

Referência Completa

Tipo da Referência Conference Proceedings

Repositório sid.inpe.br/iris@1905/2005/07.29.06.22

Metadados sid.inpe.br/iris@1905/2005/07.29.06.22.20

Site mtc-m05.sid.inpe.br


Rótulo 7920

Chave Secundária INPE-6728-PRE/2750

Chave de Citação NetoPrad:1996:StGrCa

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Grupo 1 DMC-INPE-MCT-BR

Título Study of the gravitational capture in the elliptical restricted three-body problem 

Nome do Evento International Symposium on Space Technology and Science, 20.

Ano 1996

Título do Livro Anais

Data 19-25 may 1996

Localização do Evento Gifu, JP

Palavras-Chave ENGENHARIA E TECNOLOGIA ESPACIAL, MECANICA ESPACIAL, MANOBRA ORBITAL, CAPTURA GRAVITACIONAL.

Resumo Space maneuvers using gravitational capture has a large potential of use in the near future. The fuel consumption, in a transfer maneuver using this technique, can be smaller than the one required by the Hohmann transfer. In this paper we develop a numerical algorithm to study the problem of gravitational capture in the elliptical restricted three-body problem. The effect of the true anomaly for a fixed eccentricity and the effect of the eccentricity for a fixed true anomaly are studied. We show the numerical results and we conclude that the savings increase with the eccentricity and when the true anomaly goes close to zero, as expected. The contribution of this paper is to show the quantitative results of those savings.

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Data de Acesso 28 ago. 2015

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MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

INPE-6728-PRE/2750

**STUDY OF THE GRAVITATIONAL CAPTURE IN THE
ELLIPTICAL RESTRICTED THREE-BODY PROBLEM**

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Antonio Fernando Bertachini de Almeida Prado

Paper presented at the 20th International Symposium on Space Technology and Science,
Gifu, Japan, May 19-25, 1996

INPE
São José dos Campos
1998



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20th International Symposium
on Space Technology and Science
Gifu, Japan
May 19 - 25, 1996

20th ISTS Secretariat
Hamamatsu-cho Central Building, 1-29-6 Hamamatsu-cho
Minato-ku, Tokyo 105
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STUDY OF THE GRAVITATIONAL CAPTURE IN THE ELLIPTICAL RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

Space maneuvers using gravitational capture has a large potential of use in the near future. The fuel consumption, in a transfer maneuver using this technique, can be smaller than the one required by the Hohmann transfer. In this paper we develop a numerical algorithm to study the problem of gravitational capture in the elliptical restricted three-body problem. The effect of the true anomaly for a fixed eccentricity and the effect of the eccentricity for a fixed true anomaly are studied. We show the numerical results and we conclude that the savings increase with the eccentricity and when the true anomaly goes close to zero, as expected. The contribution of this paper is to show the quantitative results of those savings.

INTRODUCTION

Gravitational capture is the phenomenon where a particle, coming from outside the sphere of influence of a celestial body, has its velocity relative to this celestial body reduced by gravitational forces¹ only. In some cases this particle can even stay in orbit around this celestial body temporary. This happens due to the change of the two-body energy (in the system formed by the particle and the celestial body) of the massless body from positive to negative. The two-body energy is constant only when the dynamics is considering only the forces that come from the gravitational attraction from the two bodies involved. It is no longer a constant if the system has more forces acting, like the gravitational attraction that comes from three or more bodies. In this case, the perturbation of the other bodies of the system causes the variation of this energy. The

importance of this study is that this phenomenon can be used to decrease the fuel expenditure for a mission going from one of the bodies to the other, like an Earth-Moon mission. This maneuver is performed by applying an impulse to the spacecraft during this temporary capture to accomplish a permanent capture. This phenomenon was studied in details in the dynamics given by the circular restricted three-body problem. Some examples of the papers that use the variation of the two-body energy to study this problem are: Yamakawa¹, Yamakawa *et al.*^{2,3,4} and Vieira Neto and Prado⁵. Some of those references show that the fuel consumption, in a transfer maneuver, can be smaller than the one required by the Hohmann transfer^{3,6,7}. In this paper we consider the problem of the gravitational capture under the model given by the elliptic restricted three-body problem. It means that it is assumed that the system is formed by two main bodies that are in elliptic orbits around their center of mass and a massless third body that is moving under the gravitational attraction of the two primaries. This third body has its motion constrained to the orbital plane of the two primaries.

There are several references that study the problem of the gravitational capture for the elliptical restricted three-body problem, like Bailey^{8,9,10} and Heppenheimer¹¹, but they do not have the same goals and the same approach that we have in the present paper.

THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM

The equations of motion for the spacecraft are assumed to be the ones valid for the well-known planar restricted elliptic three-

body problem. We also use the standard canonical system of units, which implies that:

1. The unit of distance is the semi-major axis of the orbit M_1 and M_2 ,

2. The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one;

3. The mass of the smaller primary (M_2) is given by $\mu = \frac{m_2}{m_1 + m_2}$ (where m_1 and m_2 are the real masses of M_1 and M_2 , respectively) and the mass of M_2 is $(1-\mu)$, to make the total mass of the system unitary;

4. The unit of time is defined such that the period of the motion of the two primaries is 2π ;

5. The gravitational constant is one.

There are several systems of reference that can be used to describe the elliptic restricted problem¹². In this paper the fixed (inertial) system is used, as described below

In the fixed system the origin is located in the barycenter of the two heavy masses M_1 and M_2 . The horizontal axis \bar{x} is the line connecting M_1 and M_2 (at the initial time) and the vertical axis \bar{y} is perpendicular to \bar{x} . In this system, the positions of M_1 and M_2 are:

$$\bar{x}_1 = -\mu r \cos v \quad \bar{y}_1 = -\mu r \sin v$$

$$\bar{x}_2 = (1-\mu)r \cos v \quad \bar{y}_2 = (1-\mu)r \sin v$$

where r is the distance between the two primaries, given by $r = \frac{1-e^2}{1+e \cos v}$, and v is the true anomaly of M_2 .

Then, in this system, the equations of motion of the massless particle are:

$$\bar{x}'' = \frac{-(1-\mu)(\bar{x} - \bar{x}_1)}{r_1^3} - \frac{\mu(\bar{x} - \bar{x}_2)}{r_2^3}$$

$$\bar{y}'' = \frac{-(1-\mu)(\bar{y} - \bar{y}_1)}{r_1^3} - \frac{\mu(\bar{y} - \bar{y}_2)}{r_2^3}$$

where $()''$ means the second derivative with respect to time, r_1 and r_2 are the distances from M_1 and M_2 , given by:

$$r_1^2 = (\bar{x} - \bar{x}_1)^2 + (\bar{y} - \bar{y}_1)^2$$

$$r_2^2 = (\bar{x} - \bar{x}_2)^2 + (\bar{y} - \bar{y}_2)^2$$

THE GRAVITATIONAL CAPTURE

To define the gravitational capture it is necessary to use a few basic concepts from the two-body celestial mechanics. Those concepts are:

a) Closed orbit: a spacecraft in an orbit around a central body is in a closed orbit if its velocity is not large enough to escape from the central body. It remains always inside a sphere centered in the central body;

b) Open orbit: a spacecraft in an orbit around a central body is in an open orbit if its velocity is large enough to escape from the central body. In this case the spacecraft can go to infinity, no matter what is its initial position.

To identify the type of orbit of the spacecraft it is possible to use the definition of the two-body energy (E) of a massless particle orbiting a central body. The equation is $E = \frac{V^2}{2} - \frac{\mu}{r}$, where V is the velocity of the spacecraft relative to the central body, μ is the gravitational parameter of the central body and r is the distance between the spacecraft and the central body.

With this definition it is possible to say that the spacecraft is in an open orbit if its energy is positive and that it is in a closed orbit if its energy is negative. In the two-body problem this energy remains constant and it is necessary to apply an external force to change it. This energy is no longer constant in the restricted three-body problem. Then, for some initial conditions, a spacecraft can alternate the sign of its energy from positive to negative or from negative to positive. When the variation is from positive to negative the maneuver is called a "gravitational capture", to emphasize that the spacecraft was captured by gravitational forces only, with no use of an external force, like the thrust of an engine. The opposite situation, when the energy change from negative to positive is called a "gravitational escape". In the

restricted three-body problem there is no permanent gravitational capture. If the energy changes from positive to negative, it will change back to positive in the future. The mechanism of this capture is very well explained in references 1, 2, 3, 4.

DEFINITION OF THE PROBLEM

In this paper the attention is concentrated in studying the magnitude of the energy saved for a gravitational capture under the model given by the elliptic restricted three-body problem. Numerical integrations are performed backward in time from given initial conditions (close to the celestial body considered) to follow the behavior of the trajectory. Then, we verify for which initial conditions an escape occurs. An escape in backward integration is equivalent to a capture in the forward time. The quantities that are used to specify the initial conditions are:

- 1) The eccentricity of the primaries;
- 2) The position of the secondary in its orbit around the primary (the true anomaly) when the numerical integration starts;
- 3) The mass parameter of the system (μ);
- 4) The distance between the primaries;
- 5) The initial position of the spacecraft relative to the secondary body, given by the angle α (see Fig. 1) and the periaipse distance r_p ;
- 6) The value of C_3 and the type of orbit (direct or retrograde);
- 7) The distance, from the center of the secondary body, where the effects of M_2 in the motion of M_3 around M_1 can be neglected (distance of the sphere of influence of M_2).

With those initial conditions specified, it is also necessary to make some assumptions to study this problem. The most important assumptions are:

- i) The starting point of each trajectory is 100 km from the Moon's surface ($r \approx 0.0045$ in canonical units from the center of the smallest primary). The angle α , measured from the line joining the two primaries, shown in Fig. 1 is used to specify uniquely the initial position;
- ii) The magnitude of the initial velocity is calculated from a given value of $C_3 = v^2 - 2\mu/r_2$,

where v is the velocity of the massless body relative to the smallest primary. The direction of the velocity is assumed to be perpendicular the line joining the smallest primary to the massless body in a counter-clock-wise direction (retrograde orbits are not studied in this paper, because the differences in terms of C_3 between these two types of orbits are not large);

iii) The escape occurs when (see reference 1, 2, 3 and 5) the spacecraft reaches a distance of 100,000 km for the Earth-Moon system (0.26 in canonical units for all the other systems studied in this paper) from the center of smallest primary in a time shorter than 50 days (≈ 12 in canonical units);

iv) The true anomaly (v) of the secondary body at the beginning of the numerical integration is the parameter used to study the importance of the eccentricity in the problem (see Fig. 1). It means that the true anomaly is the parameter that is varied in the numerical simulations to show the differences between the circular and the elliptic problems.

NUMERICAL RESULTS

First of all, we show the results for the Earth-Moon system (eccentricity equals to 0.055) in Fig. 2. In this picture we plot the magnitude of the C_3 (the radial variable) as a function of the angle α (the angular variable), for the true anomalies $v = 0^\circ$, $v = 90^\circ$, $v = 180^\circ$ and $v = 270^\circ$. The magnitude of the C_3 represents the savings obtained from the perturbation of the third body. C_3 equals zero is the minimum required for an escape to occur under the model given by the two-body problem.

The results show that there are significant differences in the savings obtained (more than 10% in some cases) for different values of the true anomaly. It is also possible to see that the best value for the true anomaly (largest magnitude of C_3) depends on the value of α . For example, $v = 0^\circ$ gives the optimum value in the region $0^\circ < \alpha < 30^\circ$, $v = 90^\circ$ is the best value in the region $200^\circ < \alpha < 240^\circ$, etc. Those results are compatible with the results obtained for the circular problem^{1,2,5}

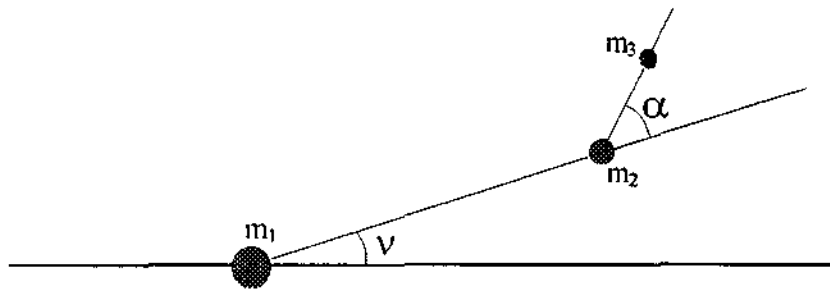


FIG. 1 CONFIGURATION OF THE BODIES AT $T = 0$ IN THE ELLIPTICAL RESTRICTED THREE-BODY PROBLEM.

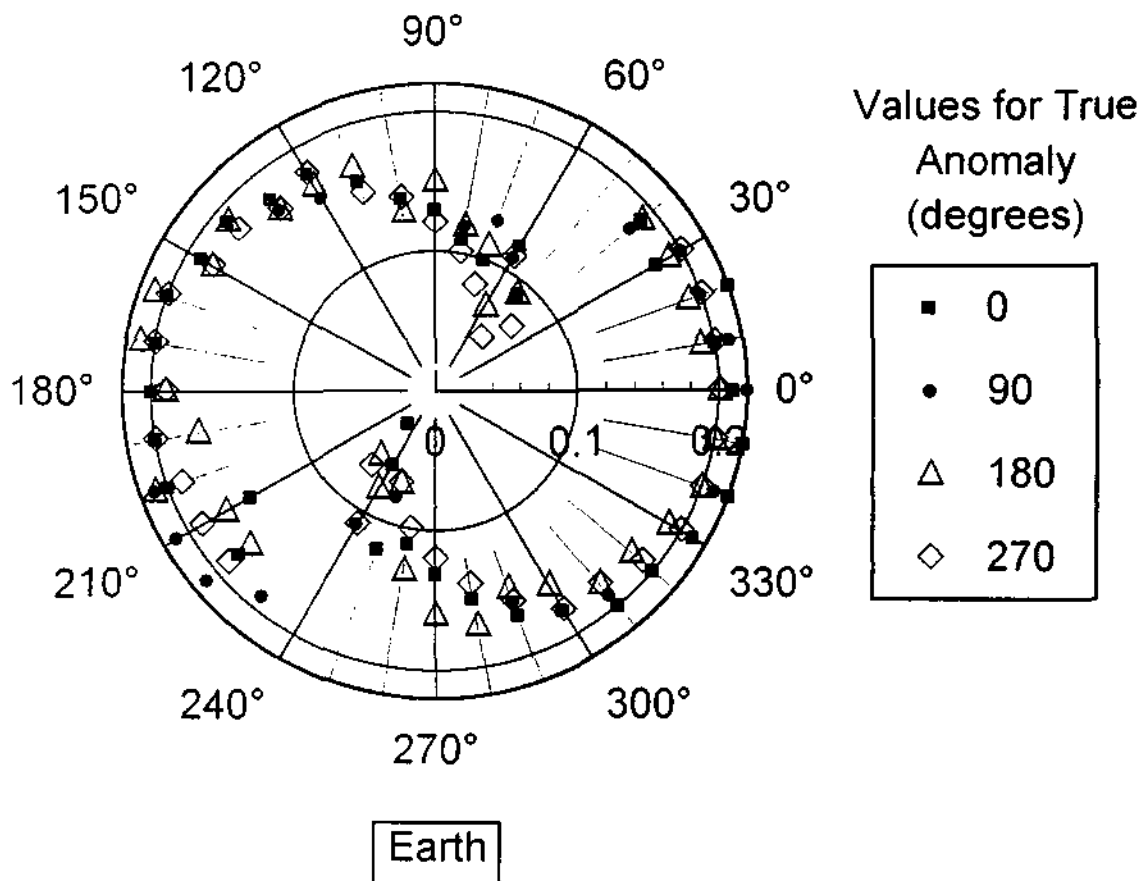


FIG. 2 - RESULTS FOR THE EARTH-MOON SYSTEM.

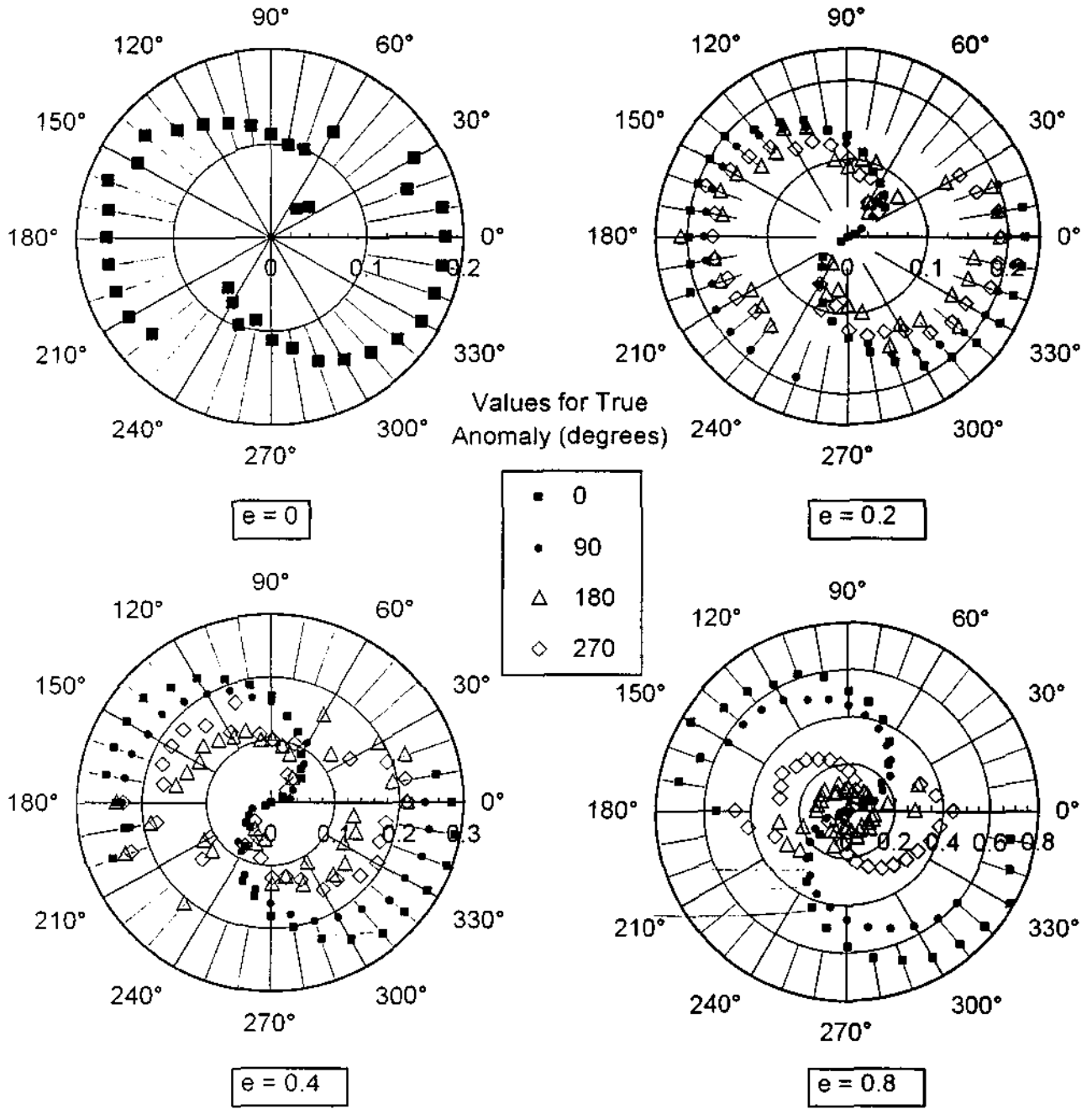


Fig. 3 MANEUVERS SIMULATED FOR $\mu = 0.01$.

Next, some hypothetical systems with larger eccentricities are studied to verify the effects of the eccentricity in this problem. In Fig. 3, we show the results for the cases where μ is equal to 0.01, the eccentricity of the primaries goes from 0.0 to 0.8 and the true anomaly assumes the values 0° , 90° , 180° , 270° . Fig. 3 also shows the numerical results in plots where the radial variable is the magnitude of C_3 and the angular variable is the angle α . The

eccentricity is 0.0 for the first plot, 0.2 for the second, 0.4 for the third and 0.8 for the fourth. We can see in all the plots that the savings are greater when the secondary body is at periaapsis ($v = 0^\circ$) and smaller when it is at the apoaapsis ($v = 180^\circ$). Values for $v = 90^\circ$ and $v = 270^\circ$ give intermediate results. This result is expected, since the smaller distance between the two primaries that occurs in this situation increases the effect of the third body perturbation (the

main cause of the savings). The objective of this research is to quantify this effect. We can also see the regions of maximum and minimum savings for each case. The increase of the eccentricity causes an increase in the differences of the magnitude of C_3 among the families $v = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ (the families become more evident). In Fig. 3 we can also see the direct effect of the eccentricity looking the regions of maximum and minimum savings and we can conclude that when the eccentricity increases, the magnitude of the savings also increases. Look at the scale of the plots for the radial variable. It goes from 0.2 when $e = 0.0$ to 0.8 when $e = 0.8$.

Those plots also show the importance of making a good choice for the angle α . The differences in the magnitude of C_3 obtained for different values of this parameter are very large.

CONCLUSIONS

In this paper a numerical algorithm to study the problem of gravitational capture in the elliptical restricted three-body problem is developed. The effect of the true anomaly of the secondary body for a fixed eccentricity is shown, as well as the regions of maximum savings. The effect of the eccentricity for a fixed mass parameter is also studied. We conclude that the savings increase with the eccentricity and when the true anomaly goes close to zero, as expected, and we give the quantitative values of those savings for several cases.

ACKNOWLEDGMENTS

We are very grateful to Dr. Hiroshi Yamakawa and Dr. Kuninori Uesugi from ISAS (Japan) for the references and suggestions that they provided for us. We also thank CNPq (National Council for Science and Technology Development), CAPES (Federal Agency for Post-Graduate Education) and INPE (National Institute for Space Research - Brazil) for supporting this research.

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AUTORIZAÇÃO PARA PUBLICAÇÃO

TÍTULO					
Study of the Gravitational Capture in the Elliptical Restricted Tree-Body Problem					
AUTOR					
Ernesto Vieira Neto, Antonio Fernando Bertachini de Almeida Prado					
TRADUTOR					
EDITOR					
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DMC	SPG	-518	7		
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