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Resumo	The gravitational capture is a very interesting phenomenon that allows a spacecraft that is in a hyperbolic orbit around a celestial body to be transferrede to an elliptic orbit without the use of any propulsive system. One of the objectives of this paper is to study the problem of gravitational capture in the restricted three-body problem. We study the importance of several of the parameters involved for a capture in the Earth-Moon system, with emphasis in the time required for the capture. To make this study, a large number of trajectories starting close to the secondary body are numerically integrated backward in time. The initial position and velocity of the spacecraft are changed and it is verified if an escape occurs for every trajectory. From the results it is possible to study the balance between the time required for the capture and the saved for each maneuver. The existence of "windows" with short time for capture, that are very sharp for values of C3 close to the minimum (- 0.2) are found. The second part of this paper has the goal of developing a set of analytical equations based in the third body perturbation, using the Planetary equations. It is shown that the gravitational capture does not exist in models using average techniques, what confirms the temporary character of this phenomenon.
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A STUDY OF THE GRAVITATIONAL CAPTURE USING AVERAGE METHODS

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ABSTRACT

The gravitational capture is a very interesting phenomenon that allows a spacecraft that is in a hyperbolic orbit around a celestial body to be transferred to an elliptic orbit without the use of any propulsive system. One of the objectives of this paper is to study the problem of gravitational capture in the restricted three-body problem. We study the importance of several of the parameters involved for a capture in the Earth-Moon system, with emphasis in the time required for the capture. To make this study, a large number of trajectories starting close to the secondary body are numerically integrated backward in time. The initial position and velocity of the spacecraft are changed and it is verified if an escape occurs for every trajectory. From the results it is possible to study the balance between the time required for the capture and the ΔV saved for each maneuver. The existence of "windows" with short time for capture, that are very sharp for values of C3 close to the minimum ($\approx$ -0.2) are found. The second part of this paper has the goal of developing a set of analytical equations based in the third body perturbation, using the Planetary equations. It is shown that the gravitational capture does not exist in models using average techniques, what confirms the temporary character of this phenomenon.

INTRODUCTION

The phenomenon called gravitational capture is a very interesting characteristic of some dynamical system, like the three- or four- body system in celestial mechanics. It is under investigation for some time now, specially by Yamakawa and colleagues[1,2,3] and the group formed by Belbruno, Miller, Krish and Hollister[4,5,6,7,8,9,10,11,12]. The basic idea is that a slightly hyperbolic orbit (with a residual positive energy) around a celestial body can be transformed in a slightly elliptic orbit (with a residual negative energy) without the use of any propulsive system. The only forces responsible for this capture are the gravitational perturbations from one or more other bodies. One of the most important applications of this property is the construction of trajectories to the Moon. In this maneuver, a spacecraft leaves a parking orbit around the Earth on its way to the Moon, makes a swing-by with the Moon to go to a distant region and then, using the perturbations of the Sun and the Earth, it comes back to the Moon for a gravitational capture. This capture is only temporary, but an impulse can be applied during this temporary capture to make it permanent. The advantage is that this impulse as a magnitude smaller than the one required for a standard maneuver without the gravitational capture, and it means that there is a saving in the fuel involved in this special type of maneuver.

In this paper we study the importance of several of the parameters involved for a capture in the Earth-Moon system, specially the time required for the capture.

The second part of this paper has the goal of developing a set of analytical equations based in the third body perturbation, using the Planetary equations. Single and double-average techniques are also considered. It is shown that the gravitational capture does not exist in models using average techniques, what confirms the temporary character of this phenomenon.

MATHEMATICAL MODEL

The model used in this part of the paper is the well-known planar circular restricted three-body problem. This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3), with negligible mass, is orbiting these two primaries. The motion of M_3 is supposed to stay in the plane of the motion of M_1 and M_2 and it is affected by both primaries, but it does not affect their motion[13]. The standard canonical system of units associated with this model is used (the unit of distance is the distance between M_1 and M_2, and the unit of time is chosen such that the period of the motion of M_2 around M_1 is 2π). Under this model, the equations of motion are:

$$\ddot{x}-2\dot{y} = x-\frac{\partial U}{\partial x} = \frac{\partial \Omega}{\partial x} \qquad (1)$$

$$\ddot{y}+2\dot{x} = y-\frac{\partial U}{\partial y} = \frac{\partial \Omega}{\partial y} \qquad (2)$$

where Ω is the pseudo-potential function given by:

$$\Omega = \frac{1}{2}\left(x^2+y^2\right)+\frac{(1-\mu)}{r_1}+\frac{\mu}{r_2} \qquad (3)$$

and x and y are two perpendicular axes with the origin in the center of mass of the system, with x pointing from M_1 (that has coordinates $x = -\mu$, $y = 0$) to M_2 (that has coordinates $x = 1-\mu$, $y = 0$).

The equations of motion given by (1-3) are correct, but they are not suitable for numerical integration in trajectories passing near one of the primaries. The reason is that the positions of both primaries are singularities in the potential U (since r_1 or r_2 goes to zero, or near zero) and the precision of the numerical integration is affected every time this situation occurs. The solution for this problem is the use of regularization, that consists of a substitution of the variables for position (x-y) and time (t) by another set of variables (ω_1, ω_2, τ), such that the singularities are eliminated in these new variables.

PRELIMINARY NUMERICAL RESULTS

To quantify the "gravitational captures", this problem is studied under several different initial conditions. The assumptions made for the numerical examples presented in the first part of this section are (some of them ~~is~~ are changed later, to generalize the results):

i) The system of primaries used is the Earth-Moon system (some fictitious systems is used later to generalize the results);

ii) The motion is planar everywhere, because the capture out of plane cannot achieve larger savings[2];

iii) The starting point of each trajectory is 100 km from the surface of the Moon (r_p = 1838 km from the center of the Moon). Then, to specify the initial position completely, it is necessary to give the value of one more variable. The variable used here is the angle α, an angle measured from the Earth-Moon line, in the counter-clockwise direction and starting in the side opposite to the Earth (see Fig. 1) (different values of r_p is used later to generalize the

results);
iv) The magnitude of the initial velocity is calculated from a given value of $C3 = 2E = V^2 - \frac{2\mu}{r}$, where E is the two-body energy of the spacecraft with respect to the Moon, V is the velocity of the spacecraft, μ is the gravitational parameter of the Moon and r is the distance between the spacecraft and the center of the Moon. The direction of the velocity is assumed to be perpendicular to the line spacecraft-center of the Moon and pointing to the counter-clockwise direction for a direct (posigrade) orbit and to the clockwise direction for a retrograde orbit (see Fig. 1);
v) To consider that an escape occurred, it is requested (following the conditions used in Refs. 1, 2 and 3) that the spacecraft reaches a distance of 100000 km (0.26 canonical units) from the center of the Moon in a time shorter than 50 days. The value for the distance cames from the equation for the limit = $(2\mu)^{1/3}$, that is well explained in Ref. 2. Fig. 1 shows the point P where the escape occurs. The angle that specifies this point is called the "entry position angle" and it is designated with the letter β. There is also a check to verify if a crash into the Moon did not happen.

Then, for each initial position, the trajectories are numerically integrated backward in time. Every escape in backward time corresponds to a "gravitational capture" in forward time. The time-of-flight until an escape occurs is obtained. The stopping criteria for the numerical integration is the one that comes first among the three possibilities: the time is longer than 50 days; the distance from the Moon is longer than 100000 km or the distance from the Moon is smaller than 1738 km (the Moon radii). The numerical simulations are performed in an IBM-PC Pentium 100 MHz using the Microsoft Fortran Powerstation 1.0. The numerical integration method used is the Runge-Kutta of fourth order. Then, the results are organized and plotted in several figures. The time-of-flight for escape in all those figures is expressed in canonical units, what means that 1 unit of time is equivalent to 4.46 days. The next subsections show the results in detail.

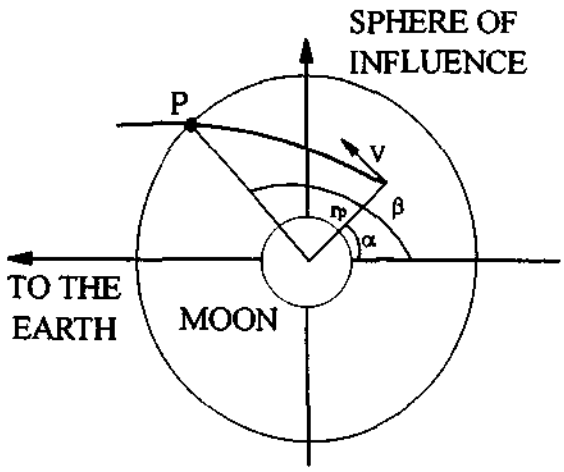

Fig. 1 - Variables to specify the initial conditions of the spacecraft.

Influence of the Parameters in the Time Required For the Capture

In this section, the numerical tools developed in this paper are used to study the influence of the parameters that govern this problem. Those parameters are: the system of primaries involved, specified by the parameter μ; the distance from the spacecraft to the secondary body in the moment that the impulse is applied to complete the maneuver (r_p); the energy C3 of the spacecraft at this moment; the direction of the velocity at this point (it is assumed to be perpendicular to the radius vector, but it is free to be posigrade or retrograde); and the departure angle α (see Fig. 1). The simulations showed the results described below. This research is a continuation of previous work performed by the authors[14,15]

i) Effects of the mass parameter. Several simulations were made to study the influence of the mass parameter in the time required for the capture. This particular parameter brings a new difficult into the problem. The value of the maximum savings increase very much when the mass parameter increases. So, studying this problem for a fixed value of C3 $\approx$ -0.2 (that is close to the maximum saving for μ = 0.01) makes the time required for the capture to be close to zero for different values of μ (μ = 0.3, 0.5, etc.). Then, the method used in this paper to solve this problem is to find and use a value of C3 that is close to the limit (minimum value that allows a gravitational capture) for a given μ. Then, the comparison is made using a fixed value of r_p = 0.004781477 in canonical units, a posigrade direction for the velocity and the value of C3 that is the one that gives the maximum saving for a given μ. The results showed that there is no general trend for the variation of this parameter. There are very large oscillations in the time required and, for every value of α, there is a different value of the mass parameter that holds the minimum time. This oscillatory behavior is due to the necessity of changing the values of C3, as explained in the beginning of the present section. The effect of this parameter is studied only to make this paper more complete and it is not a key parameter for practical applications, because in a real mission the system of primaries is always fixed in advance.

ii) Effects of the r_p. To study the importance of this parameter, simulations were made for the Earth-Moon system (μ = 0.0121285627) and for the posigrade direction of the velocity. The parameter r_p was varied in a wide range of values (1800 km $\leq r_p \leq$ 22000 km) for several values of C3. Several simulations were performed. The first fact noted is that the results for 1800 km < r_p < 7000 km are very similar to each other. It means that changing this parameter in a range of values close to the Moon does not give a significant impact in the time required for the capture. Increasing this parameter to r_p > 12000 km it is possible to see an increase in the time-of-flight. The amount of this increase changes according to the value of α. In the maximum cases, it reaches the level of three times larger than the value obtained with lower values for r_p. This situation occurs when α is between 50° and 200°. So, the general conclusion is that the increase of r_p has the effect of increasing the time for capture, but this effect is visible only for r_p $\geq$ 10000 km.

iii) Effects of the departure angle α. This is a very important parameter in this problem, because it has a strong impact in the savings obtained for the maneuver. Simulations to measure the time-of-flight as a function of α were made for several values of C3 in the range $-0.2 \leq C3 \leq 0$. Fig. 2 shows the results obtained for C3 = -0.14. This figure was build using a step of 0.1° in α, and it is representative of the others. The radial distance represents the time and the angular variable represents α. The ratio between the higher and the lower values is of the order of ten. The minimum times belong to the regions $120° \leq \alpha \leq 180°$ and $300° \leq \alpha \leq 360°$. Those results bring the most important conclusion of this section. They show that it is possible to obtain a minimum time-of-flight that are ten times shorter than the maximum without any reduction in the savings, since C3 is kept constant. The only task that has to be performed is to find the value of α that allows those savings in time. This information can be obtained from the Fig. 2.

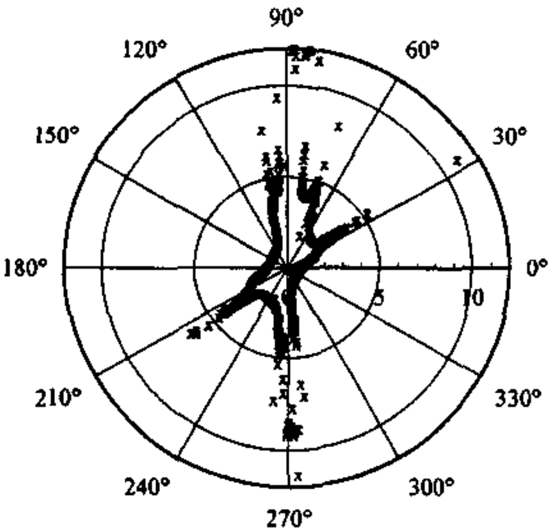

Fig. 2 - Effects of the departure angle.

iv) Effects of the direction of the velocity. Several simulations were made keeping constant the mass parameter (μ = 0.0121285627) and the periapse distance (r_p = 1838 km) and changing the values of C3 ($-0.2 \le C3 \le 0$) for posigrade and retrograde orbits. The orbit is called posigrade when the initial velocity is counter-clockwise and retrograde when it is clockwise. The simulations showed that, for values of C3 in the first half of the interval considered ($-0.1 \le C3 \le 0$), in the majority of the domain (values of α) the time-of-flight required for the capture is almost independent of the direction of the velocity. A significant difference occurs only in very specific positions ($30^\circ \le \alpha \le 70^\circ$ and $220^\circ \le \alpha \le 260^\circ$) and, in those cases, the posigrade orbits have a smaller value for the time-of-flight. But, for the most important cases (C3 about -0.2) where the savings are close to the maximum, there are significant differences in the time required for the capture for almost all the values of α. By examining in detail the results, it is possible to conclude that the posigrade orbits require a smaller time for the capture for all values of α. The ratio of those times (time required for the retrograde orbits divided by the time required for the posigrade orbits) can reach three in the region $130^\circ \le \alpha \le 180^\circ$. The posigrade (direct) orbits holds all the minimums. When C3 approach the value of -0.2, the occurrence of retrograde orbits decreases faster than the occurrence of posigrade orbits. In this situation, the posigrade orbits dominate the plots and they are the only choice in a large portion of the domain. In the small parts of the domain that have retrograde orbits, the difference in the time for capture increases.

v) Effects of C3. This is also a very important topic of investigation. To perform this research, simulations were made keeping $\mu = 0.0121285627$ and the direction of the velocity posigrade. Then, a set of simulations were performed in the whole interval $0^\circ \le \alpha \le 360^\circ$. This study wants to quantify numerically the balance that exist between consumption of fuel and time required for the maneuver. The approach to solve this problem is the following. A value of C3 is fixed and then a plot of the time-of-flight versus α is made. From this simulation, the minimum value of the time-of-flight is found. Repeating this process for several values of C3 it is possible to build a large table (omitted here), that shows the maximum magnitude of C3 obtained for every value of time-of-flight. The values of α and ΔV saved (in canonical units) can also be shown in this table. Fig. 3 shows those results in a graphic form for r_p = 1838 km. The expected result that an increase in the savings obtained causes an increase in the time-of-flight is quantified in the plot. Several others simulations changing the values of r_p, not shown here, were made and it was possible to conclude that r_p is a parameter that has little effect in this problem. It is also possible to conclude from the results that the regions where the minimum are found are always in the region close to the interval $309^\circ \le \alpha \le 345^\circ$.

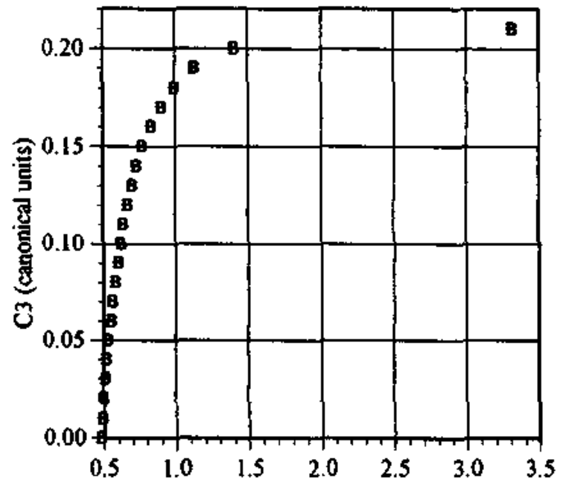

Fig. 3 - Effects of C3.

THE DISTURBING FUNCTION

This section has the goal of developing an analytical study of the perturbation caused in a spacecraft by a third body involved in the dynamics. The assumptions are the same ones made in the restricted three-body problem.

The main body with mass m_0 is fixed in the center of the reference system x-y. The perturbing body with mass m' is in a circular orbit with semi-major axis a' and mean motion n' (given by the expression $n'^2 a'^3 = G[m_0 + m']$) in the plane of the figure. The massless spacecraft m is in a generic three dimensional orbit which orbital elements are: a, e, i, ω, Ω and the mean motion is n (given by the expression $n^2 a^3 = Gm_0$).

In this situation, the disturbing potential that the spacecraft has from the action of the disturbing body is given by:

$$R = \frac{\mu'}{\sqrt{r^2 + r'^2 - 2rr'\mathrm{Cos}(S)}}$$

Using the traditional expansion in Legendre polynomials (assuming that r' >> r) the following expression can be found:

$$R = \frac{\mu'}{r'} \sum_{n=2}^{\infty} \left(\frac{r}{r'}\right)^n P_n(\mathrm{Cos}(S))$$

Simulations are made using the equations of motion obtained directly from those equations. The results are shown later in this paper.

The next step is to average those quantities over the short period satellite as well as with respect to the distant perturbing body. The standard definition for average used in this research is $\langle f \rangle = \frac{1}{2\pi}\int_0^{2\pi}(f)dM$, where M is the mean anomaly, that is proportional to time.

After performing the average over the perturbed body, we have:

$$\overline{R}_2 = \frac{\mu' a^2 n'^2}{2}\left(\frac{a'}{r'}\right)^3 \left[\left(1 + \frac{3}{2}e^2\right)\left(\frac{3}{2}(\alpha^2 + \beta^2) - 1\right) + \ldots \right.$$
$$\left. \ldots + \frac{15e^2}{4}(\alpha^2 - \beta^2)\right]$$

$$\overline{R}_3 = \frac{\mu' a^3 n'^2}{2a'}\left(\frac{a'}{r'}\right)^4 \left[\frac{15\alpha e(4 + 3e^2)}{8} - \ldots \right.$$
$$\left. \ldots - \frac{25\alpha^3 e(3 + 4e^2)}{8} + \frac{75\alpha\beta^2 e(e^2 - 1)}{8}\right]$$

$$\overline{R}_4 = \frac{3\mu' a'^3 n'^2 a^4}{64 r'^5}\left[(8 + 40e^2 + 15e^4) - 10\alpha^2(4 + \ldots \right.$$
$$\ldots + 41e^2 + 18e^4) + 35\alpha^4(1 + 12e^2 + 8e^4) - 10\beta^2(4 - \ldots$$
$$\left. \ldots - e^2 - 3e^4) + 70\alpha^2\beta^2(1 + 5e^2 - 6e^4) + 35\beta^4(e^2 - 1)^2\right]$$

Here $\alpha = (\hat{P}.\hat{r}')$ and $\beta = (\hat{Q}.\hat{r}')$, where $\hat{r}'$ is the unit vector pointing from the central body to the disturbing body and $\hat{P}$ and $\hat{Q}$ are the usual orthogonal unit vectors, functions of (i, ω, Ω), in the plane of the satellite orbit, $\hat{P}$ pointing towards the periapse.

Considering the special case of circular orbits for the disturbing body and performing the second average with respect to the disturbing body to eliminate the variable M', we have the results shown in Table 1.

Table 1 - Disturbing Functions up to P_4.

$$\langle \overline{R}_2 \rangle = \frac{\mu' a^2 n'^2}{16}\left[\left(2+3e^2\right)\left(3Cos^2(i)-1\right)+15e^2 Sin^2(i)Cos(2\omega)\right] \quad (4)$$

$$\langle \overline{R}_3 \rangle = 0 \quad (5)$$

$$\langle \overline{R}_4 \rangle = \frac{9n'^2 a^4}{65536a'^2}\left[144+720e^2+270e^4+\left(320+1600e^2+600e^4\right)Cos(2i)+\left(560+2800e^2+1050e^4\right)Cos(4i)+\ldots\right.$$
$$\ldots+\left(1680e^2+840e^4\right)Cos(2\omega)+4410e^4Cos(4\omega)+\left(2240e^2+1120e^4\right)Cos(2i)Cos(2\omega)+\ldots$$
$$\left.\ldots+\left(3920e^2+1960e^4\right)Cos(4i)Cos(2\omega)+5880e^4Cos(2i)Cos(4\omega)+1470e^4Cos(4i)Cos(4\omega)\right] \quad (6)$$

The Equations of Motion

After calculating $\langle \overline{R}_2 \rangle = \overline{\overline{R}}_2$, $\langle \overline{R}_3 \rangle = \overline{\overline{R}}_3$ and $\langle \overline{R}_4 \rangle = \overline{\overline{R}}_4$, the next step is to obtain the equations of motion of the spacecraft. They come from the Lagrange's planetary equations (Taff, 1985) in the form that depends on the derivatives of the disturbing function R with respect to the Keplerian elements.

Our first result in this section is the integration of the equations of motion generated by making $R = R_2$, $R = R_2 + R_3$ and $R = R_2 + R_3 + R_4$, successively. Figs. 4 to 6 show the results for the behavior of the semi-major axis and the eccentricity. It is clear to see that the semi-major axis always increases, what indicates that the gravitational escape in reverse time (and the capture in forward time) occurs. The speed of the process increases when more terms are considered in the model. The eccentricity oscillates with large amplitude. The others orbital elements were also computed, but they are omitted here to save space. In general, M0 always increases and the argument of the periapsis oscillates, but has a tendency to increase in the long term.

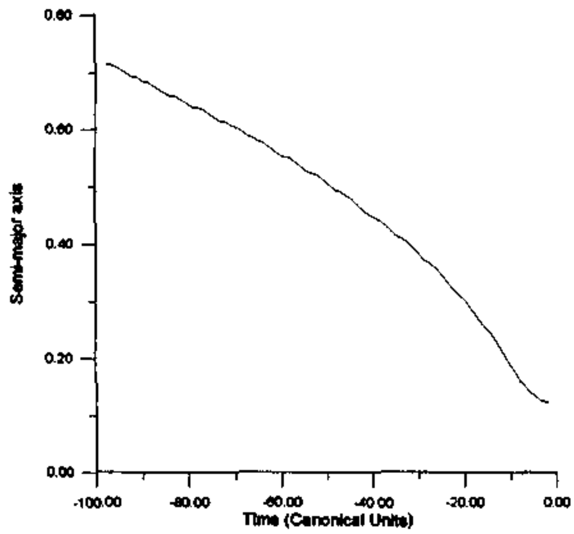

Fig. 4a - Semi-major axis vs. Time ($R = R_2$).

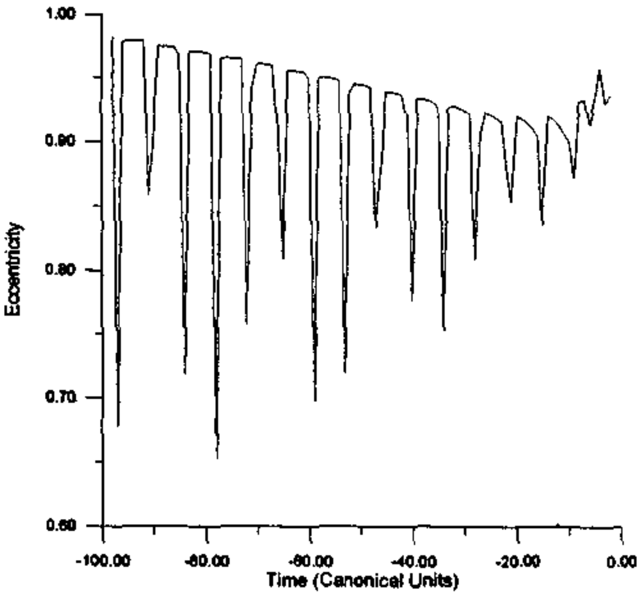

Fig. 4b - Eccentricity vs. Time ($R = R_2$).

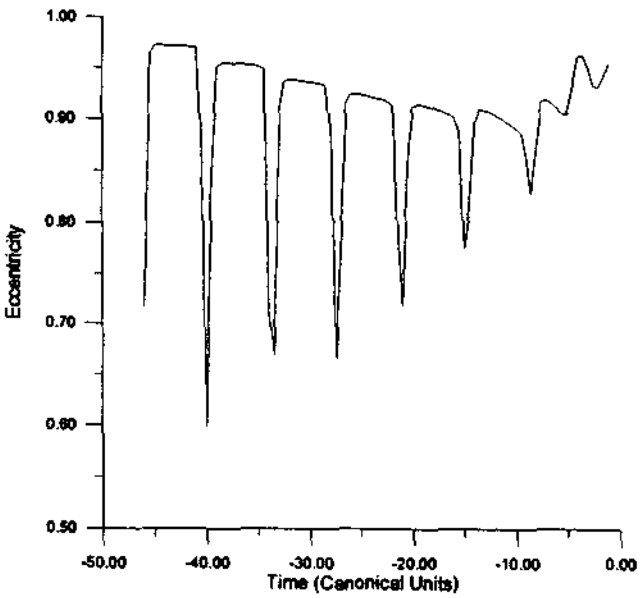

Fig. 5b - Eccentricity vs. Time ($R=R_2+R_3$).

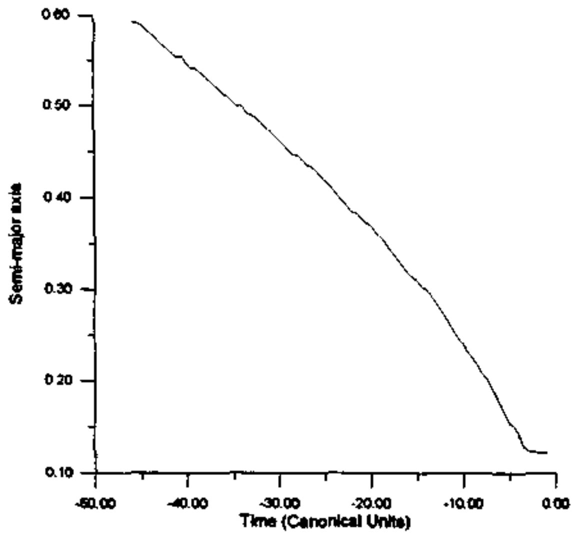

Fig. 5a - Semi-major axis vs. Time ($R=R_2+R_3$).

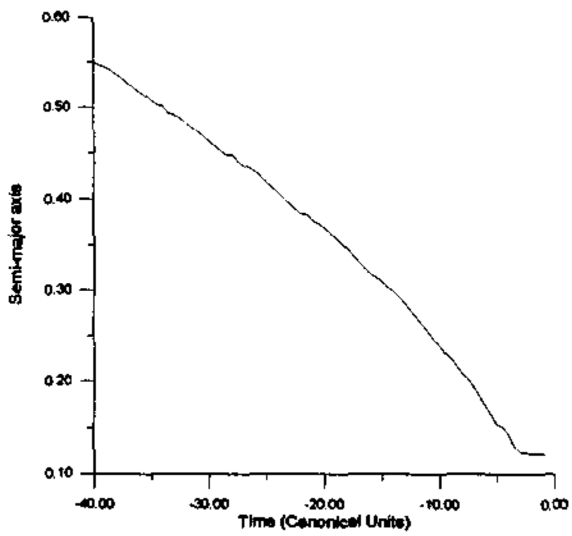

Fig. 6a - Semi-major axis vs. Time ($R=R_2+R_3+R_4$).

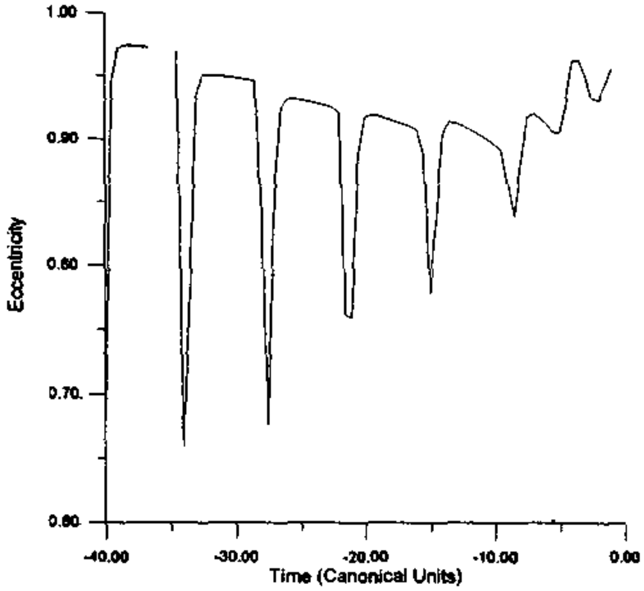

Fig. 6b - Eccentricity vs. Time ($R=R_2+R_3+R_4$).

To make a comparison, the numerical integration is performed for the full system (with no truncation) and the results are shown in terms of the Keplerian elements. Fig. 7 shows those results. It is immediate to conclude that the speed of the process is a lot larger, indicating the transitory character of this phenomenon.

The next step is to obtain the derivatives of R for the averaged models. For the second-order model the assumption $R=\overline{\overline{R_2}}$ is made and it is possible to obtain the following expression for the derivatives:

$$\frac{\partial \overline{\overline{R_2}}}{\partial a}=\frac{\partial \overline{\overline{R_2}}}{\partial \Omega}=0 \qquad (7)$$

$$\frac{\partial \overline{\overline{R_2}}}{\partial e}=K\cdot\left[6\cdot e\cdot\left(3\cdot\cos(i)^2-1\right)+30\cdot e\cdot\sin(i)^2\cdot\cos(2\cdot\omega)\right] \qquad (8)$$

$$\frac{\partial \overline{\overline{R_2}}}{\partial i}=3\cdot K\cdot\sin(2\cdot i)\cdot\left(-2-3\cdot e^2+5\cdot e^2\cdot\cos(2\cdot\omega)\right) \qquad (9)$$

$$\frac{\partial \overline{\overline{R_2}}}{\partial \omega}=-30\cdot K\cdot e^2\cdot\sin(i)^2\cdot\sin(2\cdot\omega) \qquad (10)$$

where $K=\dfrac{\mu' a^2 n'^2}{16}$ and $\mu'=\dfrac{m'}{m'+m_0}$.

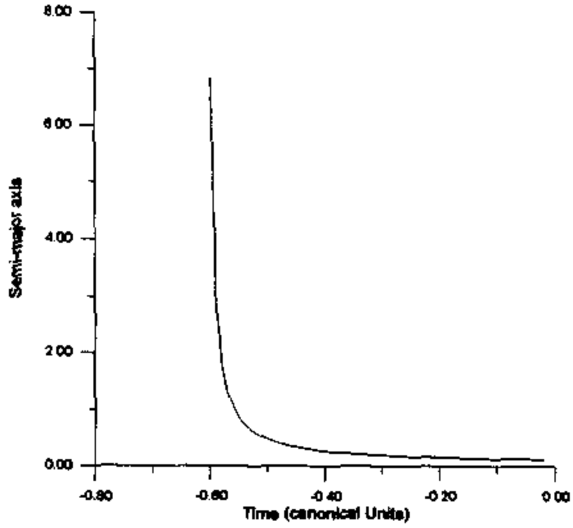

Fig. 7a - Semi-major axis vs. Time (Numerical).

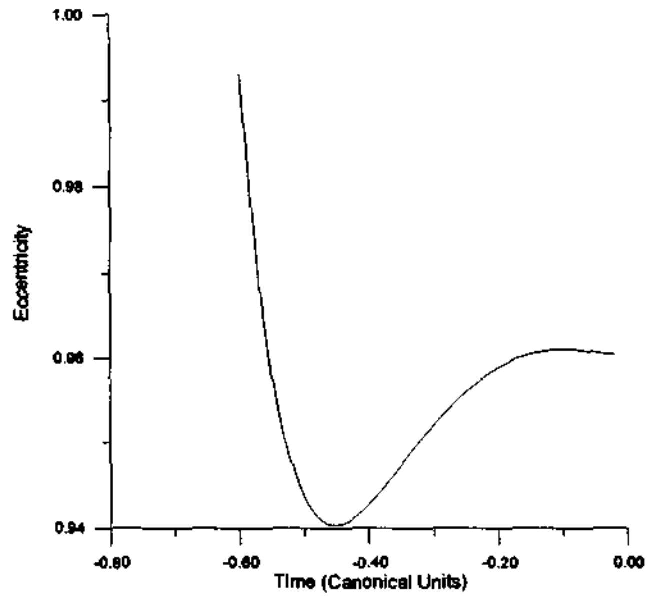

Fig. 7b - Eccentricity vs. Time (Numerical).

The equations of motion are then obtained using the Planetary equations.

For the fourth-order model the assumption made is $R = \overline{\overline{R_2}} + \overline{\overline{R_4}}$ (since $\overline{\overline{R_3}} = 0$. Then it is necessary to obtain the partials shown in Table 2.

Table 2 - Partials for the Disturbing Functions of P_4.

$$\frac{\partial \overline{\overline{R_4}}}{\partial e} = K2 \begin{bmatrix} 1440e + 1080e^3 + \left(3200e + 2400e^3\right)\cdot\cos(2\cdot i) + \left(5600e + 4200e^3\right)\cdot\cos(4\cdot i) \ldots \\ + \left(3360e + 3360e^3\right)\cdot\cos(2\cdot\omega) + 17640e^3\cdot\cos(4\cdot\omega) \ldots \\ + \left(4480e + 4480e^3\right)\cdot\cos(2\cdot i)\cdot\cos(2\cdot\omega) + \left(7840e + 7840e^3\right)\cdot\cos(4.i)\cdot\cos(2\cdot\omega) \ldots \\ + 23520e^3\cdot\cos(2\cdot i)\cdot\cos(4\cdot\omega) + 5880e^3\cdot\cos(4\cdot i)\cdot\cos(4\cdot\omega) \end{bmatrix}$$

$$\frac{\partial \overline{\overline{R_4}}}{\partial i} = -K2 \begin{bmatrix} 2\cdot\left(320 + 1600e^2 + 600e^4\right)\cdot\sin(2\cdot i) + 4\cdot\left(560 + 2800e^2 + 1050e^4\right)\cdot\sin(4\cdot i) \ldots \\ + 2\cdot\left(2240e^2 + 1120e^4\right)\cdot\sin(2\cdot i)\cdot\cos(2\cdot\omega) + 11760e^4\cdot\sin(2\cdot i)\cdot\cos(4\cdot\omega) \ldots \\ + 5880e^4\cdot\sin(4\cdot i)\cdot\cos(4\cdot\omega) \end{bmatrix}$$

$$\frac{\partial \overline{\overline{R_4}}}{\partial \omega} = 2K2 \begin{bmatrix} 72 + 360e^2 + 135e^4 + 160\cos(2\cdot i) + 800\cos(2\cdot i)\cdot e^2 + 300e^4\cdot\cos(2\cdot i) \ldots \\ + 280\cos(4\cdot i) + 1400e^2\cdot\cos(4\cdot i) + 525e^4\cdot\cos(4\cdot i) + 840\cos(2\cdot\omega)\cdot e^2 \ldots \\ + 420\cos(2\cdot\omega)\cdot e^4 + 2205e^4\cdot\cos(4\cdot\omega) + 1120\cos(2\cdot i)\cdot\cos(2\cdot\omega)\cdot e^2 \ldots \\ + 560\cos(2\cdot i)\cdot\cos(2\cdot\omega)\cdot e^4 + 1960\cos(4.i)\cdot\cos(2\cdot\omega)\cdot e^2 \ldots \\ + 980\cos(4.i)\cdot\cos(2\cdot\omega)\cdot e^4 + 2940e^4\cdot\cos(2\cdot i)\cdot\cos(4\cdot\omega) \ldots \\ + 735e^4\cdot\cos(4\cdot i)\cdot\cos(4\cdot\omega) \end{bmatrix}$$

where $K2 = \dfrac{9n'^2 a^4}{65536a'^2}$.

Then, the right-hand sides of those equations are added to the right-hand sides of the equations of motion to form de derivatives for the disturbing function with respect to the Keplerian elements. This result is then used to generate the equations of motion.

An important property of the averaged methods is that the semi-major axis always remains constant. This fact occurs because, after the averaging, the disturbing function does not depend on M_0 and the Planetary equations show that under this circumstance $\dfrac{da}{dt} = 0$. This is a proof that the gravitational capture does not occur in any of those averaged models.

7 - CONCLUSIONS

This paper studied the "gravitational capture" in the restricted three-body problem. A detailed and new study of the time-of-flight required for the gravitational capture was performed. The importance of each individual parameter was studied in detail. The existence of "windows" with short time for capture, that are very sharp for values of C3 close to the minimum ($\approx$ -0.2) were found. Some “blank regions”, where the gravitational capture is not possible, were also found. Optimal problems, like finding trajectories that ends in gravitational capture with minimum time or maximum savings can also be solved using the results available in this paper. Those results are important to mission designers willing to use this type of maneuver in real missions. Then, we performed a study using the equations of motion given by the planetary equations. We used an expansion in polynomials of Legendre up to order four. We found that the gravitational escape exists in models that do not have average, but the process is a lot slower. When any kind of average is made (single or double) the gravitational escape does not occur anymore.

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