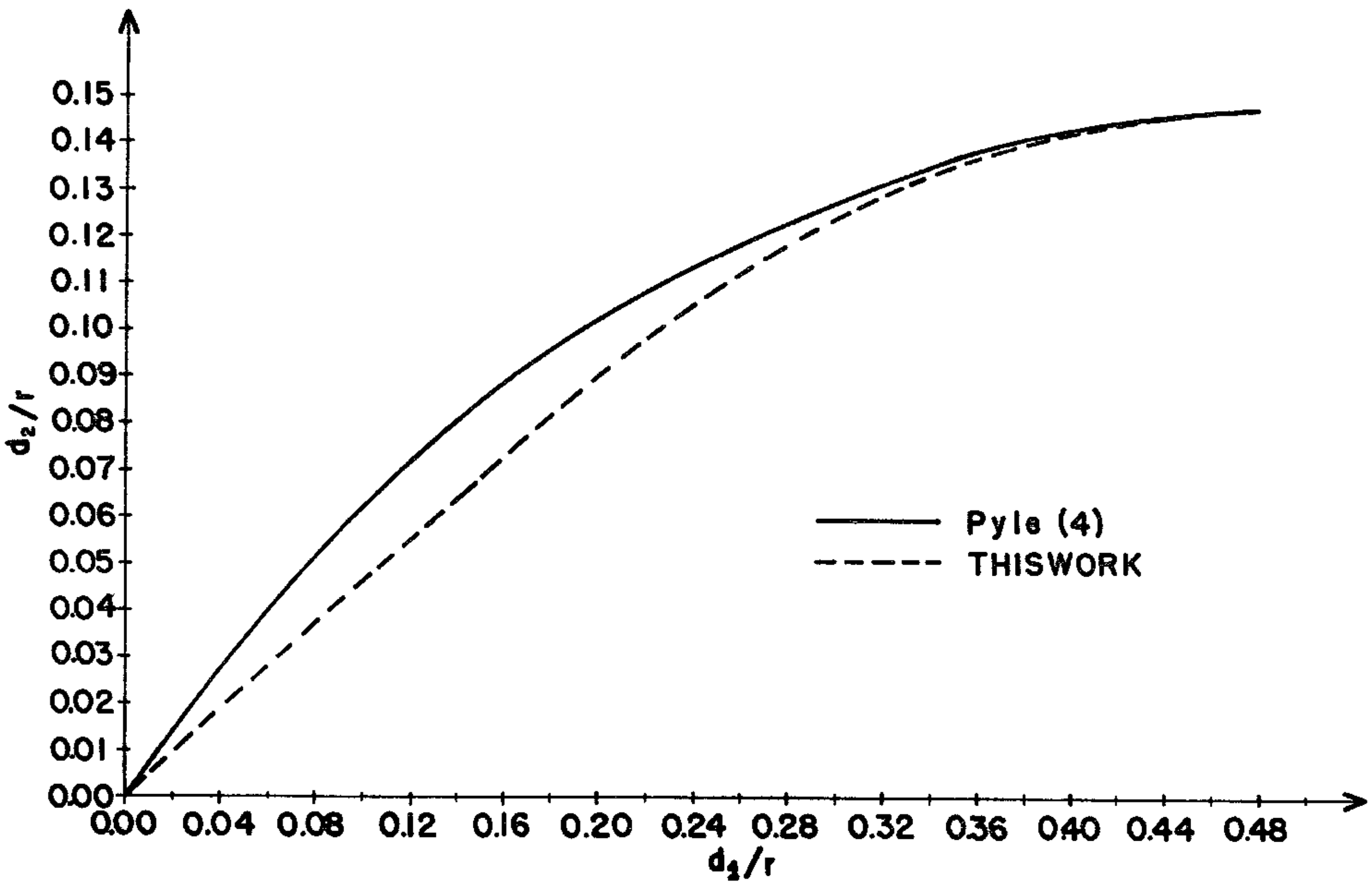
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
d2/r
0.00
0.04
0.08
0.12
0.16
0.20
0.24
0.28
0.32
0.36
0.40
0.44
0.48
d1/r
Pyle (4)
THISWORK

Figure 2

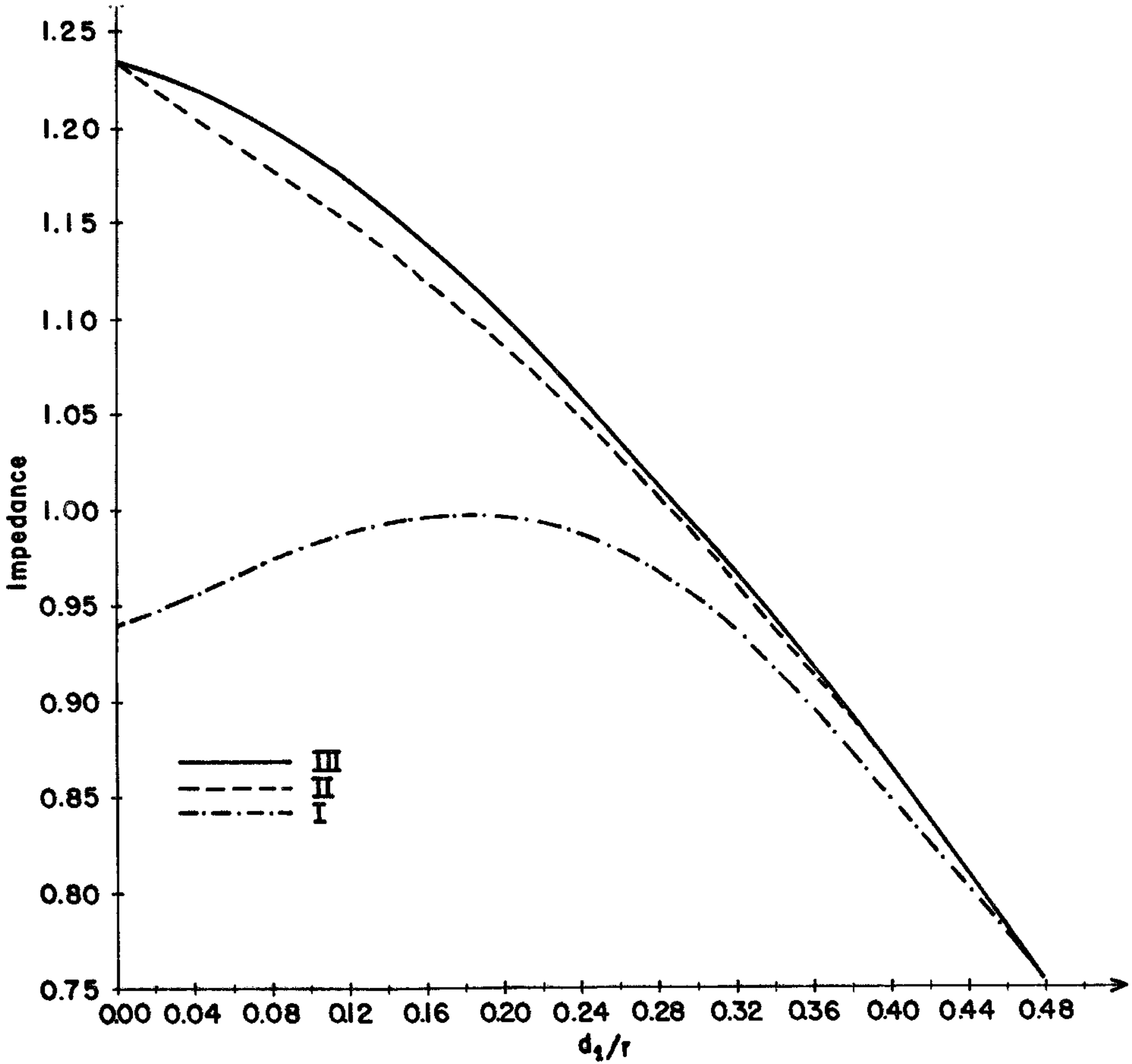
Impedance
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.00 0.04 0.08 0.12 0.16 0.20 0.24 0.28 0.32 0.36 0.40 0.44 0.48
d_1/r
III
II
I

Figure 3