EXPERIMENTAL STUDY OF INHOMOGENEOUS TURBULENCE IN THE LOWER TROPOSPHERE BY WAVELET ANALYSIS.

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Abstract. In this paper we use wavelet transform to analyse aircraft data in the case of inhomogeneous turbulence. The data was gathered during a meteorological experiment (PYREX) to study the airflow around a mountain ridge: the Pyrénées. We present the spatial distribution and the various lengthscales of the energy fluxes computed from turbulence data (thermodynamics) for four specific flows : 1) Mechanical turbulence in the lee side of the mountain; 2) Mountain lee waves and small scale turbulence; 3) Coherent structures within Tramontana wind over the Mediterranean sea and 4) Inhomogeneous and strongly turbulent valley wind (Cierzo wind). These two latter local winds are generated by the Pyrénées mountain range.

§Introduction

In geophysical flows, turbulence plays a fundamental role. It is essentially turbulence that controls the transfer of energy, particularly momentum. Therefore, turbulence is localized in areas where strong shear occurs (such as jet streams, or flows above reliefs), or in areas with high energy gradients (such as cyclones).

Close to the ground, a transition boundary layer develops which then governs the main energy transfers between the ground and the atmosphere. This area is strongly turbulent, and can be investigated "in situ", so that numerous instruments and techniques of measurement have been developed, often by analogy with laboratory systems used in fluid mechanics.

For a long time, only homogeneous turbulence was investigated. The resulting knowledge was a parameterization of the turbulent flow characteristics. This parameterization is used in numerical models of the atmospheric flow (weather forecasting or the modeling of diffusion-transport of effluents).

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Afterwards, sensors and measurement techniques for atmospheric turbulence were adapted to be embarked onboard instrumented aircraft. With such mobile-type platforms, measurements could be performed on domains lacking in experimental dataset, such as averaged properties of heterogeneous surfaces or flow distortion due to the relief. In such conditions, the thermodynamic parameters are more complex, because of the accumulation of various mechanisms which greatly disturb the steady state.

Such studies require the development of specific processing tools enabling, first of all, the precise description of the data samples as space-wavenumber structures. Secondly, these tools need to provide a means of isolating the various mechanisms simultaneously at work in the data gathered in inhomogeneous conditions.

The aim of this paper is to present some characteristics of turbulent signals analysed with wavelet technics. We will analyse mainly non-stationary signals, gathered with instrumented aircraft during the PYREX experiment [2,3], whose goal was the study of atmospheric flow around a mountain range: the Pyrénées. For comparison, we will use as a reference the turbulence signals gathered in homogeneous conditions and analysed by a more classical approach using a Fourier transform. We will then present the contribution of the wavelet transform to the study of homogeneous signals.

§Airborne measurements

Aircraft are particularly adapted to investigation of the atmosphere. Due to their mobility, they allow in situ study of the specific area where interesting atmospheric events occur. In situ measurements can be performed for various areas of atmospheric physics: turbulence, radiation, microphysics and chemistry. The mean thermodynamic parameters are temperature, water vapour content and the three wind components, computed from the measurement of a large set of raw parameters.

In France, the aircraft currently used for tropospheric turbulence studies work at about 100 $m.s^{-1}$ airspeed. The measurement system (sensors, analog filtering and digitization) provides signals with little noise up to a resolution of about 4 m.

In homogeneous atmospheric turbulence (in the atmospheric boundary layer for instance), the scales under study range from around ten meters up to one kilometer. A correct description of these scales requires a straight and constant level run of about 20 km. In disturbed flows, these runs could be as long as 200 km, like during the flights performed in the PYREX experiment.

So, according to the sampling rate and to the length of the record, a turbulent sample is defined by a time series of from 5000 to 50000 points. This resolution allows the description of a wide wavelength band (from 10 m to 20 km). The principal mathematical tool used is the Fourier transform, which allows spectra and cross-spectra computation, as well as numerical filtering and spectral coupling of the signals. The algorithm used is the Singleton' Fast

Fourier Transform (FFT) [6]. Inhomogeneous signals are studied segment-by-segment with the Short-Time Fast Fourier Transform (SFFT) and particularly by wavelet transform [1,4]. The wavelet used is essentially for determining the spectral structure of the signals (represented as a time-frequency or a space-wavenumber). So, the analysing function is a complex Morlet-type wavelet [5].

§The "reference" case: homogeneous turbulence

The classical approach

Homogeneous turbulence can be found in the atmospheric boundary layer when mechanical production prevails, *i.e.*, with strong wind and weak sensible heat flux. The turbulence characteristics are solely dependant on the altitude above ground, *i.e.*, the vertical coordinate, which is the direction of the energy fluxes. These characteristics are independent of the position and also the orientation of the sample in the horizontal plane.

An example of the fluctuations (temperature (θ') and turbulent vertical velocity (w')) is presented in Figure 1.1. The turbulence can be quantified by the various statistical moments. Among them, the second order moment $\overline{w'\theta'}$, is the vertical turbulent kinematic fluxes of sensible heat. One of the goals (which is partially achieved in atmospheric research), consists in the development of non-dimensional profiles for these quantities, as a function of characteristic scales which depend on those parameters relevant to the turbulence: heat and momentum.

The homogeneity of the samples gathered during airborne measurements is often degraded because of the natural complexity of the areas flown over. Homogeneity can be verified by the shape of the time-integral function used for calculation of moment: for instance, a 2nd order moment is computed as follows:

$$\overline{X_i' X_j'} = \lim_{T \to \infty} \frac{1}{T} \int_0^T X_i'(t) X_j'(t) dt \tag{1}$$

let us define the time-integral function f:

$$f(\tau) = \int_0^\tau X_i'(t) X_j'(t) dt \tag{2}$$

Convergence of the calculation of moment and the homogeneity of the sample can be verified by the linearity of this function f. In Figure 1.1, this function f is plotted for the turbulent heat flux.

These turbulent samples can be characterized by a classification of the size of the various events which compose them. A more widely used method is the Fourier Transform (FT), which gives us the turbulent energy distribution, as a function of the wavenumber (spectral structure). The turbulent series is described by the sum of harmonic waves, each characterized by its amplitude and its phase. These two terms do not have a physical significance, but allow an accurate mathematical description of the sample.

Successive zooms of a turbulence sample give an impression of scale invariance. This characteristic is related to a high wavenumbers subrange in which the energy decreases following a power law as a function of the wavenumber. This feature has given rise to an idea of hierarchy in turbulent vortices (inertial cascade) which perhaps does not correspond to a physical reality. The scale of invariance domain overlays the whole inertial subrange which extends well beyond the highest wavenumbers reached by airborne measurements: at aircraft altitudes, it ranges from some ten meters down to a few millimeters. In this subrange, turbulent kinetic energy is transferred, without production, towards the viscous dissipation range.

In parallel to the spectral structure of a single turbulent sample, through cross-spectral analysis we can characterize the common frequency band for two parameters. The cospectrum of w' and x' defines the frequencies (or wavelengths) which contribute to the vertical transfer of x.

Use of wavelet transform

The wavelet transform is useful even for homogeneous samples, although this kind of data was not its intended application. It allows a more accurate description of the turbulence homogeneity. In particular, it allows a more physical study of phase variability at the various frequencies.

Using the wavelet transform for the study of homogeneous samples gives a picture of the turbulence hierarchy in a space-wavenumber representation. An example of the turbulence structure is shown in Figure 1.2 where one can see 1) the amplitude diagram, 2) the location of the extrema of the amplitude and 3) the location of phase-breakings. An analysis of the phase demonstrates that the bifurcations occur simultaneously with phase-breakings. Better knowledge of turbulence homogeneity could be derived from a statistical study of the distribution of location and energy levels for these events in the space-wavenumber representation.

§Inhomogeneous samples

Homogeneous turbulence is probably an uncommon event in the lower troposphere. Indeed, the smallest mechanical instability is often sufficient to generate coherent structures within the turbulent flow. These structures could range from helical cells to convective cells, according to the respective importance of the two sources of instability (mechanical or thermal). Laboratory studies tend to show that the bifurcations between the various types of structure are governed by threshold mechanisms. Later on, we will present in this paper an example of such structures within a flow over the sea.

More generally speaking, the relief ensures significant modification of the turbulent structure of the flow, by the accumulation of mechanisms of widely various scales. The resulting structure is highly inhomogeneous. Firstly, the

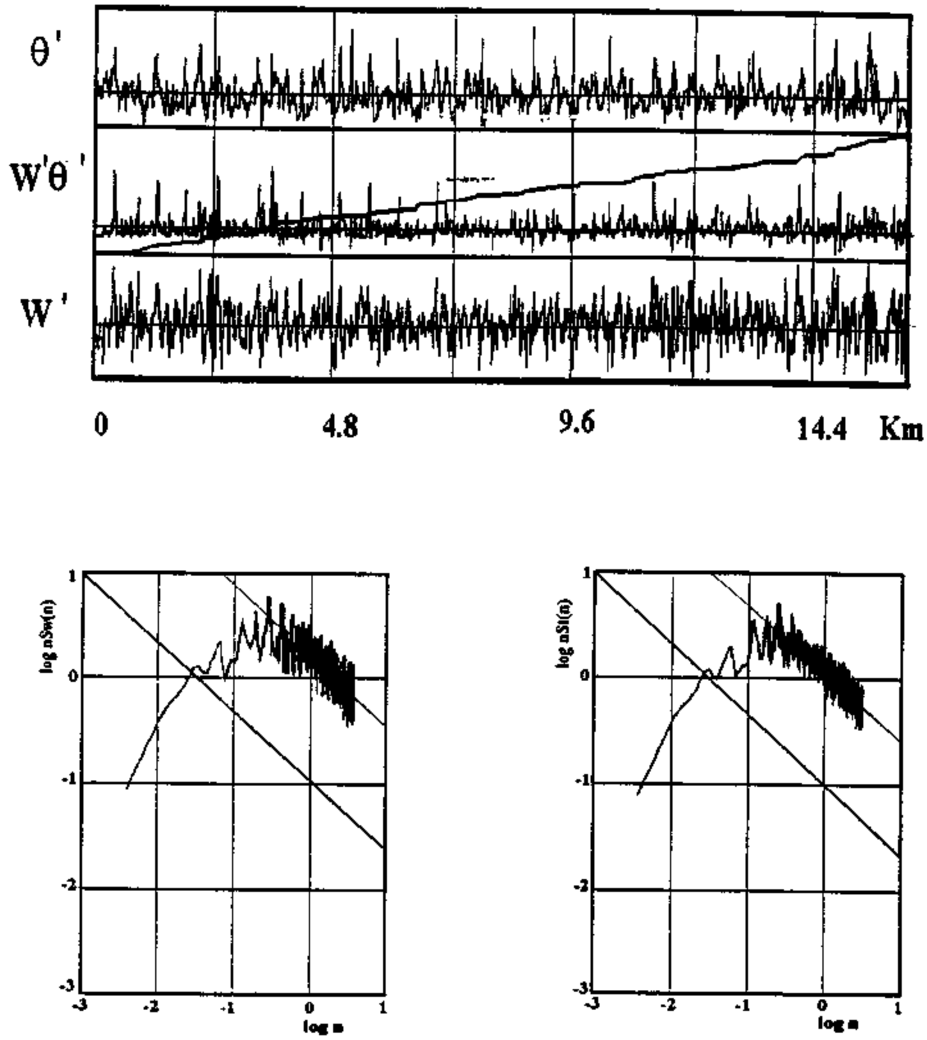

Figure 1.1 Analysis by Fourier Transform of one case of homogeneous turbulence data.

characterization of samples and then the separating and quantifying of the various mechanisms, would seem to be domains that are particularly suited to treatment by the wavelet transform.

To illustrate this, we have chosen some examples of turbulent samples measured on board instrumented aircraft in the proximity of the Pyrénées relief during the PYREX experiment. Figure 2 is a diagram of the experiment showing the main experimentation equipment. The regions of local wind are also represented. The following mechanisms will be analysed later on:

1- inhomogeneous turbulence, close to the relief on the leeward side of the mountain; in this area, rotors are generally to be found.

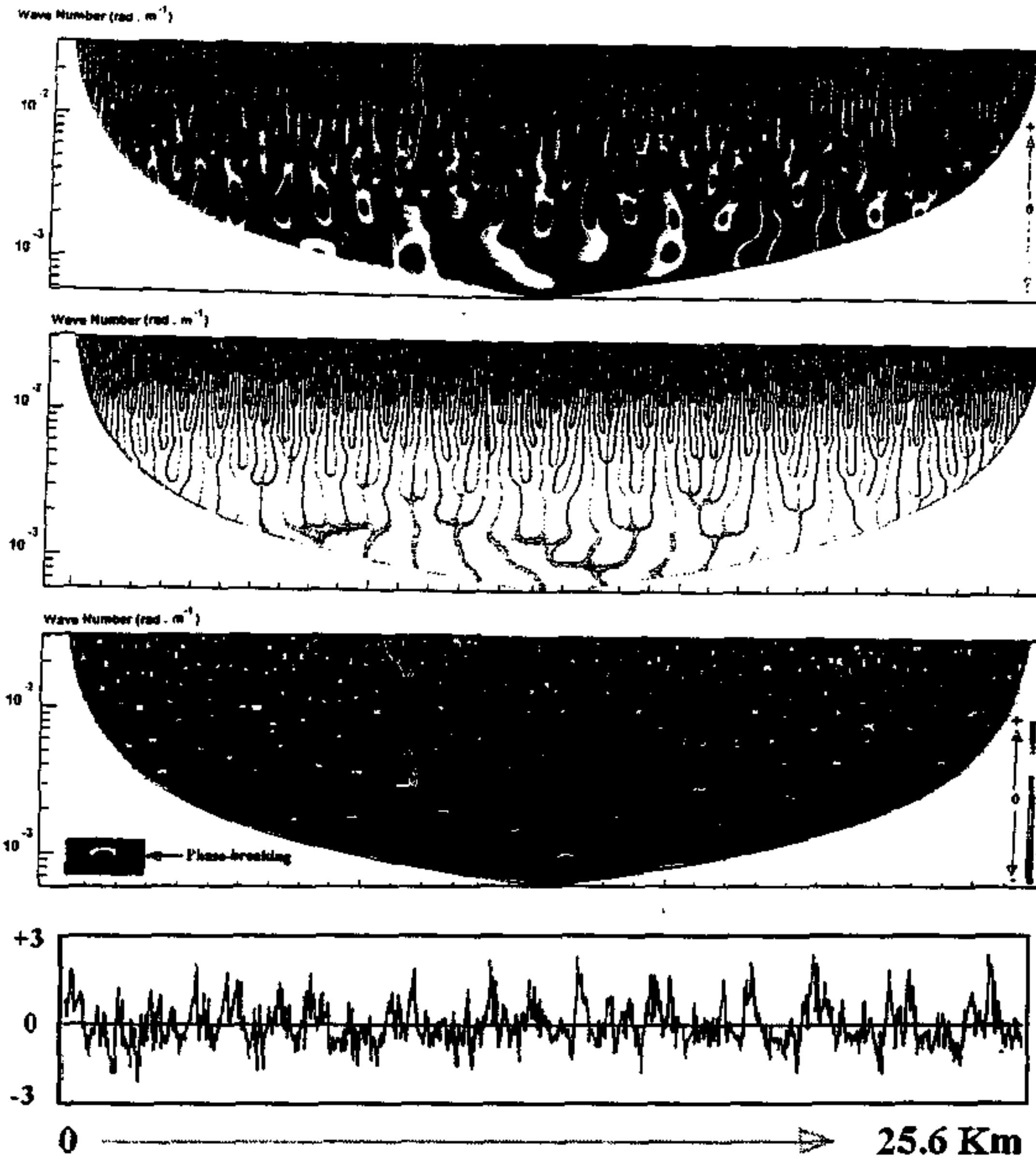

Figure 1.2 Real part of the Wavelet transform (top), Bifurcations corresponding to the extrema of the amplitude (middle), and Phase-breakings (bottom) of a homogeneous turbulent sample (vertical velocity of the air).

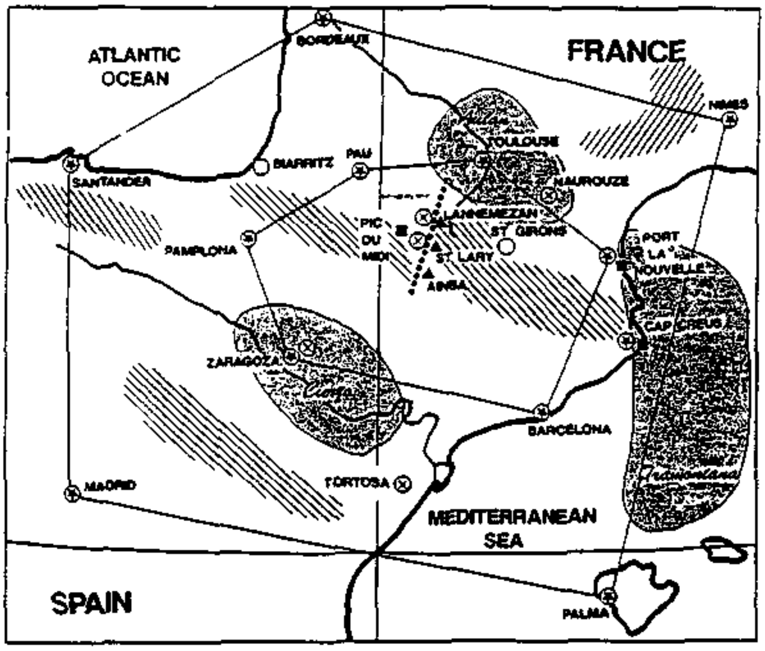

Figure 2 Synoptic view of the ground infrastructure during PYREX. The climatological range of the winds is indicated by half-tone. ⊛ Soundings; ▲ profilers; ⊗ sodar; • microbarographs; ■ constant level balloons (from Bougeault et al., 1993)[3]

2- leewaves and associated turbulence above the relief.

3- mechanical structure of the "Tramontana" wind.

4- inhomogeneous turbulence and fluxes in a valley wind (the "Cierzo")

The first two topics relate to turbulence in the close vicinity of the relief. This turbulence results from the friction of the flow on the mountain, and from the momentum flux modification.

The last two relate to local winds whose formation and evolution depend on the flow around the relief. These two latter local winds are generated by the Pyrénées mountain range.

§Lee waves and turbulence

Turbulence close to the mountain

Flight legs very close to the mountain were made by the Merlin IV aircraft of Météo-France on November 16, 1990. The synoptic meteorological situation was characterized by a northerly wind. Strongly turbulent zones associated with the roughness are spread out on the downwind side of the mountain.

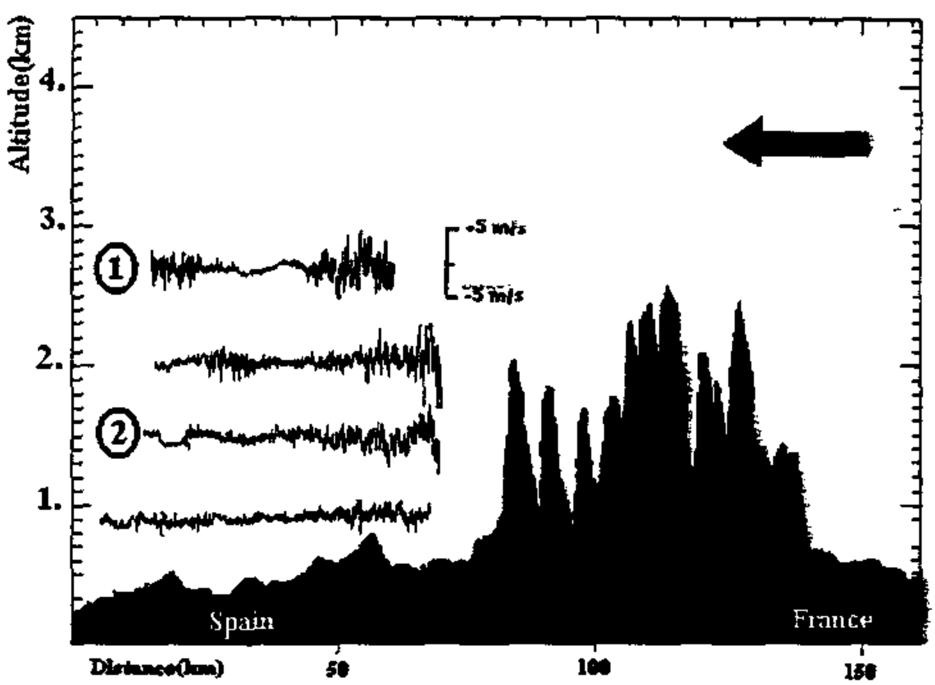

Figure 3.1 Aircraft measurements of vertical velocity of the air close to the Pyrénées on 16 November 1990 during PYREX Experiment

Figure 3.1 shows the turbulent zones observed on the vertical velocity of the air close to the relief. Figure 3.2 presents the wavelet transform (real part and energy) performed on the flight legs 1 and 2. The wavelet transform provides a spectral description of the turbulent zones and their characteristic scales. The energy diagram allows to put in evidence the link between the zones of turbulent energy production and the zones of dissipation. These diagrams show the existence of areas, of around ten kilometers in horizontal length, in which the turbulence is very strong, with fluctuations in vertical velocity of more than 5 $m.s^{-1}$. The characteristic lengthscale of this turbulence, calculated from the wavenumber of maximum energy, is about 400 m.

These turbulent areas are related to the wave motion that develop in the upper stable layer. They are called "rotors", and are dangerous for aircraft. They are associated with upper level wave clouds which present very smooth borders (laminar flow).

Wavelet analysis provides important information on how the spectral characteristics of the turbulence produced in this rotor area evolve. The evolution towards large wavenumbers enables localization of the areas where the energy will be dissipated. One can also deduce from these diagrams that dissipation is not homogeneously distributed in the turbulence areas. So, it would seem that there does not exist any local equilibrium between the production and dissipation of turbulent kinetic energy.

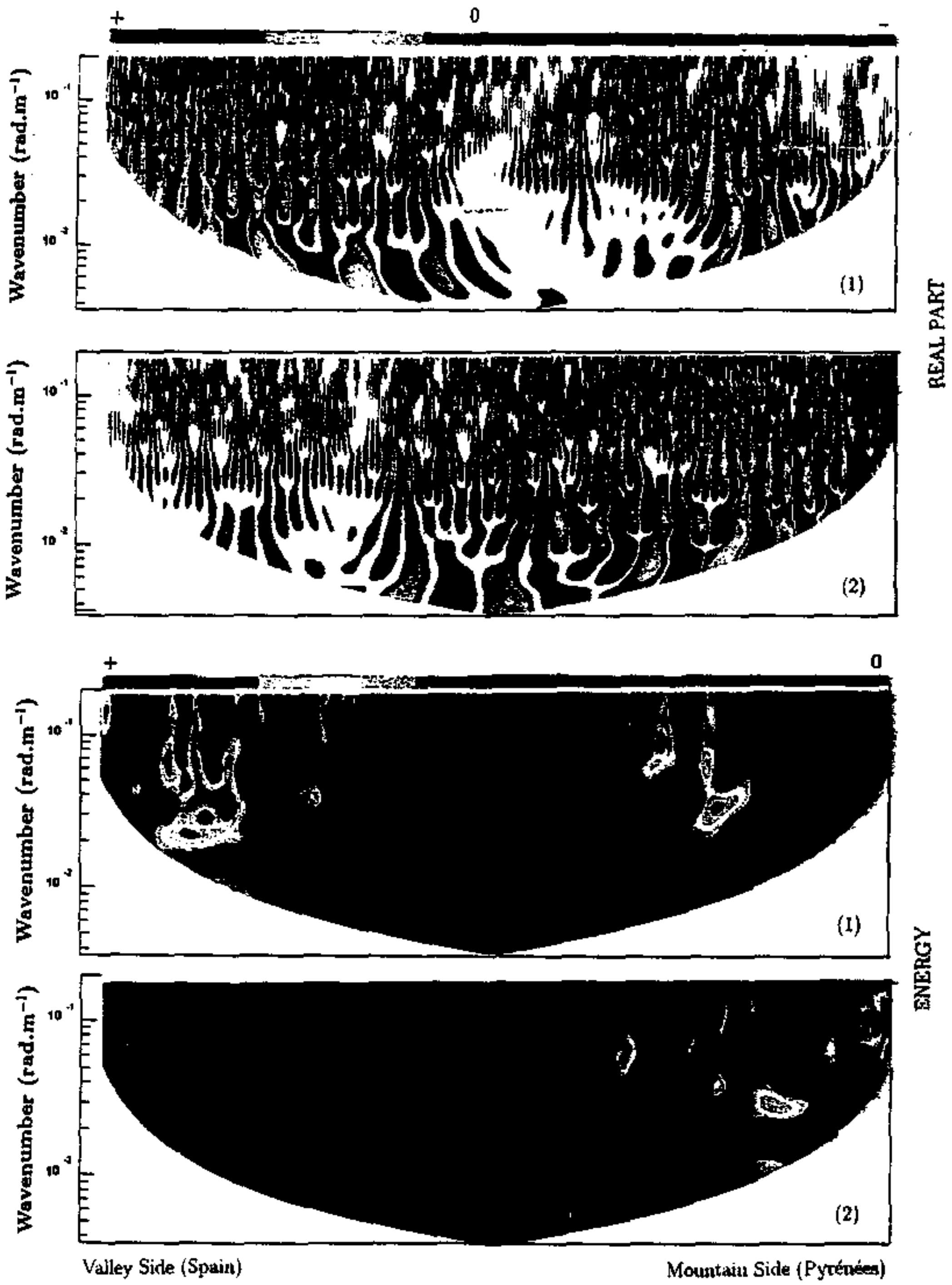

Figure 3.2 Wavelet transform performed on legs (1) and (2) shown on Figure 3.1: Real part (top); Energy (bottom). Color coding: from red to black for maximum to minimum.

Lee waves and turbulence

When clear of clouds, the middle troposphere is generally stable. Turbulence cannot develop nor remain. On the other hand, this condition of stability is favourable to the development of wave motion triggered by the crossing of the mountain by the flow (lee waves). This wave motion often propagates vertically up to the tropopause. On the lee side of the mountain, this motion can extend over more than 100 km. During their propagation, the lee waves can get amplified to a point where they become unstable and degenerate into turbulence. These lee waves stabilize the turbulence zone which develops close to the mountain.

Figure 3.3 presents a sample of data measured by the Fokker 27 aircraft. It corresponds to a straight and constant level run, 200 km long performed at 4000 m altitude. On the downwind side of the mountain, an area of very clear lee waves is observable in the air temperature and vertical velocity signals. Their horizontal extension is limited to 5 wavelengths. The wavelet transform has been applied to the fluctuations in the vertical velocity w' and in the temperature θ'.

The amplitude diagrams give a remarkable illustration of the geographical evolution of the spectral characteristics of these signals, especially as regards the lee waves: in particular, one can see, in Figure 3.3, the change in wavelength (L) as one goes further from the mountain, from 8 to 14 km, which corresponds to dL/dx=0.1 km/km. This information is fundamental for the analysis of the conditions of wave persistence. Another extremely important piece of information brought out by this analysis is the localization of turbulence and the spectral characterization of turbulent zones associated with the development and the vanishing of lee waves. The wavelet transform demonstrates quite clearly the continuity in scale between lee waves and high frequency turbulence where the waves degenerate and the turbulent kinetic energy dissipates.

The real part of the wavelet cross-spectrum clearly shows up the sensible heat flux structure. At the lee-wave frequencies, w' and θ' are in quadrature, which is shown by the symmetrical oscillations of half a wavelength on the cross-spectrum diagram. On the contrary, the higher frequencies at the downwind end of the lee waves are not symmetrical (most of the values are positive), which implies a positive correlation between w' and θ' and so a positive (upward) sensible heat flux.

§Local winds

Coherent structures within Tramontana wind

When a northerly wind bears down on the Pyrénées range, a violent regional wind blows up from the Roussillon over the septentrional basin of the Mediteranean, covering the Gulf of Lion, from the coasts of France and Spain, on towards the Balearic islands; this is the "Tramontana" wind. When moving over the sea away from the shore, the flow remains turbulent, but becomes

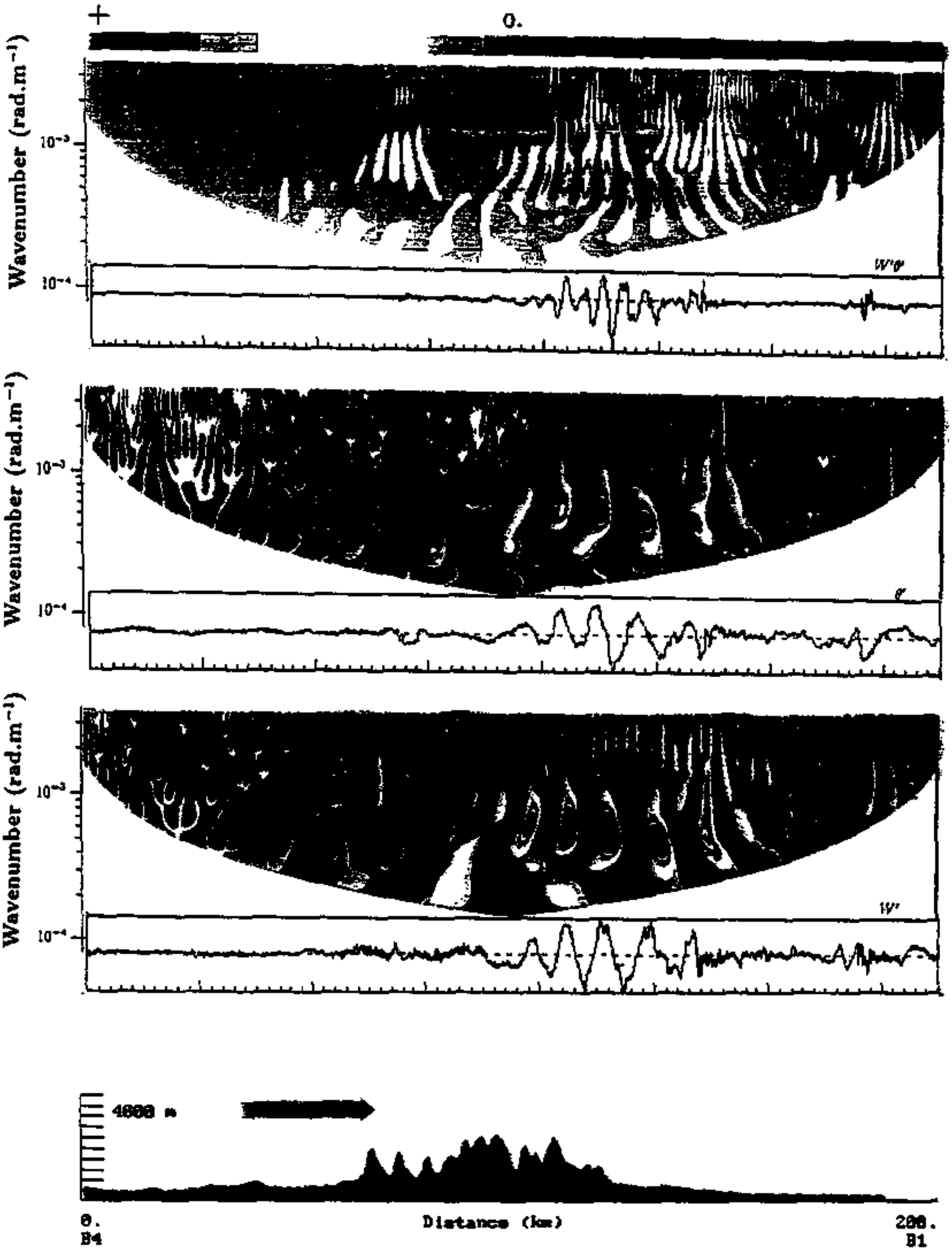

Figure 3.3 Real part of the Wavelet transform of vertical velocity (bottom) and potential temperature (middle). On the upper part of the figure is represented the corresponding wavelet cross-spectrum: case of lee waves. Color coding: see Figure 3.2.

more homogeneous. This airflow offers exceptional conditions of strong turbulence maintained by the wind stress and the buoyancy associated with surface momentum and heat flux.

Such a flow was studied during PYREX experiment, on November 4, 1990. The data was collected by the two instrumented aircraft (MerlinIV and Fokker27) on legs located in an East-West vertical plane, from the open sea to the coast of Spain. This plane is perpendicular to the Tramontana wind.

During this study, boundary conditions over the sea were the following: sensible heat flux varied from 60 W.m^{-2} to 30 W.m^{-2} from open sea to coast and evaporation was as great as 280 W.m^{-2}. The wind strength decreased from the open sea to the coast, giving a simultaneous variation in the friction velocity from 0.25 to 0.1 m.s^{-1}. These surface parameters produced exceptional conditions of stability characterized by dominant mechanical production of turbulent kinetic energy, and by an evaporation contribution to buoyancy, of the same order of magnitude than that of the sensible heat flux.

These conditions are favourable for the development of coherent structures in the atmospheric boundary layer. The structure of the marine boundary layer is illustrated in Figure 4.1 by turbulent functions and by the measurements from the LEANDRE backscattered lidar embarked on board the Fokker 27 aircraft. The turbulent series were recorded at 940, 470 and 37 m altitude on East-West runs perpendicular to the mean flow.

Such a cross-section perpendicular to the airflow shows up coherent structures that develop along the mean wind. The lidar signal, proportional to aerosol concentration, exhibits the thickness of the boundary layer. We clearly see, at the top of the boundary layer, a dome shape characterizing the top of dynamic structures, whose altitude decreases when approaching the shore. Fluctuations in water vapour, recorded close to the top of the boundary layer (at 940 m), confirm this shape due to the entrainement of dry air from the upper stable layer. In the middle and at the bottom of the boundary layer, the diagnosis parameters are, respectively, the vertical and transversal wind velocities.

The wavelet transform (fig. 4.2) is computed on the transversal velocity, measured at low altitude. It demonstrates a series of oscillations of about 5 km in wavelength. Towards the end of this series, a second organization occurs with a shorter wavelength which increases with the distance from the shore. At lower lengthscales (greater wavenumbers) the turbulence structure appears more homogeneous and normally cascades towards the viscous dissipation range (local dissipation).

Turbulence and fluxes in a valley wind

For northerly flow conditions over the Pyrénées range, a strong wind develops in the upper Ebro valley and extends up to the Mediteranean shore. This wind, called "Cierzo", has been investigated in the principal valley axis with a straight and constant level flight run of 100 km long, *i.e.*, along the whole valley. The wavelet transform (fig. 4.3) has been applied to the vertical velocity w' and to

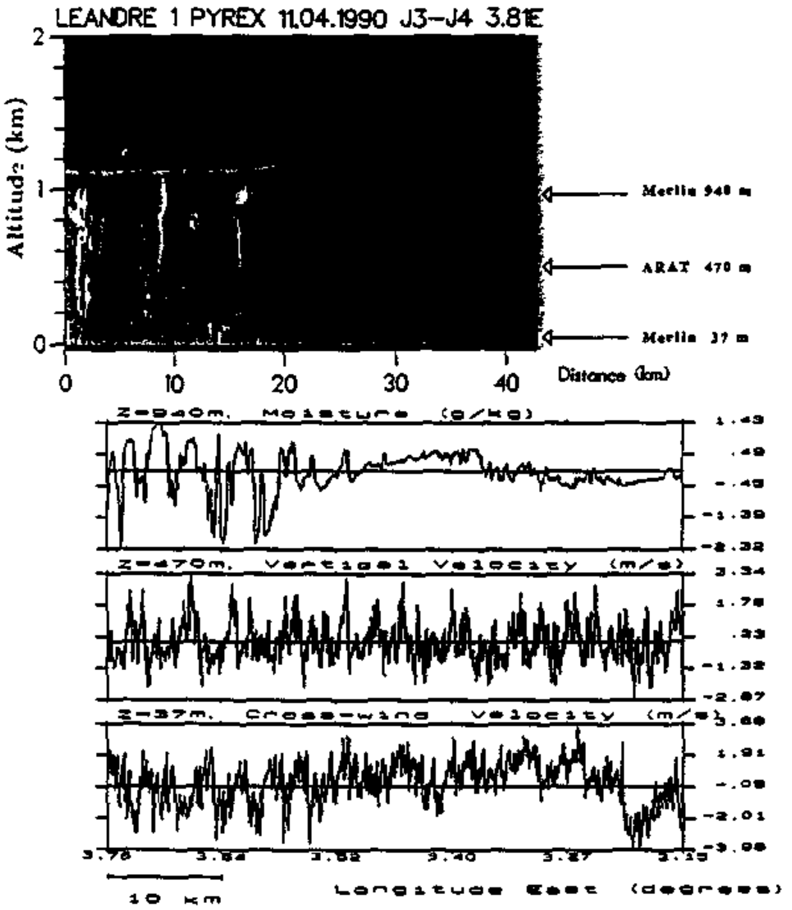

Figure 4.1 Lidar and in situ measurements along a flight track crossing the wind from east to west on 4 November. On the upper part is represented the lidar range corrected signal, obtained from the Fokker flying at 3200 m altitude. On the lower part are shown three detrended turbulent series obtained by the Fokker and the Merlin (from Bougeault et al., 1993)[3]

the air temperature θ'. Strong turbulence is shown in the upper (upwind) part of the valley. It is produced at middle and low frequencies and is transferred up to the inertial subrange (local dissipation). In the lower (downwind) part of the valley, a turbulence area occurs in which the production range is clearly shifted towards higher frequencies. The temperature is more homogeneous with a maximum of energy located around the upper quarter of the valley.

The multiplication of the two diagrams enables location of the zones of heat transfer. When energy is present at low frequencies (wavelength greater than 1 km), there is a quadrature between the two signals, as can be seen on symmetrical oscillations of the cross-spectrum diagram. Significant and organised heat transfer appears in the upper quarter of the valley. In the

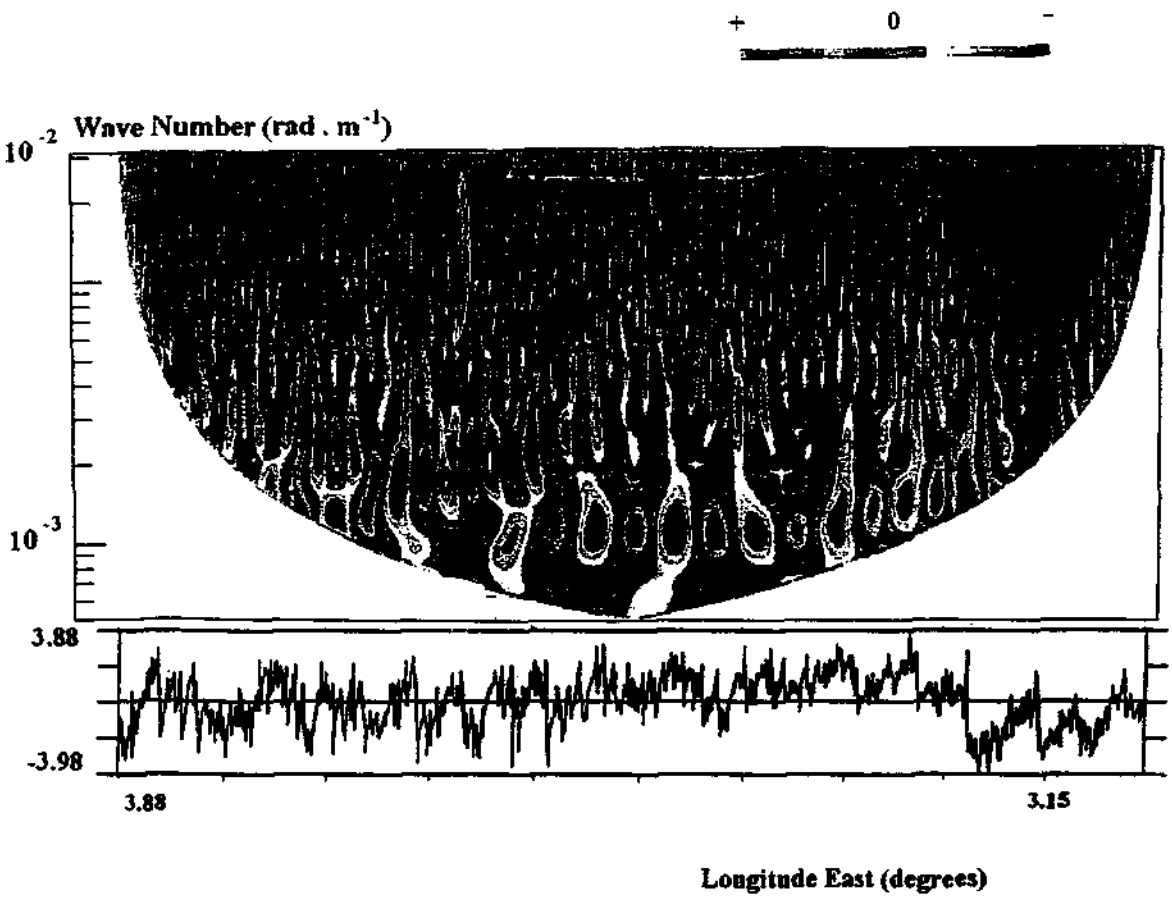

Figure 4.2 Real part of the wavelet transform applied to the transversal wind component: case of Tramontana wind. Color coding: see Figure 3.2

lower part of the valley, heat is transferred at higher frequencies. This analysis enables one to localise areas of turbulence, to quantify the energy transfers and to determine the characteristic lengthscales that contribute to these transfers.

§Conclusion

The classical determination of the characteristics of a homogeneous turbulence sample is achieved by the computation of the various turbulence moments and the use of the Fourier Transform for spectral analysis. Similarly, the cross-characterization of two variables is made using the covariant moments and cross-spectral analysis.

Experimental studies of tropospheric turbulence are often performed with instrumented research aircraft. For many practical reasons, like homogeneity defects of the flight area, or disturbed meteorological conditions, specific techniques for the analysis of unsteady turbulent samples have had to be developed.

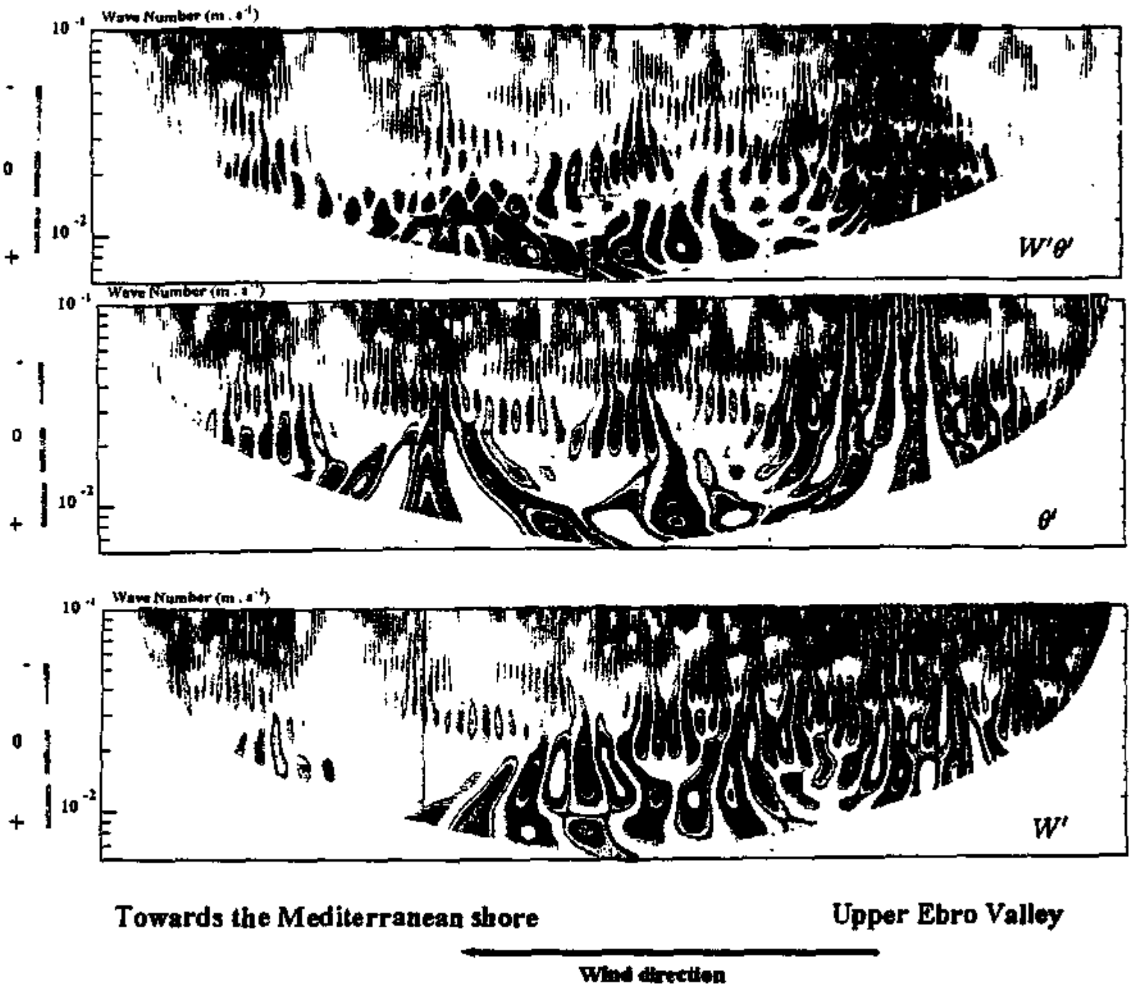

Figure 4.3 Real part of the wavelet transform of the vertical velocity (bottom), the temperature of the air (middle) and corresponding wavelet cross-spectrum (top): case of valley wind (the Cierzo). Color coding: see Figure 3.2

In this domain, the wavelet transform is a very powerful tool. With five examples, four of them being turbulence samples measured close the Pyrénées range, we have shown the capability of the wavelet transform method. In the case of homogeneous turbulence, a study of phase-breakings can lead to a quantitative analysis of the homogeneity of the sample.

The examples of non homogeneous turbulence were found in the atmospheric airflow around the Pyrénées chain: mountain turbulence, lee waves, internal circulations within tramontana wind, turbulence and fluxes in a valley wind.

The wavelet transform appears to be a powerful tool - for determination of the spatial distribution of the various events, - for the characterization of frequencies contributing to the transfer, - for isolating the physical mechanisms and quantifying their specific contribution, - for the identification of secondary structures within the turbulent flow and finally for the representation of the path followed by the turbulent kinetic energy towards viscous dissipation.

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A. Druilhet