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Research Note

A self-consistent theory of MHD parametric instabilities driven by a standing Alfvén wave in the planetary magnetosphere

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Abstract. A finite-amplitude standing Alfvén wave of circular polarization can excite MHD parametric instabilities in a low-β plasma. In the presence of a standing Alfvén pump wave, two independent gratings associated with the density fluctuations are generated. The role of the second grating in the convective and purely growing instabilities is elucidated. The intense auroral Alfvén-acoustic events observed in the planetary magnetosphere provide good experimental evidence in support of the Alfvén parametric instabilities.

Key words: Magnetohydrodynamics(MHD) – plasmas – instabilities – Earth – planets and satellites: general

Standing Alfvén waves have been detected in the magnetospheres of various planets: Earth (Fukunishi 1987; Bloch & Falthämmar 1990; Knudsen et al. 1990), Jupiter (Walker & Kivelson 1981; Glassmeier et al. 1989) and Mercury (Russell 1989). These standing Alfvén waves serve as a key electromagnetic coupling mechanism between the planetary magnetosphere and ionosphere (Southwood & Hughes 1983) as well as between the solar wind and the planetary magnetosphere (Harold & Samson 1992).

In the Earth's auroral plasma, the phenomenon of Alfvén-acoustic turbulence has been observed in connection with finite-amplitude standing and traveling Alfvén waves (Boehm et al. 1990; Knudsen et al. 1990). The electric field strengths of Alfvén waves may exceed 100 mV/m and the largest waveforms consist of step functions instead of near-sinusoidal waves. Moreover, considerable density perturbations and plasma heating are seen during these auroral events. These observed features of auroral Alfvén-acoustic turbulence are strongly indicative of nonlinear mode-mode coupling.

A theory was proposed recently by Chian & Oliveira (1994) to interpret the intense auroral Alfvén-acoustic events in terms of the magnetohydrodynamic (MHD) parametric instabilities driven by a standing Alfvén wave. According to this theory, the beating of the pump and induced Alfvén waves in a low-β plasma produces a ponderomotive force which acts on the acoustic wave to amplify the density perturbations and cause the large density cavities observed. It was shown that the Alfvén parametric instabilities in a low-β plasma can be either convective or purely growing. In addition, the MHD parametric instabilities may lead to dissipation processes, such as the Landau damping of the induced acoustic waves, whereby the Alfvén wave energy is converted to the plasma thermal energy resulting in significant temperature rise and energetic electron precipitation, in agreement with the features observed during the intense auroral Alfvén-acoustic events.

Prior to the work of Chian & Oliveira (1994), most studies of the Alfvén parametric instabilities treated the pump as a traveling wave (Galeev & Oraevskii 1963; Sagdeev & Galeev 1969; Lashmore-Davies 1976; Derby 1978; Goldstein 1978; Sakai & Sonnerup 1983; Terasawa et al. 1986; Brodin & Stenflo 1988; Kuo, Whang & Schmidt 1989; Viñas & Goldstein 1991; Jayanti & Hollweg 1993a, b). In contrast to the case of a traveling pump, the theory of parametric processes generated by a standing pump is much more complex since a larger number of waves are usually excited. A standing pump wave can be described in terms of two counterpropagating waves with the same amplitude (Chian & Alves 1988; Chian 1991). In the presence of a pair of oppositely directed Alfvén pumps, two independent gratings (i.e., low-frequency density modes) may be generated (Rizzato & Chian 1992; Glanz et al. 1993). In the work of Chian & Oliveira (1994), for the sake of simplicity only one grating was considered. In this *paper* we present a self-consistent theory of the MHD parametric instabilities driven by a circularly polarized standing Alfvén wave in a low-β plasma. The aim is to investigate the effect of the second grating on the MHD parametric instabilities and to verify the regime of validity of the one-grating theory.

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Writing the total magnetic field as $\mathbf{B} = B_0\hat{\mathbf{z}} + \mathbf{b}_0 + \mathbf{b}$, where $B_0\hat{\mathbf{z}}$ is the uniform background magnetic field, $\mathbf{b}$ is the induced transverse magnetic field and $\mathbf{b}_0$ is the Alfvén pump field consisted of two counterpropagating left(right)-hand circularly polarized Alfvén waves $\mathbf{b}_0 = \mathbf{b}_0^+ + \mathbf{b}_0^-$, with $\mathbf{b}_0^+(z,t) = \hat{\mathbf{e}}_\pm(b_0^+/2)\exp[i(k_0 z - \omega_0 t)] + c.c.$ and $\mathbf{b}_0^-(z,t) = \hat{\mathbf{e}}_\pm(b_0^-/2)\exp[i(-k_0 z - \omega_0 t)] + c.c.$, where the polarization unit vectors $\hat{\mathbf{e}}_\pm = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$ denote left(right)-hand circularly polarized shear Alfvén (fast magnetosonic) waves, respectively. For both polarizations we have, $\omega_0^2 = c_A^2 k_0^2$, where $c_A = B_0/(\mu_0\rho_0)^{1/2}$ is the Alfvén velocity. We assume all waves propagating along the ambient magnetic field. A large-amplitude Alfvén pump can parametrically couple to fluctuations in the magnetic field $\mathbf{b}(z,t)$ and plasma density $\rho(z,t)$. In the presence of two oppositely directed traveling Alfvén pump waves, two independent low-frequency density modes (i.e., gratings), with the same frequency but different wave vectors, can be induced (*c.c.* omitted)

$$\rho(z,t) = \frac{\rho_1}{2}\exp[i(k_1 z - \omega t)] + \frac{\rho_2}{2}\exp[i(k_2 z - \omega t)], \tag{1}$$

where $\mathbf{k}_1 = \pm\mathbf{k}$ and $\mathbf{k}_2 = \mp 2\mathbf{k}_0 \pm \mathbf{k}$. In this *paper*, we consider the decay-type wavevector kinematics ($b_0 \to b + \rho$), as illustrated in Fig. 1. The wave coupling involves the following four wave triplets. Firstly, the forward pump $\mathbf{b}_0^+$ interacts with the first grating $\rho_1(\omega^*, \mathbf{k})$ to generate the $\mathbf{b}_0^+$-driven Stokes Alfvén mode $\mathbf{b} = \mathbf{b}_-^+(\omega_0 - \omega^*, \mathbf{k}_0 - \mathbf{k})$. Secondly, the backward pump $\mathbf{b}_0^-$ interacts with the first grating $\rho_1^*(\omega^*, \mathbf{k}) = \rho_1(-\omega, -\mathbf{k})$ to generate the $\mathbf{b}_0^-$-driven Stokes Alfvén mode $\mathbf{b} = \mathbf{b}_+^-(\omega_0 + \omega, -\mathbf{k}_0 + \mathbf{k})$. Thirdly, the $\mathbf{b}_-^+$-mode can also be generated by the coupling of the backward pump $\mathbf{b}_0^-$ with the second grating $\rho_2(\omega^*, -2\mathbf{k}_0 + \mathbf{k})$. Fourthly, the $\mathbf{b}_+^-$ can also be generated by the coupling of the forward pump $\mathbf{b}_0^+$ with the second grating $\rho_2^*(\omega^*, -2\mathbf{k}_0 + \mathbf{k}) = \rho_2(-\omega, 2\mathbf{k}_0 - \mathbf{k})$.

The parametric interaction of the magnetic field and density fluctuations is described by a set of coupled wave equations derived from the MHD equations (Lashmore-Davies 1976; Goldstein 1978; Jayanti & Hollweg 1993a; Chian & Oliveira 1994). In the present paper, we treat the limit of low-β ($\beta \equiv c_S^2/c_A^2 \ll 1$, where the acoustic velocity $c_S = (P_0/\gamma\rho_0)^{1/2}$), which is a valid assumption for the planetary magnetospheres (Fukunishi 1987). For $\beta \ll 1$, the high-frequency magnetic field fluctuations are resonant Alfvén waves, whereas the low-frequency density waves can be either resonant ($Re[\omega] \equiv \omega_R \approx c_S k$) or nonresonant ($\omega_R \neq c_S k$) acoustic modes. The self-consistent theory of the MHD parametric instabilities driven by a circularly polarized standing Alfvén wave in a low-β plasma is governed by the following Fourier-transformed coupled wave equations (assuming $|b_0| \gg |b_-^+|, |b_+^-|$)

$$D_-^+ b_-^+ = -\frac{\omega_0^2 k_-^+}{2k_0\rho_0}(\rho_1^* b_0^+ - \rho_2^* b_0^-), \tag{2}$$

$$D_+^- b_+^- = -\frac{\omega_0^2 k_-^+}{2k_0\rho_0}(\rho_1 b_0^- - \rho_2 b_0^+), \tag{3}$$

$$D_1\rho_1 = \frac{k^2}{4\mu_0}(b_0^+ b_-^{+*} + b_0^{-*} b_+^-), \tag{4}$$

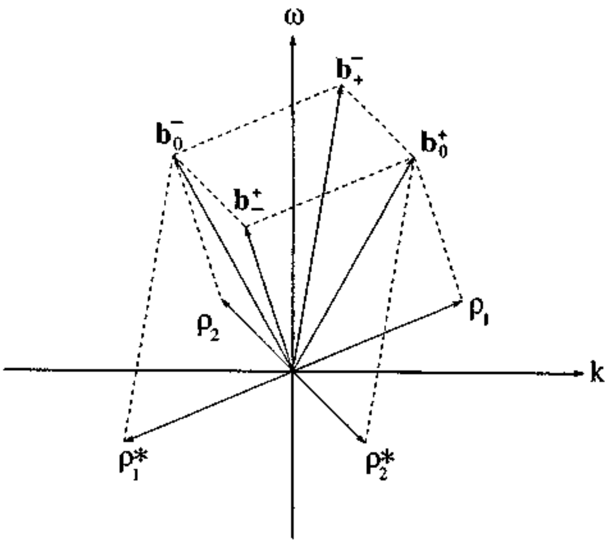

Fig. 1. Coupling diagram for the MHD parametric instabilities involving two gratings: b_0^+ and b_0^- are the Alfvén pump waves; b_-^+ and b_+^- are the Alfvén daughter waves; ρ_1 and ρ_2 are the two independent acoustic waves (gratings).

$$D_2\rho_2 = \frac{k_2^2}{4\mu_0}(b_0^{+*} b_+^- + b_0^- b_-^{+*}), \tag{5}$$

with the dispersion functions given by

$$D_-^+(\omega_-^+, \mathbf{k}_-^+) = (\omega_0 - \omega^*)^2 - c_A^2(\mathbf{k}_0 - \mathbf{k})^2, \tag{6}$$

$$D_+^-(\omega_+^-, \mathbf{k}_+^-) = (\omega_0 + \omega)^2 - c_A^2(\mathbf{k}_0 - \mathbf{k})^2, \tag{7}$$

$$D_1(\omega, \mathbf{k}) = \omega^2 - c_S^2\mathbf{k}^2, \tag{8}$$

$$D_2(\omega, \mathbf{k}_2) = \omega^2 - c_S^2\mathbf{k}_2^2, \tag{9}$$

where $\mathbf{k}_2 = -2\mathbf{k}_0 + \mathbf{k}$, $\mathbf{k}_-^+ = -\mathbf{k}_+^- = \mathbf{k}_0 - \mathbf{k}$, $\omega_-^+ = \omega_0 - \omega^*$ and $\omega_+^- = \omega_0 + \omega$. From (2)–(9), we obtain the nonlinear dispersion relation

$$D_1 D_2 D_-^+ D_+^- = \frac{\Lambda}{k^2}(k^2 D_2 - k_2^2 D_1)(D_-^+ + D_+^-), \tag{10}$$

where $\Lambda = -(\omega_0^2 k^2 k_-^+ |b_0|^2)/(8\mu_0\rho_0 k_0)$ and since we are considering a standing pump we set $b_0^+ = b_0^- \equiv b_0$.

In general, (10) is rather difficult to analyze analytically. However a good insight of its solutions can be obtained by making the resonant approximation for b_-^+ and b_+^-, and assuming ρ_2 a nonresonant mode (i.e., $D_2 \neq 0$). Under these assumptions, (10) becomes

$$(\omega^2 - c_S^2 k^2)(\omega^2 - \delta^2) = \delta W, \tag{11}$$

where the linear detuning factor $\delta = \omega_A - \omega_0$, $\omega_A = -c_A k_-^+ = c_A k_+^-$, and $W = (1 - r_\rho)\Lambda/\omega_A$ with

$$r_\rho = \frac{\rho_2}{\rho_1} = \frac{k_2^2 D_1}{k^2 D_2}. \tag{12}$$

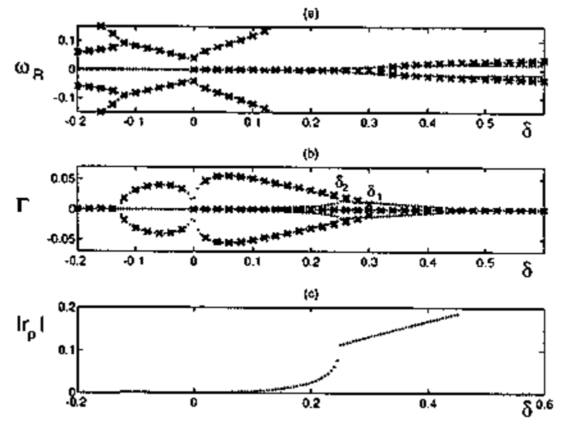

Fig. 2. Numerical solutions of the nonlinear dispersion relation for the one-grating theory (x-curves) and two-gratings theory (dotted curves); $A = 10^{-3}$ and $\beta = 4 \times 10^{-4}$.

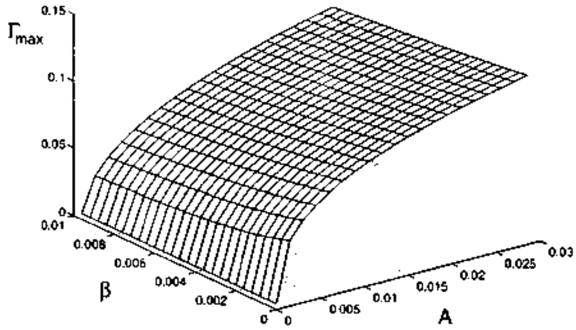

Fig. 3. The plot of the maximum growth rate Γ_{max} as a function of β and A in the convective regime ($\delta < 0$).

The complex parameter r_ρ measures the ratio of the contribution of two gratings ρ_2 and ρ_1 in the wave interactions. In the limit $r_\rho \to 0$, (11) recovers the one-grating theory (equation (7) of Chian & Oliveira 1994). Note that in that *paper*, W should be defined as $W = \Lambda/\omega_A$ with Λ as given above. Similar to the one-grating theory, (11) shows that both convective ($\omega_R = Re[\omega] \neq 0$) and purely growing ($\omega_R = 0$) instabilities can be produced. For the convective instabilities, an analysis of (12) indicates that $|r_\rho| \ll 1$ which implies that the second grating has negligible influence on the wave coupling in this regime and the one-grating theory is applicable; under these circumstances, $W \to \Lambda/\omega_A > 0$ and $\delta < 0$. For the purely growing instabilities $W > 0$ and $\delta > 0$; in this case r_ρ increases monotonically with δ and lies in the interval $0 < r_\rho < 1$ in the unstable region of the spectrum. Since (11) only differs from the one-grating theory by a factor of $1 - r_\rho$ in the expression for the pump intensity W, by comparing with the results of Chian & Oliveira (1994), we conclude that the effect of the second grating is to decrease the growth rate and increase the dissipative threshold. Setting $\omega = i\Gamma$ (Γ is the growth rate) in (11), we have

$$(\Gamma^2 + c_S^2 k^2)(\Gamma^2 + \delta^2) = \delta W, \tag{13}$$

which shows that the purely growing instabilities occur in the interval

$$0 < \delta < \frac{\omega_0 A}{8\beta}\left(1 - r_\rho\right), \tag{14}$$

where $A = b_0^2/B_0^2$. A comparison of (14) with the one-grating theory indicates that the corresponding unstable bandwidth is reduced by a factor of $1 - r_\rho$ due to the influence of the second grating.

We now discuss the general behavior of (10), with the resonant approximation for b_-^+ and b_+^- but without the nonresonant restriction for the second grating ρ_2 imposed above. Introducing the normalization $\omega \to \omega/\omega_0$, $\delta \to \delta/\omega_0$, $k \to k/k_0$ and $k_2 \to k_2/k_0$, (10) then becomes

$$(\omega^2 - \beta k^2)(\omega^2 - \beta k_2^2)(\omega^2 - \delta^2) = \frac{\delta A}{8}[k^2(\omega^2 - \beta k_2^2) - k_2^2(\omega^2 - \beta k^2)]. \tag{15}$$

Numerical solutions of (15), using the values of parameters appropriate for the planetary magnetospheres, are shown in Figs 2-4. Figure 2 compares the solutions of the one-grating theory (x-curves) and the two-gratings theory (dotted curves) for $A = 10^{-3}$ and $\beta = 4 \times 10^{-4}$. Figure 2(a) shows that $Re[\omega]$ is essentially the same for both theories. It can be seen that, for small δ's the second grating ρ_2 has small effect on the purely growing ($\delta > 0$) instability, which is confirmed by the parameter $|r_\rho|$ being very small ($|r_\rho| \ll 1$) in this region. However, for larger δ's, $|r_\rho|$ increases monotonically with δ, reaching its maximum value at the upper boundary δ_2 of the unstable region. Figure 2(b) shows that, due to the second grating, the unstable bandwidth of the purely growing instability is reduced (its upper boundary moves from the upper limit δ_1, of the one-grating theory, to δ_2) in agreement with (14). Figure 2(c) indicates that the second grating has small influence on the convective instability ($\delta < 0$) since $|r_\rho| \ll 1$ in the entire unstable convective regime. The numerical solutions demonstrate that the second grating lowers the growth rate throughout the unstable regions, as anticipated in the earlier discussion. It is important to point out that the second grating introduces a new convective unstable region, to the right of the purely growing region ($\delta > \delta_2$), which enlarges the overall bandwidth of the instabilities as seen in Fig. 2b. An overview of the variation of the maximum growth rate Γ_{max} as a function of the pump intensity A and plasma β is plotted in Figs. 3 and 4. In both convective and purely growing instabilities, for a given β, Γ_{max} increases as A increases; whereas for a given A, Γ_{max} decreases as β increases.

The self-consistent theory of the MHD parametric instabilities driven by a standing Alfvén wave presented in this *paper* confirms the existence of the convective and purely growing instabilities previously identified by the one-grating theory of

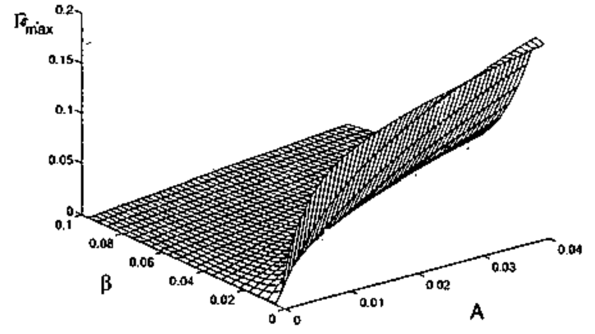

Fig. 4. The plot of the maximum growth rate Γ_{max} as a function of β and A in the purely growing regime ($\delta > 0$).

Chian & Oliveira (1994). The second grating is shown to have the following effects on the instabilities: (1) decrease of the growth rate, (2) increase of the dissipative threshold, (3) reduction of the unstable bandwidth of the purely growing instability, and (4) addition of a second convective unstable region. The one-grating theory is shown to give a fairly accurate description of the convective instability, for which the role of the second grating is not important. The results of this *paper* render further support to the theoretical interpretation of the intense auroral Alfvén-acoustic events proposed by Chian & Oliveira (1994). The observed features of the auroral Alfvén-acoustic turbulence, as mentioned in the introduction, are in good agreement with the characteristics of the Alfvén parametric instabilities. To conclude, we wish to remark that there is increasing observational and theoretical evidence that the auroral region of the planetary magnetosphere is very rich in nonlinear mode-mode coupling processes such as discussed here and in other related works (Chian, Lopes and Alves 1994a, b; Lopes & Chian 1995). Hence, the auroral plasma has proved to be an ideal laboratory for understanding the intriguing complexity of plasma turbulence.

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