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Pulsar eclipsing mechanisms: the effect of photon-beam-induced acoustic turbulence

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ABSTRACT

Pulsar eclipse due to induced three-wave interactions involving low-frequency acoustic turbulence is discussed. We consider both the case of electron acoustic waves when the ion temperature T_i is higher than the electron temperature T_e and that of ion acoustic waves when $T_e > T_i$. In each case the corresponding growth rate for low-frequency acoustic waves induced by a high-frequency photon beam is evaluated. For $T_i > T_e$, the maximum growth rate for electron acoustic waves depends strongly on T_i/T_e and may be comparable to that of a Langmuir counterpart for sufficiently large temperature ratio T_i/T_e. It is shown that induced scattering off electron acoustic waves can be important and can cause pulsar eclipse. We show also that for $T_e > T_i$ and in the small-angle approximation, growth of low-frequency ion acoustic waves due to the photon beam is always slower than Langmuir waves and the corresponding induced scattering is less effective than induced Raman scattering. Thus, induced Brillouin scattering involving low-frequency ion acoustic waves cannot be the main cause for pulsar eclipses.

Key words: plasmas – radiation mechanisms: non-thermal – scattering – turbulence – stars: neutron – pulsars: general.

1 INTRODUCTION

Induced three-wave interactions are thought to be responsible for observed pulsar eclipses (e.g. Luo & Melrose 1995a,b, and references therein). In eclipsing binary pulsar systems, e.g. PSR 1957 + 20, 1744 − 24 (Fruchter, Stinebring & Taylor 1988; Lyne et al. 1990; Fruchter & Goss 1992), intense radio waves (from pulsars) propagating through plasmas from a companion wind can induce low-frequency wave instabilities and the induced low-frequency turbulence can scatter the radio beam, resulting in eclipses. Both Langmuir (or z-mode when the ambient magnetic field is included) and electron Bernstein-mode waves are considered as viable low-frequency waves for such three-wave interactions (e.g. Gedalin & Eichler 1993; Thompson et al. 1994; Melrose 1994; Luo & Melrose 1995a,b). The previous discussion was restricted to electron plasmas in which the ion motion is ignored. There is strong observational evidence that a pulsar wind may heat the plasma of its companion wind near the eclipse region (for discussion see Fruchter et al. 1990, 1995; Thompson et al. 1994; Luo & Melrose 1995b). It is possible that the eclipsing plasma is non-isothermal, $T_e \neq T_i$, so that low-frequency ion acoustic and electron acoustic modes are excited for $T_e > T_i$ and $T_e < T_i$, respectively.

Two-temperature plasmas are not uncommon in both space and astrophysical applications when a strong heating process occurs. There is substantial evidence of the existence of ion acoustic and electron acoustic waves (or turbulence) in cosmic plasmas. For example, ion acoustic turbulence has been detected by spacecraft in the vicinity of interplanetary shock (Gurnett, Neubauer & Schwenn 1979), in the upstream region of the Earth's bow shock (Anderson et al. 1981) and in type III events in the solar wind (Lin et al. 1986). Intense broad-band electrostatic waves of short wavelength, detected upstream of the Earth's bow shock, in the high-speed solar wind, have been identified as electron acoustic waves (Marsch 1985). There is a possibility of the existence of two-temperature ($T_i \gg T_e$) accretion flows in astrophysics (Begelman & Chiueh 1988).

The purpose of this paper is to investigate whether induced scattering by low-frequency ion or electron acoustic waves can contribute to observed pulsar eclipses. We shall show that an anisotropic photon beam can induce ion acoustic (if $T_e > T_i$) or electron acoustic (if $T_e < T_i$) turbulence in the same way that Langmuir turbulence is induced by photon beams (Melrose 1994; Luo & Melrose 1995b). Anisotropy of a photon beam implies that the beam has a small angular spread as in the case of pulsar radio emission (Luo & Melrose 1995a,b). When both Langmuir and acoustic turbulence are present, induced scattering of the photon beam by either Langmuir or acoustic waves can occur. The problem is treated in the random phase approximation and all relevant waves are described by their occupation numbers. We also make the small-angle scattering approximation (Luo & Melrose 1995a,b), in which the low-frequency wavenumber is assumed to be much smaller than that of high-frequency waves so that the high-frequency waves are treated like a particle beam (i.e. the photon beam). When the low-frequency waves are electrostatic, the small-angle scattering approximation should be appropriate for any underdense and hot plasmas such as those in the pulsar eclipse region. This is due to the fact that the maximum wavenumber of low-frequency electrostatic waves is constrained by Landau damping, which is more severe for underdense and hot plasmas. We derive the growth rate for ion and electron acoustic waves as the result of instabilities induced by an anisotropic photon beam and compare it with the counterpart of photon-beam-induced Langmuir instability.

In Section 2, the pulsar eclipse model involving induced three-wave interactions is outlined. Induced Brillouin scattering by electron and ion acoustic waves is discussed in Sections 3 and 4, respectively. Application to pulsar eclipses is considered in Section 5.

2 PULSAR ECLIPSES DUE TO INDUCED THREE-WAVE INTERACTIONS

In our discussion, a beam of pulsar radio emission is described by an occupation number $N(\boldsymbol{k})$, where we use $\boldsymbol{k}$ and ω to represent wavevector and frequency, respectively. The radio beam with very high brightness temperature can induce various types of low-frequency waves through three-wave interaction. The radio beam can also scatter off the low-frequency waves through the same process. One favoured interpretation of pulsar eclipses is the scattering of the beam off low-frequency waves as the cause of pulsar eclipse. When the wavenumber of the low-frequency waves is much smaller than that of radio waves, scattering of the beam is described by the diffusion equation (Luo & Melrose 1995a,b; Luo & Melrose 1996)

$$\frac{\mathrm{d}N(\boldsymbol{k})}{\mathrm{d}t}=\frac{\partial}{\partial k_i}\left[D_{ij}(\boldsymbol{k})\,\frac{\partial N(\boldsymbol{k})}{\partial k_j}\right], \tag{1}$$

with the diffusion coefficient given by

$$D_{ij}(\boldsymbol{k})=\int\frac{\mathrm{d}^3k''}{(2\pi)^3}\,w^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')k''_ik''_jN^{\sigma''}(\boldsymbol{k}''), \tag{2}$$

where all double-primed quantities correspond to those of low-frequency modes, $N^{\sigma''}(\boldsymbol{k}'')$ is the occupation number, $w^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')=\bar{w}^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')2\pi\delta[w''(\boldsymbol{k}'')-\boldsymbol{k}''\cdot\boldsymbol{v}_g]$ is the probability of three-wave coupling, and $\bar{w}^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')$ is given by

$$\bar{w}^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')=\hbar R^{\sigma''}\,\frac{|\alpha^{\sigma\sigma\sigma''}|^2}{\varepsilon_0^3\omega^2\omega''}. \tag{3}$$

In equation (3) $R^{\sigma''}$ is the ratio of electric-to-total energy in waves of mode σ'' (the corresponding ratio for high-frequency waves is assumed to be 1/2), $\boldsymbol{v}_g=\partial\omega(\boldsymbol{k})/\partial\boldsymbol{k}$ is the group velocity, $\alpha^{\sigma\sigma\sigma''}=e_i^*e'_je''_s\alpha_{ijs}$, with $\boldsymbol{e}$, $\boldsymbol{e}'$ and $\boldsymbol{e}''$ representing the polarization vectors of the three waves, and α_{ijs} is the quadratic response tensor. The general expression for the quadratic response tensor is lengthy and some approximation must be made (Melrose 1987). For three-wave interactions involving two high-frequency waves and a low-frequency wave one has the following approximation: $\alpha_{ijs}\approx(\varepsilon_0e\omega''/m_e)k''_s\chi^{\mathrm{L(e)}}\delta_{ij}$, where we use $\chi^{\mathrm{L(e)}}$ and $\chi^{\mathrm{L(i)}}$ to denote respectively the electron and ion contribution to the longitudinal permittivity $K^{\mathrm{L}}=1+\chi^{\mathrm{L(e)}}+\chi^{\mathrm{L(i)}}$. The ion contribution to the quadratic response tensor is scaled down by a factor m_e/m_i and thus is ignored (e.g. Melrose 1986). We assume also that low-frequency waves are electrostatic and the corresponding phase velocity ω''/k'' is much less than the electron thermal velocity $V_e=(T_e/m_e)^{1/2}$. Using $|\boldsymbol{e}^*\cdot\boldsymbol{e}'|\approx 1$, we have

$$\bar{w}^{\sigma''}\approx 4\pi r_ec^2R^{\sigma''}\left(\frac{\hbar\omega''}{m_ec^2}\right)\left(\frac{k''c}{\omega}\right)^2[\chi^{\mathrm{L(e)}}]^2, \tag{4}$$

where $r_e=e^2/4\pi\varepsilon_0m_ec^2\approx 2.8\times 10^{-15}$ m is the classical radius of the electron. The approximation in equation (4) applies to any electrostatic waves with $k''\ll k$ and low phase speed.

Evaluation of the diffusion coefficient requires a specific form of $N(\boldsymbol{k})$. In the following discussion, the radio beam is modelled as $N(\boldsymbol{k})=N(k)\exp(-\bar{\theta}^2/2\theta_0^2)$, where θ_0 is the angular spread of the beam. The polar angles of $\boldsymbol{k}$ with respect to the beam axis and the ambient magnetic field $\boldsymbol{B}$ are represented by $(\bar{\theta},\bar{\phi})$ and (θ,ϕ), respectively. The corresponding polar angles of $\boldsymbol{k}''$ are double-primed. After integration over the angle $\bar{\theta}''$, the diffusion coefficient can be further written into the following form

$$D_{ij}=\int\frac{\mathrm{d}\bar{\phi}''}{2\pi}\int k''\,\mathrm{d}k''\left(\frac{\bar{w}^{\sigma''}}{2\pi c}\right)k''_ik''_jN^{\sigma''}, \tag{5}$$

where $\bar{\phi}''$ is the azimuthal angle of $\boldsymbol{k}''$ with respect to $\boldsymbol{k}$. Since $\bar{w}^{\sigma''}$ is usually expressed in terms of (θ'',ϕ''), when substituting (4) for equation (5) θ'' should be expressed in terms of θ and $\bar{\phi}''$ through a relation $\sin\theta''=(\sin^2\bar{\phi}''+\cos^2\theta\cos^2\bar{\phi}'')^{1/2}$.

In close analogy with wave–particle interaction, if saturated low-frequency wave turbulence is isotropic, we may write the diffusion coefficient as

$$D_{ij}=D_\parallel\hat{k}_i\hat{k}_j+D_\perp(\delta_{ij}-\hat{k}_i\hat{k}_j), \tag{6}$$

where $\hat{\boldsymbol{k}}=\boldsymbol{k}/k$. The coefficients $D_\parallel$ and $D_\perp$ describe the parallel and perpendicular diffusion (with respect to the beam axis), respectively. We can show that the isotropization ($D_\perp$) of the beam is always much faster than change of the spectrum (described by $D_\parallel$). From equations (1) and (6) we find

$$\begin{pmatrix} D_\parallel \\ D_\perp \end{pmatrix} = \int \frac{\mathrm{d}\bar{\phi}''}{2\pi} \int k''\,\mathrm{d}k'' \left(\frac{\bar{w}^{\sigma''}}{2\pi c}\right) N^{\sigma''}(\boldsymbol{k}'')k''^2 \begin{pmatrix} \dfrac{1}{n''^2} \\ 1-\dfrac{1}{n''^2} \end{pmatrix}, \tag{7}$$

with $n''=k''c/\omega''$. For $n''\gg 1$, we obtain

$$D_\perp \approx n''^2 D_\parallel \tag{8}$$

and therefore we have $D_\perp \gg D_\parallel$. This result is very similar to the diffusion of a particle beam due to wave–particle interactions. Since electrostatic waves usually can have large n'', a photon beam would spread well before the spectral change. If pulsar eclipses are due to the beam diffusion, there should be little change of spectrum at the entrance and exit of eclipses.

3 PHOTON-BEAM-INDUCED ELECTRON ACOUSTIC TURBULENCE

3.1 Growth due to a photon beam

From equations (2), (5) or (7), effective scattering requires a sufficient level of low-frequency wave turbulence. This can be provided by the photon beam itself through three-wave interactions. In the small-angle scattering approximation, the currently favoured low-frequency waves are Langmuir or magnetically modified Langmuir (z-mode) (Luo & Melrose 1995a,b). In general the growth of low-frequency waves can be described by

$$\frac{\mathrm{d}N^{\sigma''}(\boldsymbol{k}'')}{\mathrm{d}t} = -\Gamma^{\sigma''}(\boldsymbol{k}'')N^{\sigma''}(\boldsymbol{k}''), \tag{9}$$

where we ignore any damping processes as such processes can be included by adding the relevant damping terms into the right-hand side of equation (9). The absorption coefficient is given by

$$\Gamma^{\sigma''}(\boldsymbol{k}'') = -\int \frac{\mathrm{d}^3k}{(2\pi)^3} w^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')k_i''\frac{\partial N(\boldsymbol{k})}{\partial k_i}, \tag{10}$$

where the three-wave probability $w^{\sigma''}(\boldsymbol{k},\boldsymbol{k}'')$ is given by equation (3) or equation (4). Waves can grow provided that $\Gamma^{\sigma''}$ is negative. Using the spherical coordinates $\boldsymbol{k}=(\bar{\theta},\bar{\phi},k)$, the $\boldsymbol{k}$-derivative can be separated into the wavenumber k and angular derivatives. When the angular spread θ_0 is small and satisfies the condition $|\bar{\theta}''-\chi_0|/\theta_0 \gg \theta_0 \cot\chi_0 |k\,\mathrm{d}\ln N(k)/\mathrm{d}k|$, with $\chi_0=\arccos(\omega''/k''c)$, the Čerenkov angle, the angular derivative dominates and gives rise to $\Gamma^{\sigma''}<0$ for $\bar{\theta}''<\chi_0$ (Luo & Melrose 1996). For electrostatic waves with $n''\gg 1$ the condition for $\Gamma^{\sigma''}<0$ can be written as

$$(\alpha+3)\frac{\theta_0}{n''} \ll 1, \tag{11}$$

where $N(k)\propto k^{-(\alpha+3)}$, α being the spectral index of radio emission. In the following discussion, we assume that this condition is always satisfied. Then, the maximum growth rate is obtained as

$$\Gamma \approx \frac{1}{2\pi(\alpha+1)}\left(\frac{\pi}{2e}\right)^{1/2}\left(\frac{k^2\bar{w}^{\sigma''}}{2\pi c}\right)N(k). \tag{12}$$

The growth rate can be calculated by substituting equation (4) into (12).

3.2 Dispersion relation of electron acoustic waves

We consider first the case of non-isothermal plasmas with hot ions, that is $T_\mathrm{i}>T_\mathrm{e}$. There is a low-frequency mode called the electron acoustic mode with frequency and phase speed within the ranges $\Omega_\mathrm{i}\ll\omega''\ll\Omega_\mathrm{e}$ and $k_z''V_\mathrm{e}\ll\omega''\ll k''V_\mathrm{i}$ (with $k_z''=k''\cos\theta''$), respectively. The electron and ion thermal speeds are given by $V_\mathrm{i}=(T_\mathrm{i}/m_\mathrm{i})^{1/2}$ (with m_i the ion mass) and $V_\mathrm{e}=(T_\mathrm{e}/m_\mathrm{e})^{1/2}$ (with m_e the electron mass), respectively. Other notations Ω_e (Ω_i) correspond to electron (ion) cyclotron frequency. The frequency range implies that electrons spiral tightly around magnetic field lines but ions move as if unmagnetized. To avoid being heavily Landau-damped, the phase speed must be slower than the ion thermal speed but faster than the parallel component of electron thermal speed. In practice, this condition can be satisfied only when waves propagate nearly perpendicularly to the ambient magnetic field. The small projection of the electric field along the magnetic field forces electrons to slide along the field line and act as heavy particles. In this way, electrons provide inertia just like ions in ion acoustic waves (e.g. discussion in Section 4). The corresponding dispersion relation can be derived by regarding electrons as cold, with $\chi^{\mathrm{L(e)}}=(\omega_\mathrm{pe}^2/\Omega_\mathrm{e}^2)\sin^2\theta''-(\omega_\mathrm{pe}^2/\omega''^2)\cos^2\theta''$ (where ω_pe is the electron plasma frequency), and the ion contribution as unmagnetized, with $\chi^{\mathrm{L(i)}}=1/k''^2\lambda_\mathrm{Di}^2$. Then, from $K^\mathrm{L}=\chi^{\mathrm{L(e)}}+\chi^{\mathrm{L(i)}}=0$ we have the dispersion relation (Mikhailovskii 1974)

$$\omega''=k_z''c_\mathrm{se}/\Delta, \tag{13}$$

where $\Delta=[1+K\lambda^2\lambda_\mathrm{Di}^2+k''^2\varrho_\mathrm{i}^2(m_\mathrm{e}/m_\mathrm{i})]^{1/2}$, $\varrho_\mathrm{i}=V_\mathrm{i}/\Omega_\mathrm{i}$ is the ion gyroradius, $\lambda_\mathrm{Di}=V_\mathrm{i}/\omega_\mathrm{pi}$ is the ion Debye length, ω_pi is the ion plasma frequency and $c_\mathrm{se}=(T_\mathrm{i}/m_\mathrm{e})^{1/2}$ is the electron acoustic speed. When $k''^2\lambda_\mathrm{Di}^2\ll 1$ and $k''^2\varrho_\mathrm{i}^2\ll m_\mathrm{i}/m_\mathrm{e}$, we have $\omega''=k_z''c_\mathrm{se}$, which has the same form as for ion acoustic mode (cf. discussion in Section 4) except that the role of ions and electrons are interchanged. For $k''^2\lambda_\mathrm{Di}^2\gg 1$, we have $\omega''=\omega_\mathrm{pe}\cos\theta''$. Fig. 1 shows the dispersion relation (13) with $\omega_\mathrm{pe}/\Omega_\mathrm{e}=0.5$, $\cos\theta''=0.1(T_\mathrm{i}m_\mathrm{e}/T_\mathrm{e}m_\mathrm{i})^{1/2}$, $T_\mathrm{e}=5\times 10^5$ K. As k'' increases, we have $\omega''^2/\omega_\mathrm{pe}^2\approx\cos^2\theta''$. For a larger ratio $T_\mathrm{i}/T_\mathrm{e}$, we have larger $\cos\theta''$ as shown by the dashed curve.

3.3 Three-wave probability and growth rate

To calculate the three-wave probability, we derive the ratio of the electric-to-total energy in electron acoustic waves as (Melrose 1986)

$$R^\mathrm{se} = \left.\frac{\omega''}{\partial(\omega''^2K^\mathrm{L})/\partial\omega''}\right|_\mathrm{se}, \tag{14}$$

where $\sigma''=\mathrm{se}$ represents the electron acoustic mode. From (14) we derive $R^\mathrm{se}=\omega''^2/(2\omega_\mathrm{pe}^2\cos^2\theta'')$, where $\omega''\ll\Omega_\mathrm{e}$. Substituting (4) for (14) and using $\chi^{\mathrm{L(e)}}$ for cold electrons, we have the three-wave probability

$$\bar{w}^\mathrm{se}=2\pi r_\mathrm{e}c^2n_\mathrm{se}''^2\frac{\hbar\omega''}{m_\mathrm{e}c^2}\left(\frac{\omega_\mathrm{pe}^2}{\omega^2}\right)\eta, \tag{15}$$

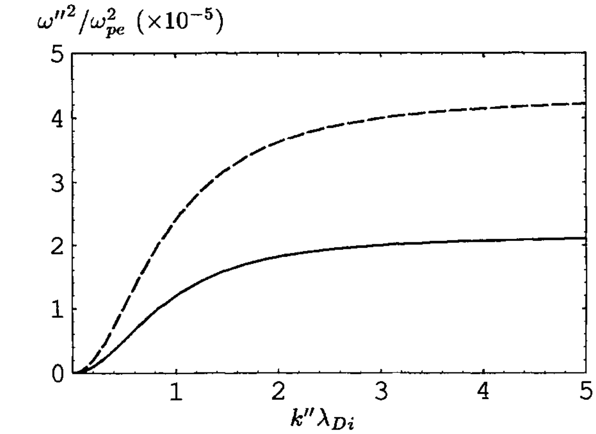

Figure 1. Dispersion relation of electron acoustic mode. The ion contribution is treated as unmagnetized. The solid and dashed curves correspond to the temperature ratio $T_i/T_e=5$ and 10, respectively.

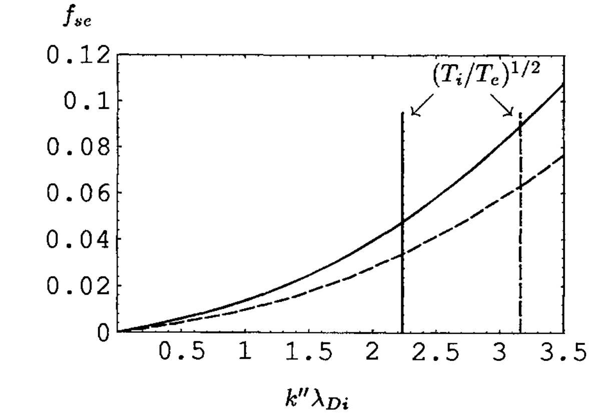

Figure 2. Plots of f_{se} vs. $k''\lambda_{Di}$ with $T_i/T_e=5$ (solid) and $T_i/T_e=10$ (dashed). Due to the Landau damping constraint the wave is undamped only if $k''\lambda_{Di} \ll (T_i/T_e)^{1/2}$. The limits are indicated by the solid $((T_iT_e)^{1/2}=\sqrt{5})$ and dashed $((T_i/T_e)^{1/2}=\sqrt{10})$ vertical lines.

where $\eta=2R^{se}(\omega''/\omega_{pe})^2(\chi^{L(e)})^2=(\omega''^2\sin^2\theta''/\Omega_e^2-\cos^2\theta'')/\cos^2\theta''$, n''_{se} is the refractive index for electron acoustic waves. For $k''^2\lambda_{Di}^2<1$, we have $k''^2c_{se}^2<\omega_{pe}^2$. The corresponding growth rate can be derived as

$$\Gamma=\frac{n''^2_{se}\eta}{\alpha+1}\frac{r_e\omega_{pe}^2}{2\pi c}\frac{T_B}{m_ec^2}\frac{\omega''}{\omega}, \tag{16}$$

where $T_B=\hbar\omega N(k)$ is the brightness tempeature of the photon beam. The expression (16) takes the same form as that for Langmuir waves (e.g. Luo & Melrose 1995a,b) except that includes η.

To compare with the growth of Langmuir waves induced by the photon beam, we write down the probability for Langmuir waves as (Luo & Melrose 1995a)

$$\bar{w}^l=2\pi r_ec^2n''^2_l\frac{\hbar\omega_{pe}}{m_ec^2}\frac{\omega_{pe}^2}{\omega^2}, \tag{17}$$

where we use l to denote the Langmuir waves and n''_l is the corresponding refractive index. The corresponding growth rate depends strongly on the electron number density and also on the refractive index n''_l. For a given number density, the growth rate favours large refractive index. However, n''_l is constrained by the Landau damping, which is weak only when the wavelength is longer than the electron Debye length. This condition can be written into the form $n''_l<\delta c/V_e$, with the parameter δ determining the ratio of damping rate Γ_d to the electron plasma frequency. For $n''_l=\delta c/V_e$ and $\delta=0.3$, the ratio is $\Gamma_d/\omega_{pe}\approx 0.02$. From equations (12), (16) and (17), we obtain

$$\Gamma_{se}=f_{se}(\Gamma_l)_{max}, \tag{18}$$

with

$$f_{se}=\frac{\bar{w}^{se}}{(\bar{w}^l)_{max}}=\left(\frac{n''_{se}}{n''_l}\right)^2\frac{\omega''}{\omega_{pe}}\eta, \tag{19}$$

where n''_{se} is the refractive index of electron acoustic waves. The maximum growth rate for Langmuir waves is given by (Luo & Melrose 1995b, equation 5.1)

$$(\Gamma_l)_{max}=(18\ \mathrm{s}^{-1})\left(\frac{T_e}{10^6\ \mathrm{K}}\right)^{-2}\left(\frac{n_e}{10^5\ \mathrm{cm}^{-3}}\right)^{3/2}\times\left(\frac{T_B}{10^{21}\ \mathrm{K}}\right)\left(\frac{\omega/2\pi}{318\ \mathrm{MHz}}\right)^{-1}, \tag{20}$$

where n_e is the electron number density. In deriving equation (20), we use $n''_l=\delta c/V_e$ and $\delta=0.3$.

Using $n''_l=\delta c/V_e$, equation (19) can be rewritten in the form

$$f_{se}=\frac{\Delta}{\delta^2}\frac{T_e}{T_i}\left(\frac{k''_zc_{se}}{\omega_{pe}}\right)\left(\frac{k''^2c_{se}^2}{\Omega_e^2\Delta^2}\sin^2\theta''-1\right)^2, \tag{21}$$

where $\cos\theta''=k''_z/k''\ll(m_e/m_i)^{1/2}(T_i/T_e)^{1/2}$. Plots of f_{se} as a function of $k''\lambda_{Di}$ with $\omega_{pe}/\Omega_e=0.5$, $T_e=5\times10^5$ K and $\cos\theta''=0.1(T_im_e/T_em_i)^{1/2}$ are shown in Fig. 2. Note that f_{se} is an increasing function of k''. The maximum k'' is constrained by Landau damping through a condition $k''\lambda_{Di}\ll(T_i/T_e)^{1/2}$. Therefore, the maximum growth rate Γ_{se} depends on the temperature ratio and in principle it can be comparable to that for Langmuir waves provided that T_i/T_e is sufficiently high.

3.4 Induced Brillouin scattering

When $N^{\sigma''}$ in the diffusion coefficients (2) or (7) corresponds to acoustic turbulence produced by the photon beam, the process is called induced Brillouin scattering. For electron acoustic waves the diffusion coefficient is evaluated to be

$$D_\perp^{se}=\frac{r_ec}{\lambda_{De}^4}\frac{\hbar\omega_{pe}}{m_ec^2}\left(\frac{\omega_{pe}}{\omega}\right)^2\left(\frac{c}{V_e}\right)^2I^{se}, \tag{22}$$

where we write $D_\perp\to D_\perp^{se}$ for electron acoustic waves ($D_\perp^l$ for Langmuir waves) and

$$I^{se}=\int\frac{d\bar{\phi}''}{2\pi}\int x^5\,dx\left(\frac{\omega_{pe}}{\omega''}\right)\eta N^{se}, \tag{23}$$

with $x = k''\lambda_{De}$ and λ_{De} the electron Debye length. The angular dependence of the integrand on $\bar{\phi}''$ is implied as it depends on θ'', which can be expressed in terms of $\bar{\phi}''$. The upper and lower limits of integration are $x = 1$ and 0, respectively. For Langmuir turbulence, we have a similar form to (22) with I^{se} replaced by

$$I^{l} = \int dx\, x^5 N^{l}, \tag{24}$$

where integration is over V_e/c and 1. We find the ratio of two diffusion coefficients as

$$\frac{D_\perp^{se}}{D_\perp^{l}} = \frac{I^{se}}{I^{l}}. \tag{25}$$

In practice, only the saturated quantities are relevant. Growth of low-frequency waves leads to diffusion of the photon beam, which then reduces the growth rate. The time-scale for a beam to spread to an angle $\Delta\theta$ is $(D_\perp/k^2\Delta\theta)^{-1}$. The saturated I^{se} and I^{l} can be estimated by equating the diffusion and the growth time-scales ($\sim\Gamma^{-1}$).

Let $(I^{se})_{sa}$ and $(I^{l})_{sa}$ represent the corresponding saturated values of I^{se} and I^{l}, respectively. We find

$$\frac{(I^{se})_{sa}}{(I^{l})_{sa}} = \frac{n_{se}''^2}{n_l''^2}\frac{\omega''}{\omega_{pe}}. \tag{26}$$

For $k''\lambda_{Di} > 1$, we have $\omega'' \approx \omega_{pe}\cos\theta''$. For Langmuir waves, the maximum refractive index is given by $(n_l'')_{max} = \delta c/V_e$. The refractive index n_{se}'' can be much larger than $(n_l'')_{max}$ for a small angle θ''. Thus, the right-hand side of (26) can be larger than 1, which follows that $D_\perp^{se}/D_\perp^{l} \gtrsim 1$ and induced scattering involving low-frequency electron acoustic waves can be more effective than induced Raman scattering provided that the low-frequency electron acoustic waves can grow to the saturated level given by (26).

4 PHOTON-BEAM-INDUCED ION ACOUSTIC TURBULENCE

For non-isothermal plasmas with hot electrons ($T_e > T_i$), ion acoustic mode is allowed. The dispersion relation can be derived from $K^L = 1 + \chi^{L(e)} + \chi^{L(i)} = 0$, with $\chi^{L(e)} = \omega_{pi}^2/(k''^2 c_s^2)$ and $\chi^{L(i)} = (\omega_{pi}^2/\omega''^2)[\cos^2\theta'' - \omega''^2\sin^2\theta''/(\omega''^2 - \Omega_i^2)]$. There are two branches of ion acoustic mode with dispersion relations given by (e.g. Stix 1962; Mikhailovskii 1974)

$$\omega_\pm''^2 = \tfrac{1}{2}(\omega_s^2 + \Omega_i^2) \pm \tfrac{1}{2}[(\omega_s^2 + \Omega_i^2)^2 - 4\omega_s^2\Omega_i^2\cos^2\theta'']^{1/2}, \tag{27}$$

where $\pm$ correspond to fast and slow ion acoustic waves, respectively. In equation (27), $\omega_s^2 = \omega_{pi}^2/(1 + k''^{-2}\lambda_{De}^{-2})$. We assume the magnetic field to be in the z-direction. To avoid heavy Landau damping the phase speed of the waves must be in the range $V_i \ll \omega''/(k''\cos\theta'') \ll V_e$ (see e.g. Stix 1962). The dispersion relation (27) as a function of $k''\lambda_{De}$ is plotted in Fig. 3 (we assume $\Omega_e/\omega_{pe} = 10$). For long or short wavelengths, (27) takes a simpler form. When the wavelength is much longer than the electron Debye length, $k''^2\lambda_{De}^2 \ll 1$ (or $k''^2\lambda_{Di}^2 \ll T_i/T_e$, where λ_{Di} is the ion Debye length), we have $\omega_s = k''c_s$, where $c_s = (T_e/m_i)^{1/2}$ is the ion sound speed. For $k''^2c_s^2 \gg \Omega_i^2$, we have $\omega_+'' \approx k''c_s$, the usual ion acoustic waves and $\omega_-'' \approx \Omega_i|\cos\theta''|$, ion cyclotron waves. For $k''^2c_s^2 \ll \Omega_i^2$, we have $\omega_+'' = \Omega_i$ and $\omega_-'' = k_z''c_s$. When the wavelength is shorter

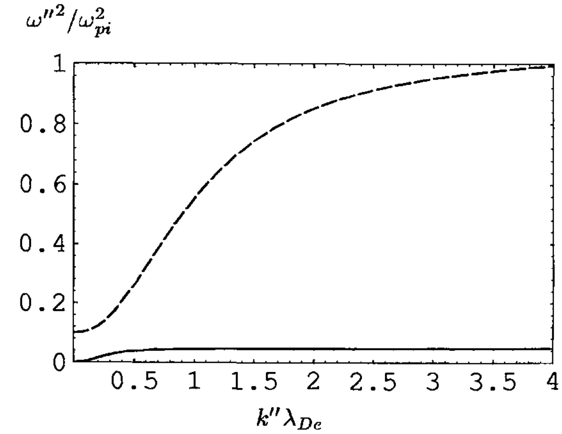

Figure 3. Dispersion relation of ion acoustic waves with $\Omega_e/\omega_{pe} = 10$, $\theta'' = \pi/4$. The dashed and solid curves correspond to the fast and slow branches, respectively.

than the electron Debye length, $k''^2\lambda_{De}^2 \gtrsim 1$ (or $k''^2\lambda_{Di}^2 \gtrsim T_i/T_e$) but $k''^2\lambda_{Di}^2 \lesssim 1$, we have $\omega_s = \omega_{pi}$. The dispersion relation for ion acoustic mode is shown in Fig. 3.

Using equation (14) for ion acoustic waves and $R^{se} \to R^s$, which gives

$$R_\pm^s = \frac{1}{2}\left(\frac{\omega_\pm''}{\omega_{pi}}\right)^2 \xi_\pm, \tag{28}$$

$$\xi_\pm = \left(1 - \frac{\Omega_i^2}{\omega_\pm^2}\right)\left(1 - \frac{\Omega_i^2}{\omega_\pm^2}\cos^2\theta'' + \frac{\Omega_i^2}{\omega_\pm^2 - \Omega_i^2}\sin^2\theta''\right)^{-1}. \tag{29}$$

For unmagnetized plasmas $\Omega_i = 0$, we have $\xi_\pm = 1$ and $R_\pm^2 = (\omega_\pm''/\omega_{pi})^2/2$. From the ratio (28), we see that lower frequency $\omega_\pm''$ gives smaller $R_\pm^s$ and the maximum ratio is $R_+^s = 1/2$ for $\omega_+'' = \omega_{pi}$.

Since waves described by (27) have phase speed much slower than the electron thermal speed, we can use approximation (4). Thus, we obtain the probability as

$$\bar{w}_\pm^s = 2\pi r_e c^2 n_s''^2 \xi_\pm \left(\frac{\hbar\omega_\pm''}{m_e c^2}\right)\left(\frac{\omega_{pi}}{\omega}\right)^2\left(\frac{\omega_\pm''}{k''c_s}\right)^4, \tag{30}$$

where $\pm$ correspond to the two branches given by equations (13). From equations (12) and (17), we obtain the growth rate for the two branches of ion acoustic waves,

$$\Gamma_\pm = \frac{n_s''^2\xi_\pm}{\alpha + 1}\frac{T_B}{m_e c^2}\frac{r_e\omega_{pi}^2}{2\pi c}\frac{\omega_\pm''}{\omega}\left(\frac{\omega_\pm''}{k''c_s}\right)^4. \tag{31}$$

Expression (31) is very similar to that for Langmuir waves (e.g. Luo & Melrose 1995a,b), except that both ω_{pi} and $\omega_\pm''$ are replaced by ω_{pe} for Langmuir waves. We may compare the growth rates for the two branches by evaluating the ratio

$$\frac{\Gamma_+}{\Gamma_-} = \frac{\xi_+}{\xi_-}\left(\frac{\omega_+''}{\omega_-''}\right)^3, \tag{32}$$

for a given wave number. Since $\omega_+'' > \omega_-''$, in general the growth rate for the fast branch is faster than that of the slow

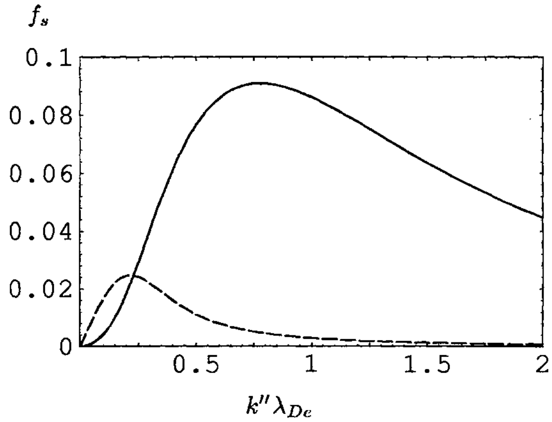

Figure 4. Plots of f_s as a function of $k''\lambda_{\rm De}$. The solid and dashed curves correspond to the fast and slow branches, respectively.

branch (see Fig. 4). To compare with Langmuir waves, we write

$$\Gamma_{\pm}=f_s(\Gamma_l)_{\rm max},$$

with

$$f_s=\frac{1}{\delta^2}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\omega''}{\omega_{\rm pi}}\right)^3\frac{\xi}{(k''\lambda_{\rm De})^2}. \tag{33}$$

Using the dispersion relation (27), the right-hand side can be expressed as a function of $k''\lambda_{\rm De}$ with $k''\lesssim\delta/\lambda_{\rm Di}$. To estimate f_s, we consider the case of $\omega_{\rm pi}^2\gg\Omega_i^2$. For eclipsing plasmas with electron plasma frequency of the same order as that of the electron cyclotron frequency, this assumption is valid. Thus, we have $\omega_s^2\gg\Omega_i^2$ and $\omega_+''^2=\omega_{\rm pi}^2/(1+k''^{-2}\lambda_{\rm De}^{-2})$ for $k''\lambda_{\rm De}\gtrsim 1$. We find $(f_s)_{\rm max}=(2/3^{3/2}\delta^2)(m_e/m_i)^{1/2}$ at $k''\lambda_{\rm De}=1/\sqrt{2}$. This gives $(f_s)_{\rm max}\approx 0.1$ for $\delta=0.3$. Therefore, the growth rate of ion acoustic waves is slower than that of Langmuir waves by a factor of 10 and can grow if the scattering length of the eclipse region is $\sim R_\odot$ (e.g. Fruchter et al. 1990; Lyne et al. 1990). Fig. 4 shows the plots of f_s versus $k''\lambda_{\rm De}$ (solid and dashed curves correspond to the fast and slow ion acoustic branches, respectively).

In analogy to (22), we have the diffusion coefficient

$$D_\perp^s=\frac{r_e c}{\lambda_{\rm De}^4}\frac{\hbar\omega_{\rm pi}}{m_e c^2}\left(\frac{\omega_{\rm pi}}{\omega}\right)^2\left(\frac{c}{c_s}\right)^2 I^s, \tag{34}$$

with

$$I^s=\int\frac{d\bar{\phi}''}{2\pi}\int x\,dx\left(\frac{\omega_+''}{\omega_{\rm pi}}\right)^3\xi_+N^s. \tag{35}$$

In (35), we consider the fast branch as an example. The upper and lower limits of integration are $(T_e/T_i)^{1/2}$ and 0, respectively. Using (24), we find the ratio of two coefficients as

$$\frac{D_\perp^s}{D_\perp^l}=\left(\frac{m_e}{m_i}\right)^{1/2}\frac{I^s}{I^l}. \tag{36}$$

The ratio of saturated I^s and I^l is given by

$$\frac{(I^s)_{\rm sa}}{(I^l)_{\rm sa}}=\left(\frac{\omega_+''}{\omega_{\rm pi}}\right)^3\frac{1}{\delta^2k''^2\lambda_{\rm De}^2}, \tag{37}$$

which gives 4.3 for $\delta=0.3$. Thus, we have $D_\perp^s/D_\perp^l\approx 0.1$, which implies that induced Brillouin scattering off ion acoustic waves is less efficient than induced Raman scattering.

5 APPLICATION TO PULSAR ECLIPSES

The observed excess delay in propagation time of a radio signal near or in the eclipse region (e.g. Fruchter et al. 1990) indicates that the eclipsing material consists of plasmas from the companion wind. Strong heating by a pulsar wind through high-energy photons or plasma wave heating may result in a hot corona-like plasma wind with T_e (or T_i) $\sim 10^6$ K and it is possible that the plasma is non-isothermal (as in the case of solar corona, electrons and ions have different temperatures). One of the important consequences of two-temperature plasmas is that acoustic turbulence can exist, which can be produced either through induced three-wave interactions as discussed in the previous section or by other mechanisms like shocks. There are two possibilities for non-isothermal plasmas: $T_e>T_i$ and $T_e<T_i$. As to which case describes best the eclipsing plasma condition: this depends on the specific heating processes. Our discussion will place emphasis on application to PSR 1957+20.

It can be shown that the small-angle scattering approximation should be a good approximation. The scattering angle can be written into the form $\theta_s=k''/k<0.3c/\lambda_{\rm De}\omega$. For $T_e>T_i$, $T_e=10^6$ K and $n_e=10^5$ cm^{-3} for PSR 1957+20 (Ryba & Taylor 1991), we have $\theta_s<0.15$. One has the same upper limit to θ_s if $T_i=10^6\,{\rm K}>T_e$. Large-angle scattering may be possible further inside the eclipsing region where the plasma density is sufficiently high.

Acoustic turbulence can have important effects on radio beam propagation through induced scattering. For $T_i\gg T_e$, we show that the maximum growth rate for electron acoustic waves induced by the photon beam can be comparable to that for Langmuir waves provided that the temperature ratio is sufficiently large. From (18) and (20), it can be shown that with the nominated physical parameters in (20) and an electron temperature, e.g. $T_e=5\times10^5$ K, the growth is fast enough to reach to saturation if the scattering region has a length-scale of order $R_\odot$ (e.g. Fruchter & Goss 1992) and the temperature ratio is $T_i/T_e\gtrsim 10$. In the previous section (cf. 26), we have shown that the induced scattering can be more effective than induced Raman scattering. In order to produce pulsar eclipses, the time-scale for induced scattering must be shorter than that for photons to traverse through the eclipse region. For induced Raman scattering, this condition appears to be satisfied (for a detailed discussion see Luo & Melrose 1995a,b). Thus, for electron acoustic waves, this condition can also be satisfied and this can cause pulsar eclipse.

For non-isothermal plasmas with hot electrons, we show that the growth rate of ion acoustic waves is slower than the Langmuir counterpart. Although ion acoustic waves may also grow to saturation under the conditions that are relevant for pulsar eclipse, the induced scattering is less important than induced Raman scattering since the diffusion time is longer than induced Raman scattering by a factor of 10. It is unlikely to be the main cause of eclipses. We should emphasize here that this conclusion is valid only in the small-angle scattering approximation.

Although the energy level of photon-beam-induced ion acoustic turbulence is lower than the corresponding Langmuir turbulence, it may still play a role in the pulsar eclipse mechanism through the non-linear coupling of Langmuir and ion acoustic waves. Similar examples can be found in space physics, e.g. type III events in the solar wind (Lin et al. 1986; Chian & Alves 1988), and the upstream region of planetary bow shocks (Anderson et al. 1981). The interaction of ion acoustic and Langmuir instabilities leads to enhancement of Langmuir wave growth, e.g. it may enhance the growth rate and widen the bandwidth of Langmuir instability (e.g. Chian & Rizzato 1994). Hence, in the application to pulsar eclipse, the photon-beam-induced acoustic turbulence may improve the efficiency of induced Raman scattering and contribute to pulsar eclipses through the coupling of ion acoustic and Langmuir waves.

There can be ion acoustic turbulence produced through mechanisms such as shocks which must be present in the interface region between pulsar wind and companion star wind or magnetosphere (for the discussion of the shocks in this region, see, e.g. Phinney et al. 1988). This is very similar to the case of interplanetary and planetary shocks where ion acoustic turbulence has been detected (Gurnett et al. 1979). If the energy density level of the pre-existing ion acoustic turbulence is sufficiently high, scattering of the beam off acoustic turbulence can produce eclipses. We may estimate the energy density required for eclipses as follows. From (36), in order for $D^{s}_{\perp} \gtrsim D^{l}_{\perp}$, we must have $(I^{s})_{sa} \gtrsim 43(I^{l})_{sa}$. Since the energy densities are $\propto \hbar\omega_{pi} I^{s}$ and $\hbar\omega_{pe} I^{l}$, respectively, this requires that the ion acoustic energy density be comparable to or higher than Langmuir turbulence.

If the ambient magnetic field plays a role in balancing the ram pressure of pulsar wind, which can be the case for PSR 1957+20 since the thermal pressure from the companion wind near the shock appears less than the ram pressure from the pulsar wind (see Thompson et al. 1994; Luo & Melrose 1995b), the electron plasma frequency can be of the same order of magnitude as the electron cyclotron frequency $\omega_{pe} \sim \Omega_e$ (e.g. Luo & Melrose 1995b). In this case, we should use the full dispersion relation (13).

In conclusion, an anisotropic photon beam such as pulsar radio emission can induce ion acoustic (if $T_e > T_i$) or electron acoustic (if $T_e < T_i$) turbulence. For $T_i > T_e$, in the small-angle scattering approximation, the maximum growth rate for electron acoustic waves can be comparable to that for Langmuir waves under similar plasma conditions (e.g. with similar plasma densities) provided that the temperature ratio is sufficiently high. For a scattering region of length-scale $R_{\odot}$, provided that the temperature ratio $T_i/T_e \gtrsim 10$, the growth rates are fast enough to reach saturation, and induced Brillouin scattering of the photon beam off electron acoustic waves can be one possible mechanism for the observed pulsar eclipses. In the case of $T_e > T_i$, induced Brillouin scattering off ion acoustic waves is much less effective than induced Raman scattering and can be ruled out as the main cause for the pulsar eclipse. However, photon-beam-induced ion acoustic turbulence may still play a role in pulsar eclipse by enhancing induced Raman scattering through coupling between Langmuir and acoustic turbulence. If ion acoustic turbulence pre-exists and has a sufficiently high level with energy density comparable to or higher than that of Langmuir turbulence, a photon beam can then be isotropized through scattering off pre-existing ion acoustic turbulence and thus produce eclipses.

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