A STUDY OF POWERED SWING-BY

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ABSTRACT *- This paper considers the problem of applying an impulsive thrust in a spacecraft that is performing a Swing-By maneuver (also called Gravity Assisted maneuver). The objective is to derive a set of analytical equations that can calculate the change in velocity, energy and angular momentum for this maneuver as a function of the three usual parameters of the standard Swing-By maneuver plus the two parameters that specify the planar impulse applied. The dynamics used to obtain those equations is the one given by the "patched-conic" approach. A study is also performed to find in which cases the impulse is more efficient when applied during that close approach or after that, in a two steps maneuver. After that, the same maneuvers are computed under the dynamics given by the restricted three-body problem and the results are compared with the ones obtained previously under the "patched-conic" dynamics.*

1 - INTRODUCTION

The Swing-By maneuver is a very popular technique used to decrease the fuel expenditure in space missions. The standard maneuver uses a close approach with a celestial body to modify the velocity, energy and angular momentum of the spacecraft. There are many important applications very well known, like the Voyager I and II that used successive close encounters with the giant planets to make a long journey to the outer Solar System [Flan 66]; the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun [Carv 86]; etc.

In this paper, a different type of Swing-By maneuver is studied, where we are allowed to apply an impulse to the spacecraft during its closest approach with the celestial body. This type of maneuver increases very much the alternatives available to mission designers to meet the requirements of many missions. New equations are derived to give us the change in velocity, energy and angular momentum as a function of the three independent parameters (required to describe the standart Swing-By maneuver) described in the next section and the new two parameters that belongs to this particular model: the magnitude of the impulse applied and the angle that this impulse makes with the velocity of the spacecraft. All those equations are derived assuming that: a) the maneuver can be modeled by the "patched conic" model (a series of Keplerian orbits); b) that the impulse is applied during the passage by the periapse and; c) that it changes the velocity of the spacecraft instantaneously; d) the motion is planar everywhere.

After that, this powered Swing-By is compared with a different maneuver, where the impulse is not applied during the close approach, but just after the spacecraft leaves the sphere of influence of the celestial body. In that way, the best position to apply an impulse in the spacecraft is investigated: during the close approach with the celestial body or after that, in a two steps maneuver.

Those maneuvers are then recalculated, using the more realistic dynamics given by the restricted three-body problem and the results are compared.

2 - THE STANDARD SWING-BY MANEUVER

The standard Swing-By maneuver consists of using a close encounter with a celestial body to change the velocity, energy, and angular momentum of a smaller body (a comet or a spacecraft). This standard maneuver can be identified by three independent parameters:

i) V_{inf-}, the magnitude of the velocity of the spacecraft when approaching the celestial body or V_p, the magnitude of the velocity of the spacecraft at periapse (those quantities are equivalent);

ii) r_p, the distance between the spacecraft and the celestial body during the closest approach;

iii) ψ, the angle of approach (angle between the periapse line and the line that connects the two primaries).

Fig. 1 shows the sequence for this maneuver and some of those and other important variables.

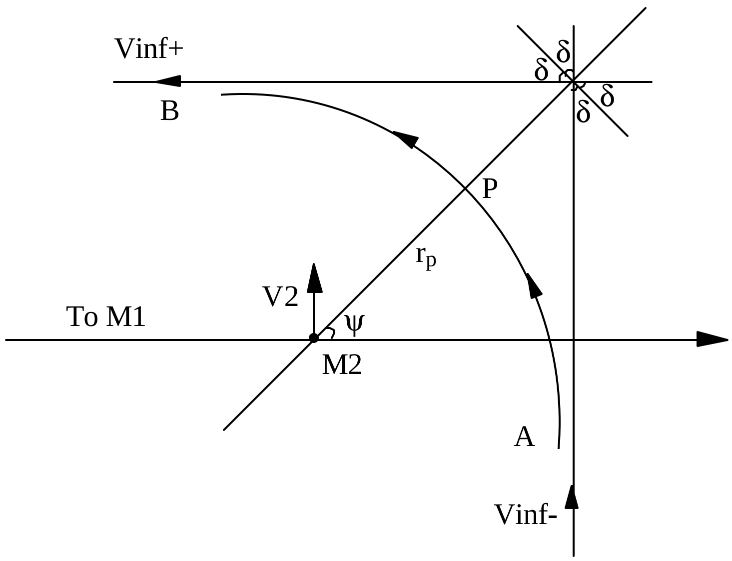


Fig. 1 - The Standard Swing-By Maneuver.

It is assumed that the system has three bodies: a primary (M1) and a secondary (M2) body with finite mass that are in circular orbit around their common center of mass and a third body with negligible mass (the spacecraft) that has its motion governed by the two other bodies. We can see that the spacecraft leaves the point A, crosses the horizontal axis (the line between M1 and M2), passes by the point P (the periapsis of the trajectory of the spacecraft around M2) and goes to the point B. We choose the points A and B in a such way that we can neglect the influence of M2 at those points and, consequently, we know that the energy is constant after B and before A (the system follows the two-body celestial mechanics). Two of our initial conditions are clearly identified in the figure: the perigee distance r_p (distance measured between the point P and the center of M2) and the angle ψ, measured from the horizontal axis in the counter-clock-wise direction. The distance r_p is not to scale, to make the figure easier to understand. In this paper only the planar motion (all the three bodies always in the same plane) is studied. The result of this maneuver is a change in velocity, energy and angular momentum in the Keplerian orbit of the spacecraft around the central body. Using the "patched conic" approximation, the equations that quantify those changes are available in the literature. Under this approximation the maneuver is considered as composed of three parts, where each of those systems are governed by the two-body celestial mechanics. The first system describes the motion of the spacecraft around the primary body before the close encounter (the secondary body is neglected). When the spacecraft comes close to the secondary body, the primary is neglected and a second two-body system is formed by the spacecraft and the secondary body. After the close encounter the spacecraft leaves the secondary body, and it goes to an orbit around the primary body again. Then, the secondary is neglected one more time. One of the best description of this maneuver and the derivation of the equations are in Broucke [Brou 88]. The equations are reproduced below.

$$\delta = \sin^{-1}\left(\frac{1}{1+\frac{r_p V_{inf-}^2}{\mu_2}}\right), \qquad \Delta E = -2 V_2 V_{inf-} \sin\delta \sin\psi \qquad (1\text{-}2)$$

$$\Delta C = -2 V_2 V_{inf-} \sin\delta \sin\psi, \quad \Delta V = 2 V_{inf-} \sin\delta \qquad (3\text{-}4)$$

In those equations δ is the total deflection of the trajectory of the spacecraft (see Fig. 1), V_2 is the linear velocity of M2 in its motion around the center of mass of the system M1-M2 and μ_2 is the gravitational parameter of M2. From those equations it is possible to get the fundamental well-known results:

a) The variation in energy (ΔE) is equal to the variation in angular momentum (ΔC);

b) If the Fly-By is in front of the secondary body, there is a loss of energy. This loss has a maximum at ψ = 90°;

c) If the Fly-By is behind the secondary body, there is a gain of energy. This gain has a maximum at ψ = 270°.

There are many publications studying the standard Swing-By maneuver in different missions. Some examples are: the study of missions to the satellites of the giant planets [Byrn 82], [D'Ama 83], [D'Ama 79], [D'Ama 81], [D'Ama 82]; new missions for Neptune and Pluto [Wein 92], [Swen 92]; the study of the Earth's environment [Farq 81], [Farq 85], [Mars 88], [Muho 85], [Dunh 85], [Efro 85], etc.

There are also some studies of the Swing-By maneuver under the model of the planar restricted three-body problem, like in the publications made by Broucke and Prado ([Brou 93a], [Brou 93b], [Prad 93], [Prad 94]).

3 - THE POWERED SWING-BY MANEUVER

The description of the powered Swing-By is the main objective of this paper. The literature presents some interesting applications of this maneuver, such as an Earth-Mars mission using a Swing-By in Venus [Stri 91]. For the present research it is assumed that the difference between this maneuver and the standard one is that it is possible to apply an impulse to the spacecraft in the moment of the closest approach between the spacecraft and the secondary body. This impulse is allowed to have any magnitude and it can have any direction that belongs to the plane of motion of the three bodies involved. Fig. 2 shows the geometry of this maneuver and defines some of the variables used.

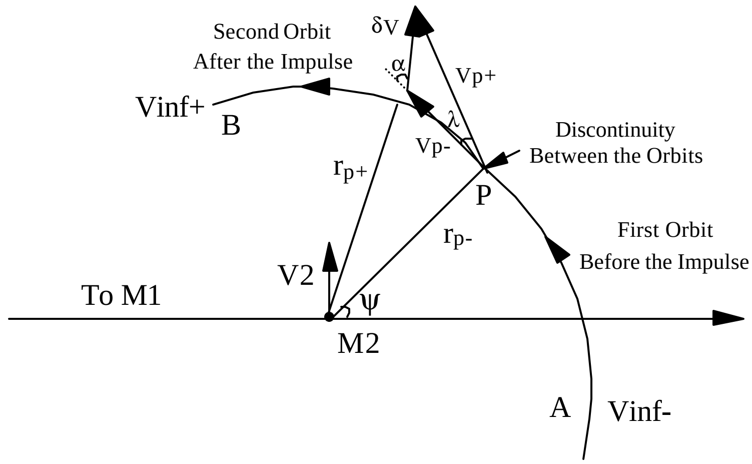


Fig. 2 - The Geometry of the Powered Swing-By.

The variables are:
i) V_{inf-}, the magnitude of the velocity of the spacecraft when approaching the celestial body;
ii) V_{p-}, the magnitude of the velocity of the spacecraft at periapse before the impulse is applied;
iii) V_{p+}, the magnitude of the velocity of the spacecraft at periapse after the impulse is applied;
iv) δV, the magnitude of the impulse applied;
v) α, angle between $\vec{V}_{p-}$ and the impulse applied. This variable defines the direction of the impulse. The range for α is $-180° < \alpha < 180°$ (positive values are measured in the clock-wise direction);
vi) λ, angle between $\vec{V}_{p-}$ and $\vec{V}_{p+}$;
v) r_{p-}, the distance between the spacecraft and the celestial body during the closest approach, when the spacecraft is in its first orbit (before the impulse);
vi) r_{p+}, the distance between the spacecraft and the celestial body during the closest approach, when the spacecraft is in its second orbit (after the impulse). Remember that the impulse changes the orbit of the spacecraft, so there are two periapses involved in this maneuver: one that belongs to the first orbit (before the impulse) and one that belongs to the second orbit (after the impulse);
vii) V_{inf+}, the magnitude of the velocity of the spacecraft when leaving the celestial body;
viii) ψ, the angle of approach (angle between the periapse line and the line that connects the two primaries).
With those variables, it is possible to develop an algorithm to evaluate all the parameters involved in this maneuver. The steps are shown next. Remember that the initial conditions (given variables) are V_{inf-}, r_{p-}, ψ for the geometry of the close approach and δV and α to specify the impulse applied.
i) Using the principle of conservation of energy it is possible to calculate V_{p-} from V_{inf-} and r_{p-}. The equation used is $V_{p-} = \sqrt{V_{inf-}^2 + \frac{2\mu_2}{r_{p-}}}$, where μ_2 is the gravitational parameter of the secondary body;
ii) The next step is to calculate V_{p+}, from V_{p-}, δV and α. The equation used is $V_{p+} = \sqrt{V_{p-}^2 + \delta V^2 - 2 V_{p-} \delta V \cos\alpha}$;
iii) Then it is necessary to calculate V_{inf+} from V_{p+} and r_{p-}. The equation is $V_{inf+} = \sqrt{V_{p+}^2 - \frac{2\mu_2}{r_{p-}}}$;
iv) The next quantity to be evaluated is the semi-major axis (a) of the orbit after the Swing-By. It is obtained from V_{inf+} by the use of the equation $a = \frac{\mu_2}{V_{inf+}^2}$;

v) Then, the quantity λ is calculated. It comes from V_{p-}, V_{p+}, δV. The equation is $\lambda = \arccos\left(\frac{\delta V^2 - V_{p-}^2 - V_{p+}^2}{-2V_{p-}V_{p+}}\right)$;

vi) Then, the angular momentum (h), the semi-lactus rectum (p) and the eccentricity (e) of the orbit after the Swing-By are calculated. They come from V_{p+}, r_{p-} and λ. The equations are $h = r_{p-} V_{p+} \sin(90° - \lambda)$, $p = \frac{h^2}{\mu_2}$ and $e = \sqrt{1 + \frac{p}{a}}$;

vii) The next step is to calculate the true anomaly (f_0) of the spacecraft in the second hyperbolic orbit (the orbit after the impulse) around the secondary body just after the impulse. It comes from e, p and r_{p-}. The equation is $f_0 = \arccos\left(\frac{1}{e}\left(\frac{p}{r_{p-}} - 1\right)\right)$;

viii) Next, it is calculated the true anomaly (f_{LIM}) of the asymptotes of the second hyperbolic orbit of the spacecraft, around the secondary body after the impulse. It comes from e. The equation is $f_{LIM} = \arccos\left(-\frac{1}{e}\right)$;

ix) Then, the total deflection for this maneuver is given by $\Theta = \delta + f_0 + f_{LIM} - 90°$.

Now, it is necessary to proceed the calculations to obtain the equations for the variation of energy, velocity and angular momentum. Fig. 3 shows the geometry of the vector addition, that provides the basic informaton to derive those equations.

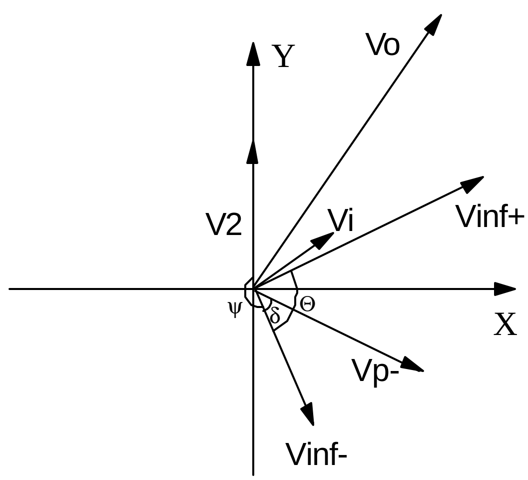


Fig. 3 - Vector Addition for the Velocities.

From that figure it is possible to obtain the analytical equations required. The horizontal and the vertical components of the velocity before the close encounter (V_{ix}, V_{iy}) and after the close encounter (V_{ox}, V_{oy}) are:

$$V_{ix} = -V_{inf-} \sin(\psi - \delta), \qquad V_{iy} = V_2 + V_{inf-} \cos(\psi - \delta) \tag{5-6}$$

$$V_{ox} = -V_{inf+} \sin(\psi - \delta + \Theta), \qquad V_{oy} = V_2 + V_{inf+} \cos(\psi - \delta + \Theta) \tag{7-8}$$

With those equations it is easy to calculate the variations in velocity, energy and angular momentum. To derive those equations it is assumed that the Swing-By maneuver is instantaneous and that the position of the spacecraft remains constant during the maneuver. The equations are:

$$\Delta V_{imp} = \sqrt{\left(V_{ox} - V_{ix}\right)^2 + \left(V_{oy} - V_{iy}\right)^2}\,, \qquad \Delta E_{imp} = \frac{1}{2}\left(V_{ox}^2 + V_{oy}^2 - V_{ix}^2 - V_{iy}^2\right) \tag{9-10}$$

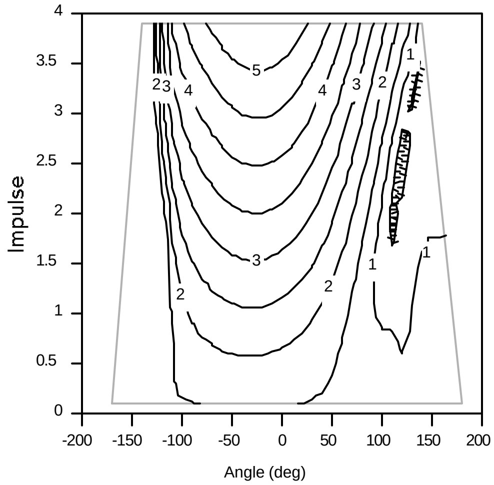
$$\Delta C_{imp} = d\left(V_{oy} - V_{iy}\right) \tag{11}$$

where d is the distance between M_1 and M_2.

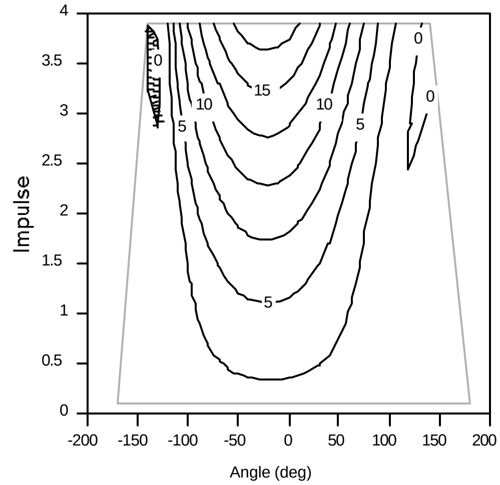
4 - RESULTS FOR THE TWO-BODY MODEL

In this section the methods explained in the previous sections are used to generate some results to understand better this maneuver. The Earth-Moon system is used as an example. A spacecraft makes a powered Swing-By with the Moon for several values of the impulse (they can have different magnitudes and directions, but they are always in the plane of the motion of the three bodies). Fig. 4 shows the variations in velocity, energy and angular momentum for the powered Swing-By. The horizontal axis represents the angle α that defines the direction of the impulse and the vertical axis represents the magnitude of the impulse. The parameters used for this maneuver are: μ_1 = 398600 kg.km^3/s^2, μ_2 = 4900 kg.km^3/s^2, V_{inf+} = 1.0 km/s, ψ = 270°, r_{p-} = 1900 km, V_2 = 1.02 km/s, d = 384400 km. This is a maneuver that generates an increase in the energy (180° < ψ < 360°). For the intervals α > 90° and α < -90° the impulse has a component opposite to the direction of motion of the spacecraft, decreasing the energy, and it is working against the Swing-By. The blanck parts of the graphics ($\alpha > \approx$150° and $\alpha < \approx$-150°) correspond to regions where the impulse caused the capture of the spacecraft by the Moon. From Fig. 4 it is clear to see that the maximum transfer of velocity and energy occurs close to α = 0° and the minimums occur close to the borders of the graphic. Note also that there is a simmetry with respect to the line $\alpha \cong$ 0°. The graphic for the variation in angular momentum shows a different pattern, with simmetries with respect to the lines $\alpha \cong$ -90° and $\alpha \cong$ 90°. But, remember that the maximum transfer is not always the goal of the mission. A very close approach may be required to get data from the celestial body, but the consequent large increase in velocity and energy may not be desired for the continuation of the mission. In that way, a chart like that can provide important information for the mission designers, that can choose the parameters of the impulse that satisfy better the goals of the mission.

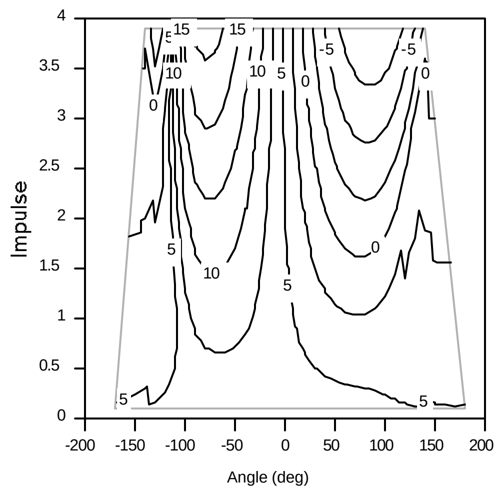
Next, Fig. 5 shows a similar maneuver, but with ψ = 90°, that is the case where the energy decreases in the standart Swing-By (0° < ψ < 180°). For the interval α > 90° and α < -90° the impulse has a component opposite to the direction of motion of the spacecraft, decreasing the energy, and it is working in favor of the Swing-By. In this case there are positive and negative values for the change in energy. In the positive regions of the plot of the variation in energy, the impulse is dominating the Swing-By and the net result is an increase in energy. In the negative regions of the same plot, the Swing-By and the impulse are working together to decrease the energy of the spacecraft. Note that positive values occur only above a certain limit in the magnitude of the impulse and that this limit decreases when α approaches zero. The variation in velocity and angular momentum present a behaviour similar to the case ψ = 270°.



Variation in Velocity

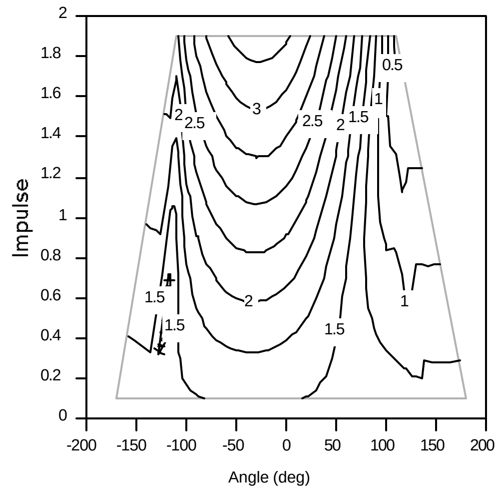


Variation in Energy

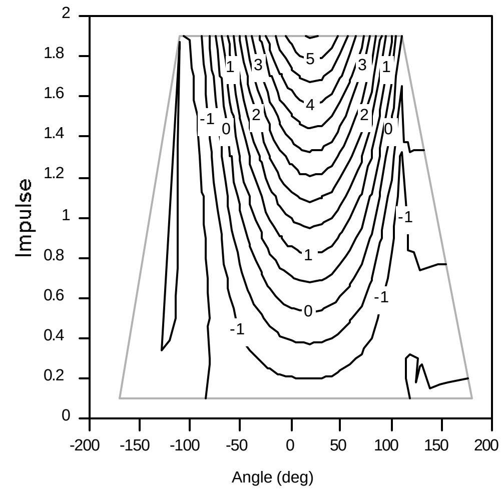


Variation in Angular Momentum x 10^{-5}

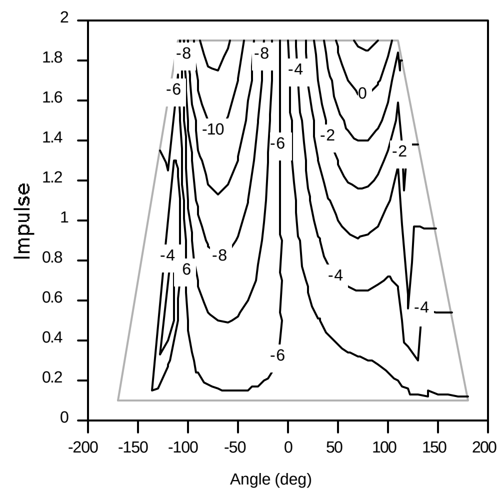
Fig. 4 - Variation in Velocity, Energy, Angular Momentum for the Powered Swing-By (ψ = 270°).



Variation in Velocity



Variation in Energy



Variation in Angular Momentum x 10^{-5}

Fig. 5 - Variation in Velocity, Energy and Angular Momentum for the Powered Swing-By (ψ = 90°).

After those first calculations, the efficiency of the powered maneuver is studied. The powered maneuver is compared with a maneuver where the impulse is applied after the Swing-By. This new maneuver has two main steps: a) A standard non-propelled Swing-By with the same parameters of the powered maneuver (the same V_{inf-}, r_p, ψ); b) Then, in a second step, an impulse (with the same magnitude δV of the impulse used in the powered Swing-By) is applied after the spacecraft leaves the secondary body. This impulse is assumed to be applied in a direction that extremize the transfer of energy. For the maneuvers where the goal is to increase the energy (180° < ψ < 360°), this impulse is posigrade (applied in the direction of the motion of the spacecraft) and for the maneuvers where the goal is to decrease the energy (0° < ψ < 180°), this impulse is retrograde (applied in the direction opposite to the motion of the spacecraft). Fig. 6 shows the results for ψ = 90° and 270° and for V_{inf-} = 1.0 and 2.0 km/s. The quantity plotted is $\left|\Delta E_{imp}\right| - \left|\Delta E_{impafter}\right|$, where ΔE_{imp} is the energy variation obtained by the powered Swing-By and $\Delta E_{impafter}$ is the energy variation of the maneuver that applies the impulse after the close approach. The system of axis has r_p (distance of closest approach, in km) in the horizontal axis and ψ (the angle of approach, in degrees) in the vertical axis.

It means that a positive value for this quantity indicates that the application of the impulse during the close approach is more efficient (in terms of causing a variation in energy of larger magnitude) than the application of an impulse with the same magnitude after the close approach.

To obtain the numerical value for the $\Delta E_{impafter}$ it is necessary to follow the steps shown below.

i) Evaluate the energy before the close approach (E_i) from the equation $E_i = \frac{1}{2}\left(V_{ix}^2 + V_{iy}^2\right) - \frac{\mu_1}{d}$, where μ_1 is the gravitational parameter of the primary body and d is the distance M_1-M_2;

ii) Next, the energy after the standart Swing-By maneuver is obtained, directly from the expression $E_o = E_i - 2V_2 V_{inf} \sin\delta \sin\psi$;

iii) Then, the magnitude of the velocity after the standard Swing-By maneuver is calculated from the energy, using the expression $V_o = \sqrt{2\left(E_o + \frac{\mu_1}{d}\right)}$;

iv) Finally, the $\Delta E_{impafter}$ is obtained from $\Delta E_{impafter} = \frac{1}{2}(V_o + \Delta V)^2 - \frac{\mu_1}{d} - E_i$, if $180° < \psi < 360°$; or $\Delta E_{impafter} = \frac{1}{2}(V_o - \Delta V)^2 - \frac{\mu_1}{d} - E_i$, if $0° < \psi < 180°$.

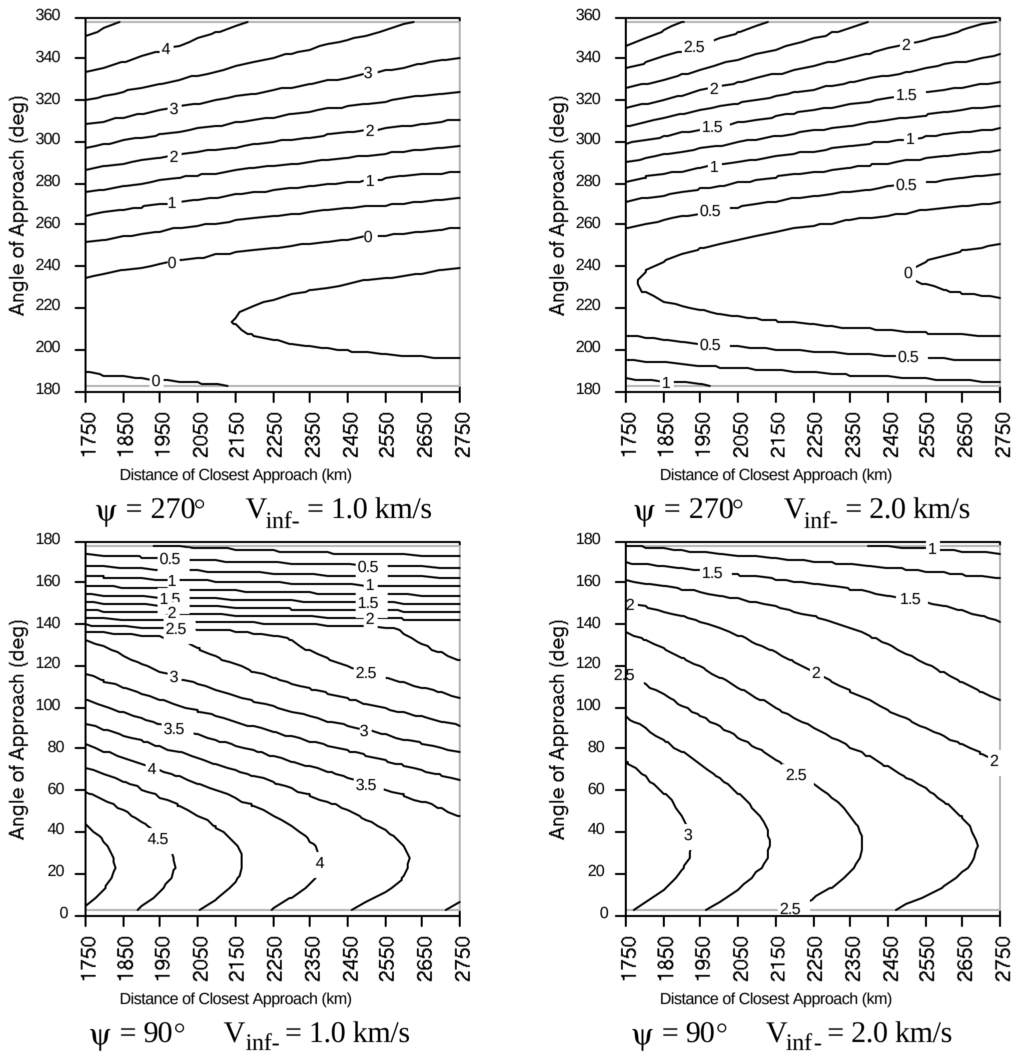


Fig. 6 - Efficiency of the Powered Swing-By.

It is clear to see that the efficiency is highly dependent on the angle of approach and that it has little dependence on the distance of closest approach. We can also see that the efficiency has the same behaviour for the cases V_{inf-} = 1.0 km/s and 2.0 km/s. There is only a shift on the values. It is also visible that there is a relatively small area of negative values, what means that an impulsive Swing-By maneuver is a better choice for most of the cases. Note also that the plots match very well at $\psi = 180°$, although the goal of the maneuver (gain or lose of energy) changes at this point.

5 - RESULTS USING THE RESTRICTED THREE-BODY PROBLEM

The goal of this section is to reproduce the maneuvers calculated in the previous sections using the well-known planar circular restricted three-body problem as the dynamical model. This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3), with negligible mass, is orbiting these two primaries. The motion of M_3 is supposed to stay in the plane of the motion of M_1 and M_2 and it is affected by both primaries, but it does not affect their motion [Szeb 1967]. The canonical system of units is used, and it implies that: i) The unit of distance (l) is the distance between M_1 and M_2; ii) The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; iii) The mass of the smaller primary (M_2) is given by $\mu = \frac{m_2}{m_1 + m_2}$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is $(1-\mu)$, so the total mass of the system is one; iv) The unit of time is defined such that the period of the motion of the primaries is 2π; v) The gravitational constant is one.

Then, the equations of motion in the rotating frame are:

$$\ddot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x}, \quad \ddot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \tag{12-13}$$

where Ω is the pseudo-potential given by:

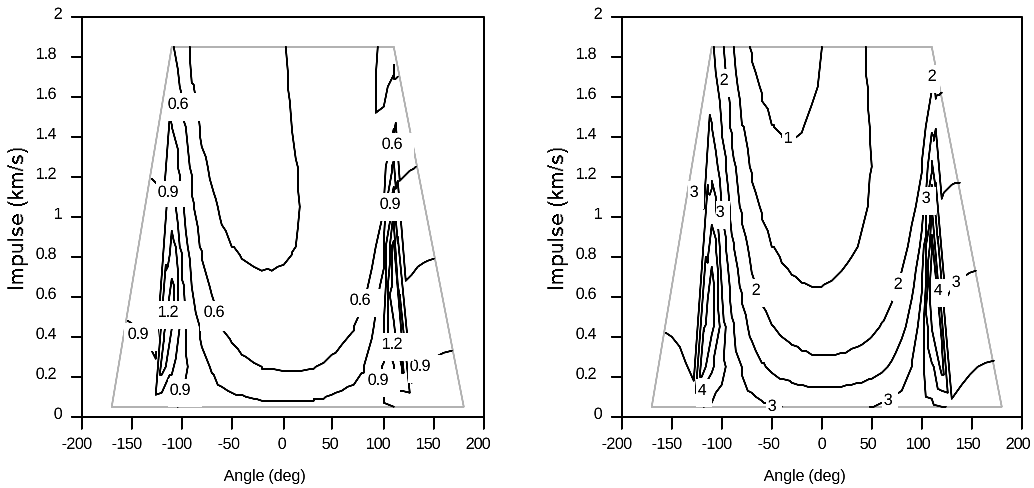
$$\Omega = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{14}$$

This system of equations has no analytical solutions, and numerical integration is required to solve the problem.

The equations of motion given by equation (12-14) are right, but they are not suitable for numerical integration in trajectories passing near one of the primaries. The reason is that the positions of both primaries are singularities in the potential V (since r_1 or r_2 goes to zero, or near zero) and the accuracy of the numerical integration is affected every time this situation occurs.

The solution for this problem is the use of regularization, that consists of a substitution of the variables for position (x-y) and time (t) by another set of variables (ω_1, ω_2, τ), such that the singularities are eliminated in these new variables. Several transformations with this goal are available in the literature ([Szeb 1967], chapter 3), like Thiele-Burrau, Lamaître and Birkhoff. They are called "global regularization", to emphasize that both singularities are eliminated in the same time. The case where only one singularity is eliminated at a time is called "local regularization". For the present research the Lamaître's regularization is used. More details are available in [Szeb 1967].

Fig. 7 shows the difference between the results obtained using the two-body celestial mechanics and the new results obtained using the restricted three-body problem for the maneuver with $\psi = 90°$. Results are similar for other cases studied and are not shown here to save space. The quantities shown are defined as: (Value for the two-body model) - (Value for the three-body model). To make the plots more clear, the results for the energy is multiplied by 10 and the results for the angular momentum is divided by 10^4. The magnitudes of the differences go from very close to zero until 0.15 (in energy) and $4*10^4$ (in angular momentum). Those numbers represents maximum errors in the order of a few percent (less than 10) in both cases. The errors are smaller in the interval $-90° < \alpha < 90°$ and they grow up close to the border of the graphics. It means that the two-body approximation of this maneuver gives better results when $-90° < \alpha < 90°$. It is also possible to conclude that this approximation increases in quality when the magnitude of the impulse increases.



Energy　　　　Angular Momemtum

Fig. 7 - Comparison Between the Two Models Used (Two and Three Bodies).

6 - CONCLUSIONS

A method to calculate the variations in velocity, energy and angular momentum for the powered Swing-By is developed based in the "patched-conic" approximation. Numerical examples are calculated for the Earth-Moon system to test and validate the algorithm developed. Then, the powered Swing-By maneuver is compared with a different maneuver that is performed in two steps: i) a non-propelled Swing-By; ii) an impulsive thrust applied after the Swing-By. In that way, it is possible to investigate the best position to apply the impulse. The results shown that for the majority of the cases studied the powered Swing-By is a better choice. Next, the maneuvers are reproduced under the dynamical model give by the restricted three-body problem. The differences between the results are shown. It is possible to conclude that the two-body problem gives a better approximation in the interval $-90° < \alpha < 90°$ and that this approximation increases in quality when the magnitude of the impulse increases.

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