GPS-BASED NAVIGATION SOLUTION AND SPIN-AXIS ATTITUDE DETERMINATION: NUMERICAL RESULTS OF ON THE GROUND EXPERIMENT

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ABSTRACT

In previous studies the authors applied parameter estimation procedures to satellite orbit and spin-axis attitude determination from GPS observables.This work adapts and applies those algorithms for the first time to real data from an experiment conducted at INPE in collaboration with UFPR. The experiment and the procedures are briefly presented and their numerical results shown. The attitude of a spinning bar with two offset antennas linked to independent receivers was determined from the double-difference phase observable. The solution of the integer ambiguity problem takes advantage of the antenna baseline motion. The navigation solution is performed by ORBEST algorithm, which considers C-code pseudorange from all GPS satellites at sight. The work is a preliminary step towards the in flight Brazilian experiments foreseen for the near future.

1 INTRODUCTION

INPE's policy for next Brazilian space missions includes the use of the Global Positioning System (GPS) for orbit and attitude determination purposes. As a consequence, on an exploratory phase, the data communication satellite SCD-3, a stratosphere balloon with γ-ray telescope MASCO, and a national sub-orbital platform, all of them scheduled to be launched in this year, shall be equipped with GPS receivers to perform in flight experiments.

Several works have been published about GPS-based attitude determination, most of them concerned with three-axes stabilized satellites ([1],[2] and [3], for example). The spin stabilized case nevertheless presents some interesting characteristics which can be explored to make easier the attitude determination task. For this reason, and as a preparatory task in the context of the Brazilian space program, a first GPS experiment [4] has been carried out on the ground, jointly by the National Institute for Space Research (INPE) and Federal University of Paraná (UFPR) regarding spin axis attitude determination. Specifically, the actual efficacy of a simple algorithm [5] to solve the integer ambiguity and the accuracy of both GPS estimated aspect angle and spin axis estimated attitude are analised.

On board autonomous attitude determination from GPS carrier phase interferometry requires the knowledge of the satellite position, which of course can be provided by the GPS pseudorange observable. So, envisaging the actual implementation of the algorithm, a brief experiment was performed by INPE to complement the first one. The real performance of ORBEST algorithm [6], [7] for navigation solution is compared with that one from a PDOP-based [8] conventional procedure using the experiment data.

Unlike most of GPS-based navigation solution algorithms, ORBEST can take into account C-code pseudorange observations from all GPS satellites at sight on a least squares optimum sense. Theoretically, it should achieve equal or better accuracy than the usual PDOP-based algorithm which selects a subset of four GPS satellites from those at sight.

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Table 1: GPS EXPERIMENT CONDITIONS

Local coordinates (WGS84)	latitude 23°12′43″E
	longitude 45°51′36″
	altitude 613m
Date	September 26, 1996
Initial time	8h52min pm, local time
Spin axis direction	to the Zenith
Initial azimuth	baseline towards North
Sampling rate:	2Hz
Sampling size:	100s
Antenna baseline:	60cm

In this paper both experiments are described and the numerical results are presented together with their respective data processing algorithms. The experiment represents the first and successfull Brazilian reported work in order to qualify GPS-based algorithms for space applications.

2 THE EXPERIMENT SCENARIO

The aim of the attitude experiment was to get insight about the GPS carrier phase observables, regarding further applications on attitude determination of satellites. With this purpose, both L1 and L2 carrier phase interpherometric observations from a pair of GPS antennas axi-symmetrically placed over a spinning bar were recorded and later processed in order to estimate the spin axis attitude and its phase reference angle.

The experiment equipments were (see Figure 1): two sets of GPS receiver Ashtech Z-XII3 (with battery) and its Geodetic L1/L2/P antenna; a step motor and its associated electronic (power supply; pulse generator; and a programmable timer; and one aluminium bar 80cm long.

Some short essays were carried out with three different purposes. The step motor was off during the static essays intended to verify the stability of clock receiver. The step motor rotated steadly at 1.2rpm during the continuous essays intended to be the ordinary test condition for attitude determination. Finally, the step motor turned precisely 3.6° and stopped for a while before repeating the sequence at the intermitent essays intended to simulate a more accurate spin rate, if it turns to be needed.

Table 1 presents the GPS experiment general conditions corresponding to a continuous essay whose representative results will be shown in a later section.

Table 2: GPS COMPLEMENTARY EXPERIMENT CONDITIONS

Local coordinates (WGS84)	latitude 23°12′43″E
	longitude 45°51′36″
	altitude 613m
Date	March 10, 1997
Initial time	18h07min43.72s, GMT
Sampling rate:	2Hz
Sampling size:	100s
Receiver clock accuracy	.001s
Locked satellites PRN	14, 15, 16, 18, 22, 25, 29, 31

A complementary experiment took place some months later at INPE. A C-code data sample from a single GPS Builder-2 GEC PLESSEY receiver was recorded during a single session. The data were later processed to compute a navigation solution. The antenna stood up close to the site of the first experiment, and the conditions of Table 2 applied.

3 CARRIER PHASE DATA REDUCTION

Attitude determination from GPS carrier phase interpherometry of a body under pure rotational motion, like the spinning bar on the GPS experiment, may be performed in three steps, provided an accurate estimate of the spin rate is available. The first one consists of a carrier phase data reduction, in order to compress the data base to a few representative coefficients. In the second step, these coefficients are handled to generate the attitude observations, namely the GPS line of sight unit vectors or just their aspect angles, if only the spin axis attitude is of interest. The third step falls into an ordinary attitude determination problem, given a set of unit vectors at the external frame and their respective attitude observations, always referred to the body frame. The last step is less relevant to this study since it can always be solved by any well known attitude determination method ([9] for example). This section describes the data reduction step for the GPS attitude experiment.

The integer part of carrier phase was neglected because it is a relative information often affected by cycle slips, thus being meaningless for the present purpose. Due to the relative motion of each antenna with respect to the GPS satellites, the remaining part of carrier phase presents a random-like pattern and is apparently useless. Hopefully, most of delays and undesirable components of carrier phase cancel each other when between-antenna single-difference phase observable is computed. In fact, under the ef-

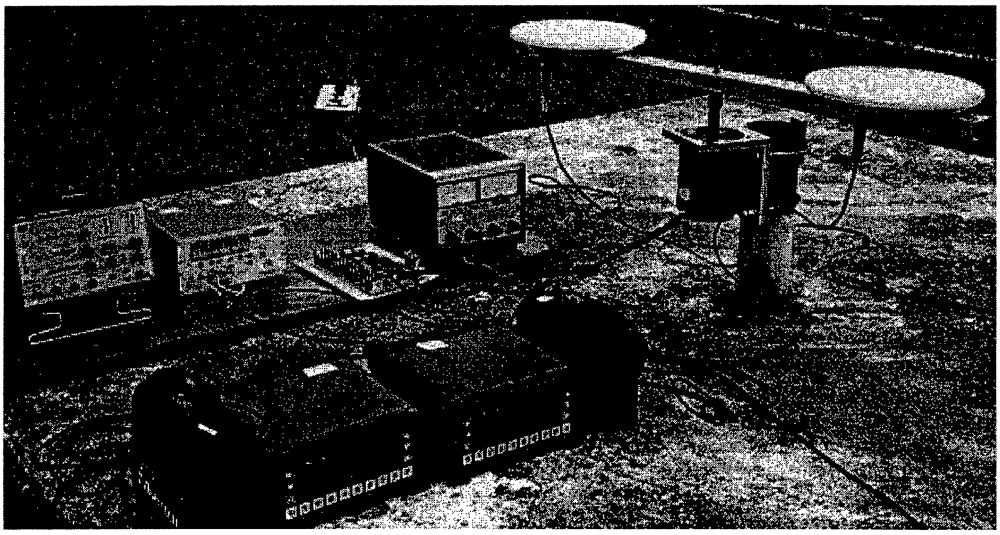

Figure 1: **The GPS Experiment With Two Antennas Over a Spinning Baseline.**

fect of the spin motion, it would present a sinusoidal pattern only masked by the integer ambiguity phenomenon. Such mask is easily removable by a simple pre-processing algorithm given in [5], since the discontinuities in this case always amount either one or two entire cycles. As the sinusoidal amplitude and phase are directly related with the GPS line of sight unit vectors as well as with their aspect angles, the attitude observation evaluation is straightforward.

This however is only part of the history. On space applications the two antennas are supposed to be linked to a single GPS receiver. In such case, the effect of user clock unstability vanishes on the between-antenna single-difference phase observable, and so does not require any concern. Nevertheless, this was not the case for the performed GPS experiment. Actually, the GPS receiver used for the experiment did not offer that facility. Therefore, each antenna had to be linked to a distinct receiver with its own distinct oscillator. As a result, computed the single-difference phase observable from the RINEX-2 recorded data, the useless random pattern still persisted.

The contingency was overcome by computing the double-difference phase observable. After the already referred pre-processing, the searched sinusoidal pattern could be recovered, as shown by Figure 2 (see the collage pattern of crude data masking the sinusoid).

The double-difference phase observable model is described below. The p-th GPS satellite line of sight

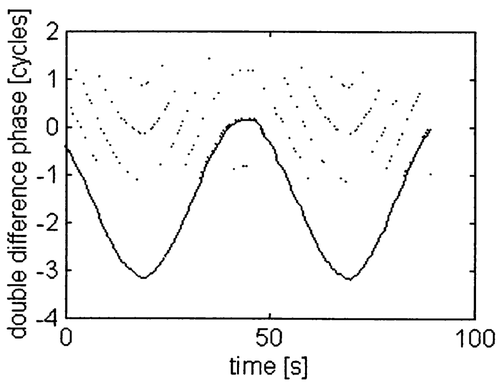

Figure 2: Double-difference L1 phase observable between GPS satellites PRN#02 and 09: ∘ crude data; —— pre-processed data.

unit vector coordinates at body frame is given by:

$$w^p = \left\{ \begin{array}{c} \sin\theta^p \cos[\omega(t-t_0) - \alpha^p] \\ -\sin\theta^p \sin[\omega(t-t_0) - \alpha^p] \\ \cos\theta^p \end{array} \right\} , \quad (1)$$

where: θ^p and α^p are the p-th GPS line of sight aspect angle and azimuth respectively; ω is the spin rate; t is the time and t_0 is a reference time. The between-antenna single-difference phase observable is given by:

$$\begin{aligned} \phi^p_{1,2} &\equiv \phi^p_1 - \phi^p_2 \\ &= \frac{b}{\lambda_k} \sin\theta^p \cos[\omega(t-t_0) - \alpha^p] \end{aligned}$$

$$+ \quad \tau_{1,2}(t) + N^p(t) + v^p(t) \ , \qquad (2)$$

where: ϕ_j^p is the carrier phase at j-th antenna from p-th GPS satellite; b is the antenna baseline; λ_k is the Lk (for $k = 1$ or 2) carrier phase wave lenght; $\tau_{i,j}$ is the between-receiver single-difference clock error; N^p is the integer ambiguity; and v^p is a small noise which represents the effect of the remaining disturbances sources such as multipath.

The double-difference phase observable is therefore:

$$\begin{aligned} \phi_{1,2}^{p,o} &\equiv \phi_{1,2}^p - \phi_{1,2}^o \\ &= \frac{b}{\lambda_k}\left[C^p \cos\omega(t-t_0) + S^p \sin\omega(t-t_0)\right] \\ &+ \ N^{p,o}(t) + v^{p,o}(t) \ , \ \forall p \neq o \ , \end{aligned} \qquad (3)$$

where $o \in \mathcal{P}$ represents the arbitrarily chosen master satellite while the $p \neq o$ are the slave ones; $N^{p,o}$ and $v^{p,o}$ have the immediate meaning; and C^p and S^p are the coefficients

$$C^p = \sin\theta^p \cos\alpha^p - \sin\theta^o \cos\alpha^o \ , \qquad (4)$$
$$S^p = \cos\theta^p \cos\alpha^p - \cos\theta^o \cos\alpha^o \ . \qquad (5)$$

After pre-processing $\phi_{1,2}^{p,o}$, the integer ambiguity $N^{p,o}(t)$ is reduced to a constant shift $N_0^{p,o}$, which can be estimated together with the coefficients C^p and S^p by curve fitting. This complete the data reduction step.

4 ATTITUDE OBSERVATION

The computation of a set of attitude observations $\{\theta^p, \alpha^p, \ \forall p \in \mathcal{P}\}$ from the set of coefficients $\{C^p, S^p, \ \forall p \neq o, \ p \in \mathcal{P}\}$ is not straightforward. Indeed, at a first glance, the problem would seem to have less equations than unknowns! Hopefully, there is an extra source of information from the fact that the scalar product is invariant to rotations. So, one has

$$\begin{aligned} \cos\psi^p &\equiv u^p \cdot u^o = w^p \cdot w^p \qquad (6) \\ &= \cos\theta^p \cos\theta^o + 2\sin\theta^p \sin\theta^o \cos(\alpha^p - \alpha^o) \ , \end{aligned}$$

where u^p is the p-th GPS satellite line of sight unit vector coordinates at the external frame, which are supposed to be known from the GPS ephemeris and the user position. Adding $\{\psi^p, \ \forall p \neq o, \ p \in \mathcal{P}\}$ to the input data set makes the problem overdetermined.

Regardless the estimation errors, the sinusoidal amplitude of the double-difference phase observable obeys

$$\begin{aligned} A^{p^2} &\equiv C^{p^2} + S^{p^2} \ , \qquad (7) \\ &= \sin^2\theta^p + \sin^2\theta^o - 2\sin\theta^p \sin\theta^o \cos(\alpha^p - \alpha^o) \end{aligned}$$

which in view of Equation 6 yields

$$\begin{aligned} A^{p^2} + 2\cos\psi^p &= \sin^2\theta^p + \sin^2\theta^o - 2\cos\theta^p \cos\theta^o \\ &= 2 - (\cos\theta^p - \cos\theta^o)^2 \ . \end{aligned} \qquad (8)$$

From Equations 1,4,5 and 8 one may write:

$$w^p(t_0) = w^o(t_0) + M^p \ , \qquad (9)$$

where

$$M^p \equiv \left\{ \begin{array}{c} C^p \\ S^p \\ \sigma^p \left\{ 2(1 - \cos\psi^p) - A^{p^2} \right\}^{1/2} \end{array} \right\} \ , \qquad (10)$$

with $\sigma^p \in \{-1, 1\}$ representing a signal ambiguity.

Now, w^p must be a unit vector. By imposing this condition to Equation 9 it results:

$$M^{p^T} w^o(t_0) = -(1 - \cos\psi^p) \ , \ \forall p \neq o \ , p \in \mathcal{P} \ . \quad (11)$$

For a given set $\{\sigma^p, \ \forall p \in \mathcal{P}\}$, the overdetermined linear set of Equations 11 can be solved for $w^o(t_0)$ by the least squares method. Of course, due to the inaccuracies of the whole process, the unconstrained least squares solution $w^o(t_0)$ may not be a unit vector. The hindrance is overcame by rewriting Equation 11 as a function of θ^o and α^o and linearizing it around the unconstrained solution. This yields

$$M^p H \left\{ \begin{array}{c} \Delta\theta^o \\ \Delta\alpha^o \end{array} \right\} = -\left[M^p w^o(t_0) + (1 - \cos\psi^p)\right] \ , \qquad (12)$$

with

$$H \equiv \left[\begin{array}{cc} \cos\theta^o \cos\alpha^o & -\sin\theta^o \sin\alpha^o \\ \cos\theta^o \sin\alpha^o & \sin\theta^o \cos\alpha^o \\ -\sin\theta^o & 0 \end{array} \right] \ . \qquad (13)$$

Solving the linear set of Equations 12 for $\Delta\theta^o$ and $\Delta\alpha^o$ by the least squares method gives the iterative correction for θ^o and α^o, and consequently for $w^o(t_0)$. Once the iterative algorithm converges, the attitude observation set referred to t_0 could be completed from Equation 9.

The total number of candidate solutions due to the signal ambiguity is 2^{n-1}. Half of them correspond to GPS signals getting the antenna to the bottom, and of course shall be rejected. The remaining signal

Table 4: ATTITUDE ERRORS

GPS frequency	Attitude Error [arc. min.] x-axis	y-axis	z-axis
L1	39	54	5
L2	6.1	5.7	6.5

ambiguity is solved by the following empirical criterion:

$$\mathrm{Min} \sum_{p \in \mathcal{P}} [|w^p(t_0)| - 1]^2 \ . \tag{14}$$

A more elegant criterion is still under investigation, but the present one worked. At last, the slave attitude observations $w^p(t_0)$ are normalized.

5 ATTITUDE DETERMINATION RESULTS

The procedure described in the previous two sections was implemented by using MATLAB software. The GPS satellite PRN#02 was arbitrarily chosen as the master one. Data from both L1 and L2 frequencies were processed separatelly and the resulting attitude compared with the true one. The intermediate results are shown in Table 3. The angular errors around each axis (x to the North; y to the West; and z to the Zenith) are shown in Table 4.

As one can see, both frequencies yielded a fine accuracy level on azimuth. The spin axis attitude error was significantly less accurate when computed from L1. Based on this single result it would be premature to claim about quantitative performance of the procedure. Such task asks for a bigger amount of essays. The concept however was clearly proven.

6 ORBEST ALGORITHM PERFORMANCE

The problem of navigation fix from GPS pseudorange observables by the least squares method can be stated as [6]:

$$\begin{aligned} \text{Min.} \quad & L^*(r, \rho_i, \Delta y) = \frac{1}{2} \sum_{i \in \mathcal{P}} a_i |r - (R_i + \rho_i)|^2 \\ & + \frac{1}{2} a^* \Delta y^2 \ , & (15) \\ \text{Subj. to} \quad & \rho_i^T \rho_i = (y_{p_i} + \Delta y)^2, \ \forall i \in \mathcal{P} \ , & (16) \\ \text{Given} \quad & \{a^*, (R_i, y_{p_i}, a_i), \ \forall i \in \mathcal{P}, \ n \geq 4\} \ , \end{aligned}$$

where r is the position vector of the user receiver; R_i is the i-th GPS satellite position vector; ρ_i is the relative position vector (slant range) of the user satellite with respect to the i-th GPS satellite; y_{p_i} is the pseudorange measurement from the i-th GPS satellite; Δy is a constant to be added to y_{p_i} to cope with the user clock bias; $\mathcal{P}$ is the subset of GPS satellites at sight; n is the number of elements of $\mathcal{P}$; and a^* and a_i are positive weights related to the input uncertainty and satisfying the Normality Constraint:

$$a^* + \sum_{i \in \mathcal{P}} a_i = 1 \ . \tag{17}$$

A derivation of ORBEST iterative algorithm to solve this problem as well as its initialization linear solution was presented in [7]. In that work, numerical results were shown based on digital simulation only and compared with those from a conventional least PDOP [8] algorithm. Regarding the solution accuracy, the comparison indicated that ORBEST would present equal or better performance than the least PDOP algorithm, the equivalence holding for $n = 4$ and the advantage increasing with n.

Such comparison is repeated here with real data from the complementary experiment. Preliminary results, summarized in Figure 3, indicates a good agreement with the simulation ones in terms of the relative uncertainty level and the better accuracy of ORBEST algorithm. On the other hand, absolute errors with respect to the geodetic point DMC/INPE could not be well determined. The causes are still under investigation and possibly more time for receiver stabilization than it was taken in this very short experiment is necessary.

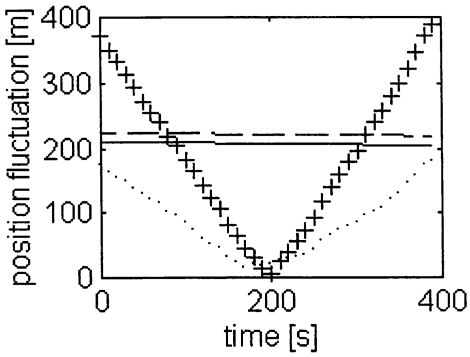

Figure 3: Relative error of navigation performance: ∘ full GPS constellation at sight (ORBEST algorithm); + least PDOP GPS sub-constellation (conventional algorithm); —— ORBEST predicted accuracy ; — — Least PDOP predicted accuracy.

Table 3: ATTITUDE OBSERVATIONS FROM GPS CARRIER PHASE

PRN #	Reference vectors			Observation vectors from L1			Observation vectors from L2		
	U_x	U_y	U_z	W_x	W_y	W_z	W_x	W_y	W_z
02	0.3646	−0.8183	−0.4443	−0.0661	−0.2998	0.9517	−0.0768	−0.3071	0.9486
07	0.6026	−0.4872	0.6321	0.8811	0.1052	0.4610	0.8744	0.0950	0.4759
10	−0.2238	−0.9448	−0.2392	0.0044	−0.8155	0.5787	−0.0148	−0.8196	0.5727
16	0.0150	−0.9999	0.0045	0.2977	−0.6823	0.6677	0.2835	−0.6912	0.6647
18	0.9396	−0.0734	0.3342	0.5991	0.6317	0.4920	0.5969	0.6274	0.5000
19	0.7359	0.1867	−0.6509	−0.4388	0.6667	0.6024	−0.4548	0.6567	0.6016
27	0.3977	−0.2575	−0.8806	−0.6142	0.1176	0.7803	−0.6201	0.1089	0.7769

Data from two among the eight GPS satellites at sight (PRN#22 and 29) were rejected by a residual analysis. The effect on the pseudorange due to the time spent by the signal to get the receiver and effect on the ephemeris due to GPS time correction coefficients were taken into account according to [10]. Stratosphere and troposphere delays were not considered.

In every case, starting from the linear approximate solution [5], ORBEST achieved a submillimeter convergence criterion after three steps. A comparison between the two algorithms execution time is out of the scope of the present work because the developed code has not been optimized in this sense yet.

7 CONCLUSIONS

The experiment primary results have proven the basic concept about determining the spin axis attitude (spin direction plus phase angle) from GPS double-difference phase observable. The data preprocessing algorithm was able to cope with the integer ambiguity and recovered the sinusoidal signal from which a redundant set of attitude observations was computed with arc-minute error.

The experiment results have also shown that the actual performance of ORBEST algorithm for navigation fix based on GPS C-code pseudorange observable is in a good agreement with its theoretical performance in terms of relative errors. The benefit of processing a redundant set of GPS satellites at sight could not be remarkable because it amounted only six effective satellites after data rejection.

The results encourage further experiments in order to characterize the statistical performance of both algorithms.

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