SPIN AXIS TILT EVALUATION FROM AN OPTIMIZATION APPROACH

Roberto V. F. Lopes*

Instituto Nacional de Pesquisas Espaciais
CP 515, 12201-090 São José dos Campos, SP, Brazil
phone: +55 12 325-6192 fax:+55 12 325-6226 e-mail:rvfl@dem.inpe.br

ABSTRACT

The spin axis tilt due to asymmetries on a spin stabilized satellite equipped with long wire cable booms is evaluated. The problem is stated as a constrained optimization one, since the equilibrium state minimizes the kinetic energy under the angular momentum conservation law, besides the geometrical constraints of the booms. Three numerical procedures are presented to solve the non-linear optimum necessary condition: a direct iteration; a Newton-Raphson; and a predictor-corrector. Numerical results indicate that the first one works fine for the ordinary, slightly tilted case only. Convergence problems under arbitrarily high tilted conditions are properly dealt with the other two procedures.

1 INTRODUCTION

Spin stabilization has been a cheep and effective way for attitude stabilization of many artificial satellites since the beginning of the space missions. With the ever increasing accuracy requirements from both attitude control system and mission payload, deep and overall analysis of satellite spin dynamics have been carried out (see [1-3]). Case studies based on observed data from real missions (see [4], for instance) have also bring relevant insight to the subject. The specific topic of prediction and estimation of a satellite spin axis tilt has though received comparatively less attention in the international specialized literature, in spite of being possibly a common practice at space agencies.

The spin axis tilt (**sat**) of a spinning stabilized satellite is the angle between its geometric z-axis as determined by probe draft and the true spin axis at steady state, which coincides with the satellite's major principal axis of inertia. Nominally, these two axes are supposed to be aligned by design. Mechanical inaccuracies, non-gravity and thermal effects, vibration during launch phase and asymmetry on moving parts and fuel tanks, and even fault on deployment devices may, however, drift the major principal axis of inertia away from the geometric z-axis, thus resulting a non-zero **sat** . Once the phenomenon starts, it is self amplified. The satellite moving parts like cable booms, spinning about the tilted axis of inertia, will look for a new equilibrium position, so affecting the satellite mass distribution in a hopefully convergent closed loop process.

During the last two decades, a series of studies has been motivated by actual missions, specially by those ones which have long cable booms. Booms are useful as antennas or as a mean of protection against photo-electric-magnetic interference from the satellite main body, for on board high sensitivity equipments placed at the booms tip. Janssens [5-7] begun by exploring the analogies of a simplified dynamic model and afterwards stated a general approach to evaluate the equilibrium position of the booms from force equilibrium considerations. Brenner [8] evaluated the effect of fuel consumption on the fuel tanks moment of inertia. Goodwin and Massart [9] derived the V-slit sun sensor measurements model for the ISPM satellite in the presence of a **sat** while Van der Beken [10] added considerations about statistics and observability. An iterative algorithm for computation of the sat due to both axial (rigid) and radial (wire) booms of ULYSSES was proposed by Gienger [11]. Then Hugo [12] presented a detailed derivation of the dynamic model for a spinning satellite with wire booms from both Newton/Euler and Lagrange methods. The accurately phased spin reference pulse specified for ULYSSES required its few arc-minute **sat** to be estimated [13] and taken into account by the control system. More recently, Mortary and Arduini [14] analyzed the attitude dynamics excited by orbit thermal transitions of a spin stabilized satellite carrying four equatorial antennas with a tip mass.

The present work follows and extends the works of Janssens [7] and Gienger [11], and is an alternative analysis to the more complete study of Delgado

*Supported by a grant from FAPESP

and Miguel [15]. The spin axis tilt due to asymmetries on a spin stabilized satellite equipped with long wire cable booms is evaluated. The problem is stated as a constrained optimization one, since for a spinning satellite on a torque free motion the equilibrium state minimizes the kinetic energy under the angular momentum conservation law, besides the geometrical constraints of the booms. This was found to be a suitable, general and systematic approach, even in the presence of highly asymmetric equilibrium state. The resulting non-linear equilibrium conditions were briefly presented by an earlier work [16]. Now, three procedures to solve the non-linear optimization problem are described and numerical results from simulation are presented.

2 NOMENCLATURE

c.o.m. center of mass.

l_i Distance from the i-th wire boom *c.o.m.* to its attached point at the rigid body part of the satellite (see Figure 1).

m_i Mass of the i-th wire boom.

$\bar{m}_i$ Tip mass of the i-th wire boom.

p_i Coordinates of the i-th wire boom *c.o.m.*, at stowed state, on $[\bar{X} : \bar{Y} : \bar{Z}]$ frame.

r_i Coordinates of the i-th wire boom *c.o.m.* at deployed state on $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

r_{0i} Coordinates of the i-th wire boom *c.o.m.*, at stowed state on $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

s Coordinates of the satellite c.o.m. at deployed state on $[\bar{X} : \bar{Y} : \bar{Z}]$ frame.

sat Spin axis tilt.

u_i Coordinates of the unit vector on the deployment direction of the i-th wire boom on $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

v Dummy vector on $\mathbf{R}^3$.

w Coordinates of the satellite angular velocity vector on the $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

z Coordinates of the satellite spin axis unit vector at deployed state on $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

H Satellite angular momentum.

I Identity matrix on $\mathbf{R}^{3\times 3}$.

J Inertia tensor of the satellite at deployed state.

J_0 Inertia tensor of the satellite at stowed state.

$\bar{J}$ Inertia tensor of the rigid body part of the satellite.

J_i Inertia tensor of the i-th wire boom at deployed state.

J_{0i} Inertia tensor of the i-th wire boom at stowed state.

K Kinetic energy of the satellite rotational motion around its center of mass.

L_i Length of the deployed i-th wire boom.

M Total mass of the satellite.

$\bar{M}$ Mass of the rigid body part of the satellite.

$\mathcal{P}(v)$ Vectorial double product operator: $v \wedge (v \wedge \bullet)$.

R Coordinates of the rigid body *c.o.m.* on $[\tilde{X} : \tilde{Y} : \tilde{Z}]$ frame.

$\bar{R}$ Coordinates of the rigid body *c.o.m.* on $[\bar{X} : \bar{Y} : \bar{Z}]$ frame.

U Matrix containing the set of boom directions: $[u_1 : u_2 : \cdots]$.

V Eigen-vector of J.

$[\bar{X} : \bar{Y} : \bar{Z}]$ Body fixed right handled frame, aligned with the satellite principal axes of inertia at stowed state, and centered on its *c.o.m.*, being $\bar{Z}$ the major principal axis of inertia.

$[\tilde{X} : \tilde{Y} : \tilde{Z}]$ Body fixed frame, parallel to $[\bar{X} : \bar{Y} : \bar{Z}]$ frame, but centered on the satellite *c.o.m.* at deployed state.

λ Eigen-value of J.

μ_i Relative mass of the i-th wire boom.

ρ_i Linear mass density of the i-th wire boom.

σ_i Inertia ratio of the i-th deployed wire boom.

ν_i Lagrange multiplier for a wire boom constraint.

$[[v]]$ Vectorial product operator: $v \wedge \bullet$.

$\wedge$ Vectorial product.

$'$ Transposition operator.

$*$ Indicates optimal value.

3 ASSUMPTIONS

The following assumptions are made:

- The satellite consists of a rigid body part and a given set of wire booms attached to it;
- The satellite at deployed state has achieved a minimum energy steady-state, and is spinning under a torque-free motion.
- Each wire boom at stowed state is a point mass.
- Each wire boom at deployed state is a straight rigid line, (stiffless and with neglectable cross section) at unconstrained direction, with homogeneous linear mass density and a point tip mass at its free extremity;
- M, J_0, $\{\bar{m}_i, \rho_i, L_i, p_i,$ for $i = 1, 2, \cdots\}$ are the exactly known inputs.
- The satellite major principal axis of inertia remain unambiguous throughout the deployment of the wire booms.

4 THE MINIMUM ENERGY APPROACH

Under the assumptions, the problem to be solved may be stated as:

Find u_i^*, s^* and z^* which minimize the rotational kinetic energy of the satellite subjected to the physical constraint due to the angular momentum conservation law for a rigid body torque-free motion and to the geometrical constraints:

$$\begin{aligned} \text{Min:} \quad & K(w,U) = \frac{1}{2} w' J(U) w, && (1)\\ \text{Subj. to:} \quad & [J(U)w]'[J(U)w] = H^2, && (2)\\ & u_i' u_i = 1. && (3) \end{aligned}$$

The necessary extremum condition can be obtained from the Lagrange multipliers method. Let be the extended cost function K_e:

$$\begin{aligned} K_e(w,U,\lambda,\nu) \;\equiv\;& K(w,U) + \frac{1}{2\lambda}[H^2 - w' J^2(U) w] \\ &+ \sum_i \frac{\nu_i}{2}[u_i' u_i - 1]. \qquad (4) \end{aligned}$$

The partial derivatives of K_e with respect to w and u_i must vanish at its extremum. This leads straightforwardly to the following equations:

$$\begin{aligned} J(u^*)w^* &= \lambda_i w^*, && (5)\\ \frac{1}{2}\frac{\partial}{\partial u_i}[w^{*\prime} J(U^*) w^*] &= \nu_i u_i^{*\prime}, && (6) \end{aligned}$$

which, in view of Equations 1 and 2 yields:

$$K(w^*, U^*) = \frac{H^2}{2\lambda}. \qquad (7)$$

From Equation 5, w^* is one of the eigen-vectors of $J(U^*)$. From Equation 7 one may easily conclude that it must be that one whose eigen-value is the biggest, which means:

$$w^* = \frac{H}{\lambda} z \qquad (8)$$

5 THE INERTIA TENSOR

In order to solve Equation 6, one need to express the inertia tensor J as a function of the directions u_i. From the assumptions, geometrical considerations and center of mass definition, one has:

$$\begin{aligned} M &= \bar{M} + \sum_i m_i, && (9)\\ \bar{R} &= R + s, && (10)\\ p_i &= r_{0i} + s, && (11)\\ r_i &= r_{0i} + l_i u_i, && (12) \end{aligned}$$

$$\begin{aligned} \bar{M}\bar{R} + \sum_i m_i p_i &= 0, && (13)\\ \bar{M} R + \sum_i m_i r_i &= 0. && (14) \end{aligned}$$

From Equations 11 to 14 it follows:

$$r_i = p_i - s + l_i u_i, \qquad (15)$$

$$\bar{M}(\bar{R} - R) + \sum_i m_i (p_i - r_i) = 0. \qquad (16)$$

Then, direct substitution of Equations 9, 10 and 15 into Equation 16 leads to a simple expression to the satellite center of mass at a deployed state:

$$s = \sum_i \mu_i l_i u_i, \qquad (17)$$

where μ_i is the relative mass of the i-th wire boom, defined by:

$$\mu_i \equiv \frac{m_i}{M}. \qquad (18)$$

The quantities m_i and l_i, as well as the inertia ratio σ_i, may be easily obtained as a function of the input parameters $\bar{m}_i$, p_i, L_i and ρ_i.

Now, by the Steiner theorem one can write:

$$J_0 = [\bar{J} - \bar{M}\mathcal{P}(\bar{R})] + \sum_i [J_{0i} - m_i \mathcal{P}(p_i)], \quad (19)$$

$$J = [\bar{J} - \bar{M}\mathcal{P}(R)] + \sum_i [J_i - m_i \mathcal{P}(r_i)], \quad (20)$$

where $\mathcal{P}$ is the vectorial double product operator, defined by:

$$\mathcal{P}(v) \equiv -(v'v)I + vv', \quad (21)$$

in such way that:

$$\mathcal{P}(v)v_1 = v \wedge (v \wedge v_1), \forall v_1 \in \mathbf{R}^3. \quad (22)$$

Since J_{0i} is zero and J_i is:

$$J_i = -m_i \sigma_i^2 \mathcal{P}(u_i), \quad (23)$$

after some algebraic handling the following expression can be obtained from Equations 19 to 23:

$$\frac{J}{M} = \frac{J_0}{M} + \mathcal{P}(s) - \sum_i \mu_i \{(l_i^2 + \sigma_i^2)\mathcal{P}(u_i) + l_i ([[p_i]][[u_i]] + [[u_i]][[p_i]])\} \quad (24)$$

6 THE ORTHOGONAL CONDITION

Once the inertia tensor J has been conveniently expressed as a function of U, one can solve the necessary extremum condition, Equation 6, which in view of Equation 8 can be rewritten as:

$$\frac{1}{2}\frac{\partial}{\partial u_i}\left[z'\frac{J(U^*)}{M}z\right] = \tilde{\nu}_i u_i^{*\prime}, \quad (25)$$

with $\tilde{\nu}_i$ given by:

$$\tilde{\nu}_i = \frac{\lambda^2}{MH^2}\nu_i. \quad (26)$$

Now, from Equation 24 one has:

$$z'\frac{J(U)}{M}z = z'\frac{J_0}{M}z + s'\mathcal{P}(z)s - \sum_i \mu_i [2 l_i p_i' \mathcal{P}(z) u_i + (l_i^2 + \sigma_i^2) u_i' \mathcal{P}(z) u_i]. \quad (27)$$

Having in mind that s represents an explicit function of u given by Equation 17, Equation 25 yields:

$$(I - zz')[l_i(p_i - s^*) + (l_i^2 + \sigma_i^2)u_i^*] = \tilde{\nu}_i u_i^*. \quad (28)$$

Pre-multiplying its both sides by z' it results:

$$\tilde{\nu}_i z' u_i^* = 0, \quad (29)$$

which holds if $\tilde{\nu}_i$ vanishes or if u_i^* is normal to z either. In the first case, Equation 28 implies that vector β_i defined by:

$$\beta_i \equiv p_i - s^* + \left(1 + \frac{\sigma_i^2}{l_i^2}\right) l_i u_i^*, \quad (30)$$

lies on the z-axis. In the second possibility, Equation 28 implies that:

$$(I - zz') l_i (p_i - s^*) = [\tilde{\nu}_i - (l_i^2 + \sigma_i^2)] u_i^*, \quad (31)$$

which means that u_i^* is normal to $z \wedge (p_i - s^*)$, besides of being normal to z. These orthogonal conditions, together with the normal constraint, form a set of nonlinear equations which determine u_i unless by a signal ambiguity. They may be compactly rewritten as:

$$g_i(u_i^*, s^*, z) = 0, \quad (32)$$

where g_i is the vectorial function defined by:

$$g_i(u_i, s, z) \equiv \left\{ \begin{array}{c} u_i' u_i - 1 \\ z' u_i \\ \{[[z]](p_i - s)\}' u_i \end{array} \right\}, \quad (33)$$

So, there are four candidate solutions at whole for the direction of each wire boom, u_i. From force equilibrium simple considerations, those two of them with $\tilde{\nu}_i = 0$ are unstable and correspond to local maxima of the kinetic energy. From those with u_i normal to z, the one with $u_i'(p_i - s) < 0$ corresponds to a saddle point of the kinetic energy, stable around $z \wedge u_i$ but unstable around the z axis. The other one is the stable equilibrium direction which corresponds to the minimum kinetic energy that the work is concerned about.

The solutions here presented could be obtained from physical considerations about the centrifugal force, but the minimum energy approach fits better for the purpose of numerical procedure development, in the sense that it offers the natural background to the interpretation of the iterative solutions. Furthermore, its is a systematic approach, from which every possible solution comes up naturally.

As a final comment to this section, one should note that if $p_i = 0$ for the whole set of wire booms, then Equation 31 holds for any set of directions u_i normal to z such that s remains null. The equilibrium of the set of wire booms becomes indifferent to rotations around z. As a consequence, one can infer a weak stability around z if $p_i << l_i$.

7 THE NUMERICAL PROCEDURES

Although Equation 33 has a simple geometrical interpretation, it repersents a nonlinear equation system, which in the general case could be solved by numerical procedures only. Three of them are described through this section. The first and simplest one is similar to the iteractive procedure of Gienger [11], where convergence was told to be achieved in five steps at most for the minute **sat** magnitude predicted to ULYSSES due to its mechanical tolerances. It is a direct optimization procedure, which takes into account virtual displacements only, towards the minimum energy eqiulibrium state for the wire booms.

Nevertheless, for highly tilted asymmetric configurations, the situation may be quite different as it will be shown. Actually, for some numerical examples, after hundreds of steps convergence has not been achieved at all. A possible explanation seems to be related with the conjecture that the Lipschitz constant of the algorithm should be the ratio of the first two eigen-values of the satellite inertia tensor [11], which of course may variate considerably along the iterative process. However a large **sat** is unlikely, it may happen due to failure on deployment devices. Therefore, at least for those odd, but nonetheless possible cases, the direct iteration approach is not suitable.

The conceptual problem with the direct iteration procedure is that it disregards the closed loop effect of a correction on the boom direction. The proper way of taking it into account is to solve the non linear set of necessary conditions by the Newton-Raphson method. Such indirect optimization method is the second proposed procedure.

Even though it never failed to find the equilibrium solution in all numerical examples, convergence of the Newton-Raphson procedure has not been theoretically assured. So, a third procedure is proposed as an extra tool to deal with strongly ill conditioned cases. That is the predictor-corrector procedure, whose basic idea is to split the original problem into a convergent sequence of similar ones, the solution of each of them being used to predict a close enough initial guess to the next one. The sequence starts with the stowed state and stops at the final deployed state. Each elementary problem of the sequence could be solved by any of the two previous procedures, on the so called corrector step. A linear predictor is then applied to find the initial guess to the next elementary problem. The number of intermediate states may always be chosen high enough to assure convergence.

8 NUMERICAL RESULTS

The algorithms described on the section above have been implemented using MATLAB. To compare their performances under different asymmetry levels, several boom deployment failure cases were simulated, where two among four wire booms deployed only 20% of their full lenghts. The satellite input parameters are:

$$M = 350\text{Kg}\,, \quad L = \{\ 7\text{m} \quad 35\text{m} \quad 7\text{m} \quad 35\text{m}\ \},$$

$$J = \begin{pmatrix} 360\text{Kg/m}^2 & 0 & 0 \\ 0 & 110\text{Kg/m}^2 & 0 \\ 0 & 0 & 430\text{Kg/m}^2 \end{pmatrix},$$

$$p = \begin{bmatrix} 1.5\text{m} & -1.5\text{m} & 0 & 0 \\ 0 & 0 & .3\text{m} & -.3\text{m} \\ .4\text{m} & .4\text{m} & -.4\text{m} & -.4\text{m} \end{bmatrix}.$$

The boom mass was varied from less then 100g up to few Kilograms and the results are sohwn in Table 1. The number of iterations and elapsed time refers to the following convergence criteria: the corrections on both spin axis tilt and *c.o.m.* offset are smaller then 1 arc second and 0.1mm, respectively.

9 CONCLUSIONS

Three different procedures to evaluate the spin axis tilt of a spin stabilized satellite with a general set of wire booms have been tested on a boom deployment failure case. The Direct Iteration Procedure was found the simplest and fastest one, suitable for a slightly tilted satellite (up to 13 degrees for the simulated case). Nevertheless, possibility of failure on the boom deployment system may yield a strong mass asymmetry around the spin axis. In such cases, the Direct Iteration Procedure may not converge. By the other hand, in all of the study cases, the Predictor-Corrector Procedure was of no benefit, since the Newton-Raphson Procedure was always able to find a solution faster then it. Anyway, it remains as a proper tool to assure convergence.

As a final remark, the minimum energy approach offered a suitable background and a systematic way to find the equilibrium condition of the wire boom problem, which suggests it would also be able to deal with other non rigid devices like a fuel tank.

Table 1: PERFORMANCE OF THE TILT EVALUATION PROCEDURES

tip mass [Kg]	cable linear density [g/m]	tilt	*c.o.m.* offset [mm]	Direct Iteration		Newton Raphson		Predictor-Corrector		
				# steps	time [s]	# steps	time [s]	# steps		time [s]
.025	2	37′	7	11	.3	16	.9	11	16	3.1
.250	10	3°59′	44	50	.6	86	4.4	56	79	7.2
.500	25	7°50′	83	90	1.2	158	7.8	112	142	13.0
.750	50	11°36′	119	127	1.5	225	11.0	177	196	18.6
1.000	50	12°23′	128	140	1.6	248	11.7	201	213	20.3
1.250	50	13°03′	136	179	2.0	267	12.9	224	227	23.5
1.500	50	13°39′	143	*	–	285	13.6	245	239	23.9
2.000	50	14°38′	156	*	–	314	14.7	243	219	22.4

* Convergence not achieved after 5000 steps

Acknowledgement

The author is indebted to Dr. L. Fraiture of ESOC.

References

[1] Wertz, J.R. (Ed.) *Spacecraft attitude determination and control.* D.Reidel, London, 1978. (Astrophysics and Space Sciences Library)

[2] Kane, T.R., Likins, P.W. and Levinson, D.A. (eds.) *Spacecraft Dynamics.* Mc Graw-Hill, New York, 1983.

[3] Hughes, P.C. *Spacecraft attitude dynamics.* John Wiley & Sons, New York, 1986.

[4] Fraiture, L.; Wiengarn, N. and Chambaz, B. "Irregularities in the motion of spin stabilized earth satellites." *Proceedings of the ESA Symposium on Spacecraft Flight Dynamics.* Darmstadt, May 18-22, 1981.

[5] Janssens, F. "Some elementary considerations about the motion of long cable booms of a spinning satellite." *OAD/ESOC Working Paper* 37, ESOC, Darmstadt, Germany, Aug. 1975.

[6] Janssens, F. "Some elementary considerations about the motion of long cable booms of a spinning satellite. Part 2." *OAD/ESOC Working Paper* 37, ESOC, Darmstadt, Germany, Aug. 1975.

[7] Janssens, F. "Consequences of boom or central body asymmetries for the equilibrium configurations of GEOS." *OAD/ESOC Working Paper* 43, ESOC, Darmstadt, Germany, Oct. 1975.

[8] Brenner, H. "Formulas for the fuel consumption and its influence on the moment of the inertia of a satellite." *OAD/ESOC Working Paper* 98, ESOC, Darmstadt, Germany, Aug. 1977.

[9] Goodwin, A. and Massart A. "ISPM sun aspect angle determination and sun aspect angle deadband setting in the presence of a spin axis tilt." *OAD/ESOC Working Paper* 244, ESOC, Darmstadt, Germany, Jan. 1984.

[10] Van der Beken, Ch. "ISPM spin axis tilt measurements." *OAD/ESOC Working Paper* 282, ESOC, Darmstadt, Germany, Nov. 1984.

[11] Gienger, G. "The effect of antenna booms on the spin axis tilt of ULYSSES." *OAD/ESOC Working Paper* 331, ESOC, Darmstadt, Germany, June 1988.

[12] Hugo, D.v. "Detailed dynamical model for spinning rigid s/c with wire booms." *OAD/ESOC Working Paper* 422, ESOC, Darmstadt, Germany, Apr. 1990.

[13] Gienger, G. "ULYSSES principal axis tilt determination and C-loading." *Proceedings of the ESA Symposium on Spacecraft Flight Dynamics.* Darmstadt, Sep. 30 - Oct. 4, 1991.

[14] Mortari, D. and Arduini, C. "Attitude dynamics induced by thermal transitions on a spin stabilized cable boom system." (AAS 94-104) *Spaceflight Mechanics* 1994, Vol 87, Part I, *Advances in the Astronautical Sciences.* J.E. Cochran Jr.; C.D. Edwards Jr. S.J. Hoffman and R. Holdaway (eds.). *Proceedings of AAS/AIAA Spaceflight Mechanics Meeting,* Cocoa Beach, Florida, Feb. 14-16, 1994. (Univelt, San Diego, CA, 1994.) pp. 53-65.

[15] Delgado, I. and Miguel, J. "Study on Spin Axis Tilt on Stabilized S/C - Part I." *Final Report GMVSA* 2057/92 V2/93 Grupo de Mecánica del Vuelo, Madrid, Spain, February, 1993.

[16] Lopes, R.V.F. "Configurações de equilíbrio para a atitude de satélites artificiais equipados com extensões lineares e estabilizados por rotação." *Anais, VI Colóquio Brasileiro de Dinâmica Orbital,* seção 6. (Abstract) IGCE/UNESP, Águas de São Pedro, 23 a 26 de novembro de 1992.