MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

INPE-8056-PRE/3872

SPACE TRAJECTORIES FOR A SPACECRAFT TRAVELLING UNDER THE GRAVITATIONAL FORCES OF TWO BODIES

Antonio Fernando Bertachini de Almeida Prado

Published at the "Book of Abstract of the 3rd International Conference on Non-Linear Problems in Aviation and Aerospace".

INPE
São José dos Campos
2001

Space Trajectories for a Spacecraft Travelling Under the Gravitational Forces of Two Bodies

Antonio Fernando Bertachini de Almeida Prado

Instituto Nacional de Pesquisas Espaciais
São José dos Campos - SP - 12227-010 - Brazil
Phone 55-(12)345-6201 - Fax 55-(12)345-6226

ABSTRACT: This paper shows a survey on space trajectories in the circular restricted three-body problem. In this situation, a spacecraft moves under the gravitational forces of two bodies, that are assumed to be in circular orbits. First of all, there is a search for orbits that can be used to transfer a spacecraft from one body back to the same body or to transfer a spacecraft from one body to the respective Lagrangian points L_4 and L_5. The method employed is to solve the Two-Point Boundary Value Problem. The close approach between the spacecraft and the celestial bodies involved is also studied in the three-dimensional space. Then, the gravitational capture is studied. It is a characteristic of some dynamical systems, like the three- or four- body system, where a hyperbolic orbit around a celestial body can be transformed in an elliptic orbit without the use of any propulsive system.

KEYWORDS: Astrodynamics, Orbital Maneuvers, Restricted Problem, Gravitational Capture, Swing-By, Lagrangian Points.

1. INTRODUCTION

This paper has the goal of showing a survey of trajectories to make orbital transfers of a spacecraft that is traveling in space under the gravitational forces of two bodies. It shows some results available in the literature, as well as some unpublished results in that topic.

First of all, it is considered the problem of finding transfer orbits in the restricted problem. Several situations are studied individually. A family of transfer orbits that can transfer a spacecraft from the Moon back to the Moon again (passing close to the Lagrangian point L_3 in most of the cases) with minimum fuel consumption is considered. The family of transfer orbits from the Moon back to the Moon that requires an impulse with magnitude lower than the escape velocity from the Moon is studied and explained separately. Then, an extension is made to study similar trajectories for the Sun-Earth system, including a new suggestion to build a cycler transportation between the Earth and the Lagrangian point L_4.

After that, a swing-by maneuver in three dimensions is studied. The swing-by maneuver is a very popular technique used to decrease fuel expenditure in space missions. The most usual approach to study this problem is to divide the problem in three phases dominated by the "two-body" celestial mechanics. Other models used to study this problem are the circular restricted three-body problem (Broucke [1], Broucke and Prado [2], Prado [3]) and the elliptic restricted three-body problem (Prado [4]). In the present paper, the swing-by maneuvers are also studied under the model given by the three-dimensional circular restricted three-body problem. Particular attention is given to study the inclination change due to this maneuver.

Finally, the problem of gravitational capture in the regularized restricted three-body problem is studied. For gravitational capture it is understood a phenomenon where a massless particle changes its two-body energy around one of the primaries from positive to negative. This capture is always temporary and, after some time, the two-body energy changes back to positive and the massless spacecraft leaves the neighborhood of the primary. The importance of this temporary capture is that it can be used to decrease the fuel expenditure for a mission going from one of the primaries to the other, like an Earth-Moon mission (Yamakawa [5]). The goal is to apply an impulse to the spacecraft during this temporary capture to accomplish a permanent capture. Since the goal of this impulse is to decrease the two-body energy of the spacecraft, its magnitude will be smaller if applied during this temporary capture.

2. MATHEMATICAL MODEL

For the research performed in this paper, the equations of motion for the spacecraft are assumed to be the ones valid for the well-known three-dimensional restricted circular three-body problem. The standard dimensionless canonical system of units is used, which implies that: the unit of distance is the distance between M_1 and M_2; the mean angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; the mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1 + m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_2 is $(1-\mu)$; the unit of time is defined such that the period of the motion of the two primaries is 2π and the gravitational constant is one. There are several systems of reference that can be used to describe the three-dimensional restricted three-body problem (Szebehely [6]). In this paper the rotating system is used. In the rotating system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis (x) is the line that connects the two primaries at any time. It rotates with a variable angular velocity in such way that the two massive primaries are always on this axis. The vertical axis (y) is perpendicular to the (x) axis. In this system, the positions of the primaries are: $x_1 = -\mu$, $x_2 = 1-\mu$, $y_1 = y_2 = 0$. In this system, the equations of motion for the massless particle are (Szebehely [6]):

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \quad (1)$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \quad (2)$$

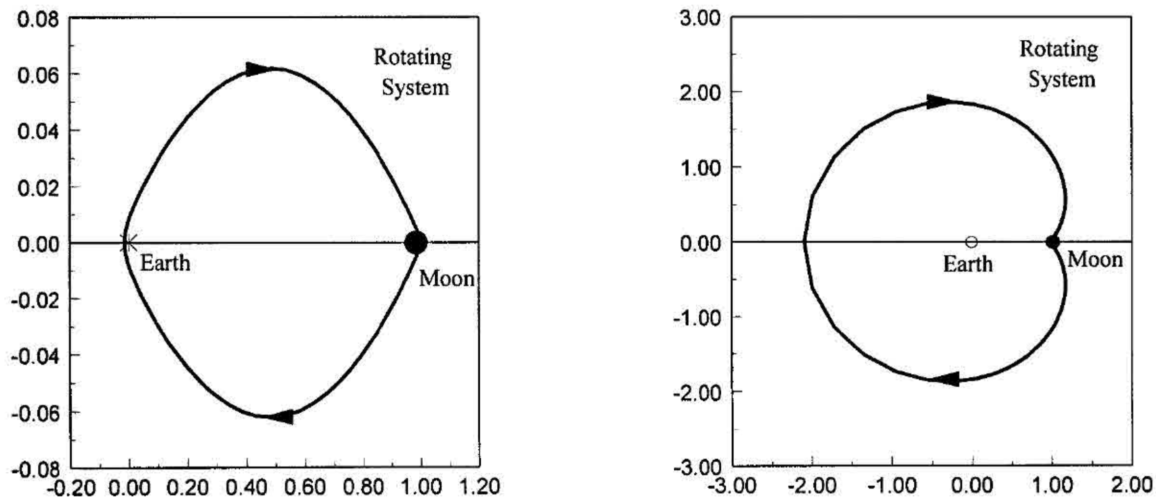
$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \quad (3)$$

where r_1 and r_2 are the distances from M_1 and M_2.

3. TRANSFER ORBITS IN THE RESTRICTED PROBLEM

The problem considered here is the problem of finding trajectories between two points that are fixed in the rotating coordinate system. This is the famous TPBVP (two point boundary value problem). There are many orbits that satisfy this requirement, and the way used in this research to find families of solutions is to specify a time of flight for the transfer. Then, the problem becomes the Lambert's three-body problem, that can be formulated as: "Find an orbit (in the three-body problem context) that makes a spacecraft to leave a given point A and go to another given point B, arriving there after a specified time of flight". Then, by varying the specified time of flight, it is possible to find a whole family of transfer orbits and study them in terms of the ΔV required, energy, etc. The transfers considered here are all restricted to the plane of motion of the two primaries.

Several families made of different trajectories to make a transfer from M_2 back to the M_2 and to the Lagrangian points are shown in Prado and Broucke [7] for the Earth-Moon system and in Prado and Broucke [8] for the Sun-Earth system. A new pair of transfers from the Moon back to the Moon is showed in Fig. 1, one involving a hyperbolic transfer (faster) and one involving an elliptic transfer (slower).



Hyperbolic Transfer Orbit (t = 1.74 days) Elliptic Transfer Orbit (t = 24.78 days)

Figure 1 - Transfer Orbits in the Earth-Moon System, as Seen in the Rotating Frame.

An interesting type of transfer was found (Prado and Broucke [7]) that requires an impulse with a magnitude lower than the escape velocity from the Moon. This possibility is opened by the restricted three-body problem model. It uses the perturbation of the third-body (the Earth in this case) to help the massless particle to escape from the Moon (the first body), and it decreases the ΔV required for the maneuver. Remember that the escape velocity is defined as the velocity required to escape one celestial body considering the system governed by the "two-body" celestial mechanics. Figure 2 shows one of those transfers. The initial conditions in canonical units for this trajectory are: x = 0.987871437, y = -0.004786681, $\dot{x}$ = 2.220000000, $\dot{y}$ = 0.000000000, Vesc = 2.251139608, ΔV-Vesc = -0.031139608, where Vesc is the escape velocity from the Moon.

Figure 3 shows a transfer from the Moon to the Lagrangian points and back to the Moon, also from (Prado and Broucke [7]). This trajectory pass twice by the Lagrangian points visited. The initial conditions in canonical units for this trajectory are: x = 0.987871437, y = -0.004500000, $\dot{x}$ = -0.100000000, $\dot{y}$ = -3.063600000, Vesc = 2.321739099, ΔV-Vesc = 0.743349025.

Several trajectories to transfer a spacecraft between the Earth and the Lagrangian points with minimum ΔV are shown in Prado and Broucke [8]. In the present paper one of them is shown in details, because it generates results for the construction of a cycler transportation between the triangular Lagrangian points and the Earth. The spacecraft leaves the Earth and visits the Lagrangian points in the order L_4 (in 1.81 years), L_3 (in 5.49 years), L_5 (in 9.20 years) and then it returns to the Earth's neighborhood (in 11.00 years). Fig. 4 shows the first two revolutions of this trajectory. The particular important point of this orbit is that after the close approach with the Earth (in the end of the first revolution) the spacecraft starts a new tour to the Lagrangian points, in the reverse order. Integrating this trajectory for a longer time it is possible to see that the first five revolutions have alternating directions of motion.

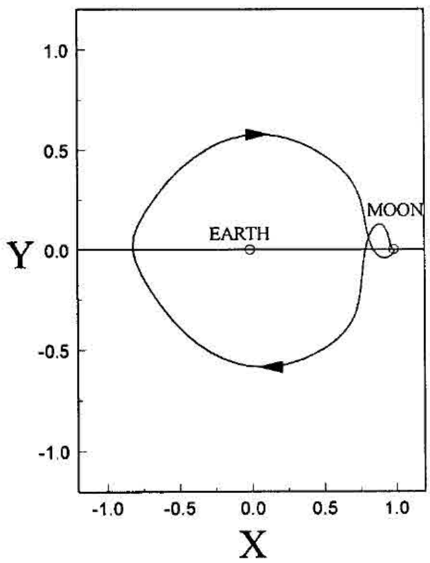


Figure 2 - Transfer from the Moon back to the Moon with ΔV-Vesc < 0.

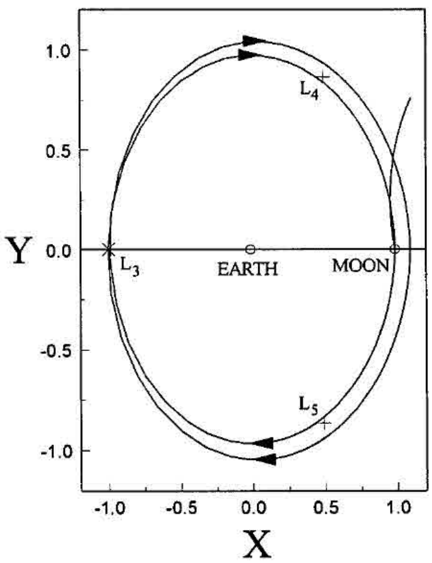


Figure 3 - Transfer from the Moon to the Lagrangian points.

The "swing-by" that reverses the direction of motion discovered in this trajectory can be used to build a cycler transportation system between the Earth and the Lagrangian point L_5 as shown in Prado and Broucke [8]. In the present paper, an extension is made to build a cycler system for the Lagrangian point L_4, by using the mirror image theorem (Miele [9]). It is necessary to find the mirror image of the trajectory linking the Earth and the Lagrangian point L_5. Fig. 5 shows this trajectory. Note that the mirror image of the legs for an Earth-bound trip in now a L_4-bound trip and the mirror image of the L_5-bound leg is now the Earth-bound leg. The time-line for a complete cycler is: t = 0: The spacecraft leaves L_4 from rest (as seen in the rotating frame) with an impulse of $\Delta V = 0.0274$ (816 m/s); t = 5.82 years: The spacecraft arrives at the Earth, makes a swing-by to reverse the sense of motion and it starts going back to L_4; t = 7.62 years: The spacecraft arrives at L_4. A new impulse of $\Delta V = 0.0377$ (1003.8 m/s) is applied to send it back to the Earth and to start the cycler again.

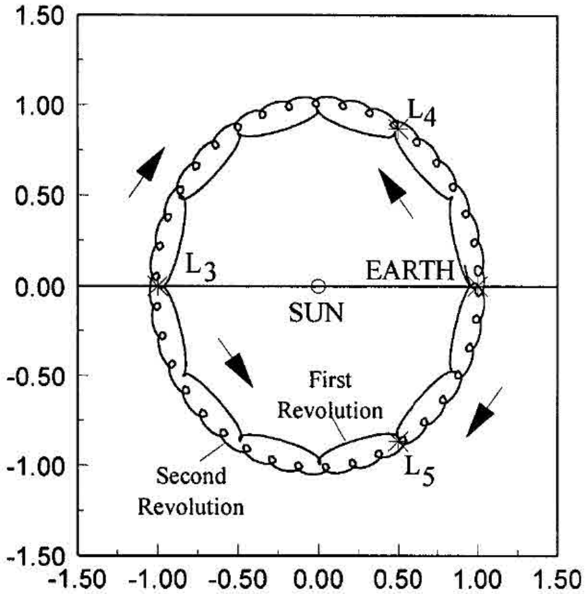


Figure 4 – Trajectory linking the Earth and the Lagrangian points.

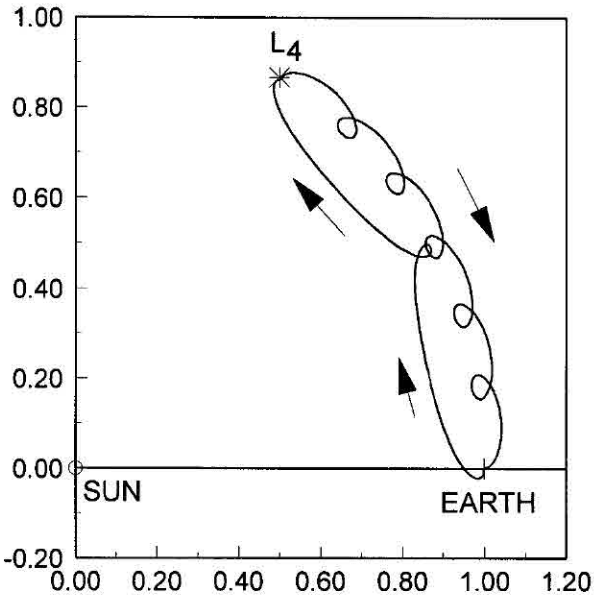


Figure 5 - The Cycler System Between the Earth and L_4.

4. THE SWING-BY IN THREE DIMENSIONS

The three dimensional swing-by maneuver consists of using a close encounter with a celestial body to change the velocity, energy, and angular momentum of a smaller body (a comet or a spacecraft). Fig. 6 shows the sequence for this maneuver and some important variables.

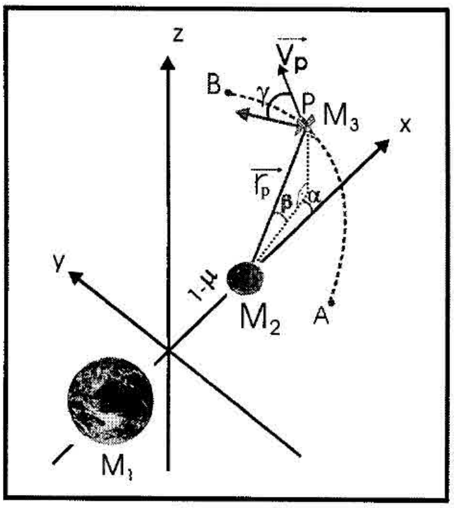


Figure 6 - The Swing-By in Three Dimensions

It is assumed that the system has three bodies: a primary (M_1) and a secondary (M_2) body with finite masses that are in circular orbits around their common center of mass and a third body with negligible mass (the spacecraft) that has its motion governed by the two other bodies. The spacecraft leaves the point A, passes by the point P (the periapsis of the trajectory of the spacecraft in its orbit around M_2) and goes to the point B. The points A and B are chosen in a such way that the influence of M_2 at those two points can be neglected and, consequently, the energy can be assumed to remain constant after B and before A (the system follows the two-body celestial mechanics). The initial conditions are clearly identified in Fig. 6: the periapsis distance r_p (distance measured between the point P and the center of M_2), the angles α and β and the velocity V_p. The distance r_p is not to scale, to make the figure easier to understand. The result of this maneuver is a change in velocity, energy, angular momentum and inclination in the keplerian orbit of the spacecraft around the central body. A numerical algorithm to solve the problem has the following steps: 1) Arbitrary values for the parameters r_p, V_p, α, β and γ are given; 2) With these values the initial conditions in the rotating system are computed. The initial position is the point (X_i, Y_i, Z_i) and the initial velocity is (V_{xi}, V_{yi}, V_{zi}), where:

$$X_i = 1 - \mu + r_p \cos(\beta)\cos(\alpha) \tag{4}$$

$$Y_i = r_p \cos(\beta)\sin(\alpha) \tag{5}$$

$$Z_i = r_p \sin(\beta) \tag{6}$$

$$V_{Xi} = -V_p \sin(\gamma)\sin(\beta)\cos(\alpha) - V_p \cos(\gamma)\sin(\alpha) + r_p \cos(\beta)\sin(\alpha) \tag{7}$$

$$V_{Yi} = -V_p \sin(\gamma)\sin(\beta)\sin(\alpha) + V_p \cos(\gamma)\cos(\alpha) - r_p \cos(\beta)\cos(\alpha) \tag{8}$$

$$V_{Zi} = V_p \cos(\beta)\sin(\gamma)\,; \tag{9}$$

3) With these initial conditions, the equations of motion are integrated forward in time until the distance between M_2 and the spacecraft is larger than a specified limit d. At this point the numerical integration is stopped and the energy (E+) and the angular momentum (C+) after the encounter are calculated; 4) Then, the particle goes back to its initial conditions at the point P, and the equations of motion are integrated backward in time, until the distance

d is reached again. Then the energy (E_-) and the angular momentum (C_-) before the encounter are calculated.

An interesting question that appears in this problem is what happens to the inclination of the spacecraft due to the close approach. Some results regarding this question for the Earth-Moon system are shown in Felipe and Prado [10]. To investigate this fact the inclination of the trajectories were calculated before and after the closest approach. To obtain the inclinations the equation $\cos(i) = Cz/C$ is used, where C_z is the Z-component of the angular momentum and C is the magnitude of the total angular momentum. Fig. 7 shows some new results for the case $\gamma = 0$ in the Sun-Jupiter system. This constraint is assumed, because it is the most usual situation in interplanetary research, since the planets have orbits that are almost coplanar. The horizontal axis represents the angle α, and the vertical axis represents the angle β. The variation in inclination is shown in the contour plots. All the angles are expressed in degrees.

Several conclusions come from those results: i) when $\beta = 0°$ (planar maneuver) the variation in inclination can have only three possible values: $\pm 180°$, for a maneuver that reverse the sense of its motion, or 0° for a maneuver that does not reverse its motion. Those numerical results agree with the physical-model, since the fact that $\beta = 0°$ implies in a planar maneuver that does not allow values for the inclination other than 0° or 180°; ii) when $\beta = \pm 90°$ the variation in inclination is very close to zero; iii) when $\alpha = 0°$ or $\alpha =$ 180° there is no change in the inclination. This is in agreement with the fact that a maneuver with this geometry does not change the trajectory at all; iv) when the periapsis distance or the velocity at periapsis increases, the effects of the swing-by in the maneuver are reduced. In the plots shown, this can be verified by the fact that the area of the regions where the variation in inclination is close to zero increases.

There is also analytical expressions for the variations in velocity ($\Delta\vec{V}$), energy (ΔE), angular momentum ($\Delta\vec{C}$) and inclination (Δi) available in the literature (Prado [11]). They are:

$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_i = -2V_\infty \sin\delta(\cos\alpha\cos\beta, \cos\beta\sin\alpha, \sin\beta) \tag{10}$$

$$\Delta V = \left|\Delta\vec{V}\right| = 2V_\infty \sin\delta \tag{11}$$

$$\Delta E = \frac{1}{2}\left(V_0^2 - V_i^2\right) = -2V_2 V_\infty \cos\beta\sin\alpha\sin\delta \tag{12}$$

$$\Delta\vec{C} = \vec{C}_0 - \vec{C}_i = 2L\,V_\infty(0, \sin\beta\sin\delta, -\cos\beta\sin\alpha\sin\delta) \tag{13}$$

$$\left|\Delta\vec{C}\right| = 2LV_\infty \sin\delta\left(\cos^2\beta\sin^2\alpha + \sin^2\beta\right)^{1/2} \tag{14}$$

Using the definition of angular velocity $\omega = \frac{V_2}{L}$ it is possible to get (for the z-component of the angular momentum C_Z and for the inclination before (i_i) and after (i_o) the swing-by):

$$\omega\Delta C_Z = -2V_2 V_\infty \cos\beta\sin\alpha\sin\delta = \Delta E \tag{15}$$

$$\mathrm{Cos}(i_i) = \frac{C_{iZ}}{|\vec{C}_i|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta \sin\delta + \cos\beta \cos\delta \sin\gamma}{\dfrac{V_2}{V_\infty} + \cos\alpha \cos\delta \cos\gamma + \cos\beta \sin\alpha \sin\delta - \cos\delta \sin\alpha \sin\beta \sin\gamma} \right)^2}} \quad (16)$$

$$\mathrm{Cos}(i_o) = \frac{C_{oZ}}{|\vec{C}_o|} = \frac{1}{\sqrt{1 + \left(\dfrac{\sin\beta \sin\delta - \cos\beta \cos\delta \sin\gamma}{\dfrac{V_2}{V_\infty} + \cos\alpha \cos\delta \cos\gamma - \cos\beta \sin\alpha \sin\delta - \cos\delta \sin\alpha \sin\beta \sin\gamma} \right)^2}} \quad (17)$$

For the planar maneuver ($\beta = \gamma = 0°$), those equations are reduced to the well known results:

$$\Delta E = -2V_2 V_\infty \sin\alpha \sin\delta \quad (18)$$

$$\Delta V = 2V_\infty \sin\delta \quad (19)$$

$$\Delta C = 2LV_\infty \sin\alpha \sin\delta \quad (20)$$

Where $\vec{V}_i$ and $\vec{V}_0$ are the velocity of the spacecraft with respect to the inertial frame before and after the swing-by, respectively, $\vec{C}_i$ and $\vec{C}_0$ are the correspondent angular momentum, L is the distance between the secondary body and the center of mass of the system, V_∞ is the velocity of the spacecraft with respect to the secondary body when the approach starts, δ is half of the total rotation angle described by the velocity vector during the maneuver.

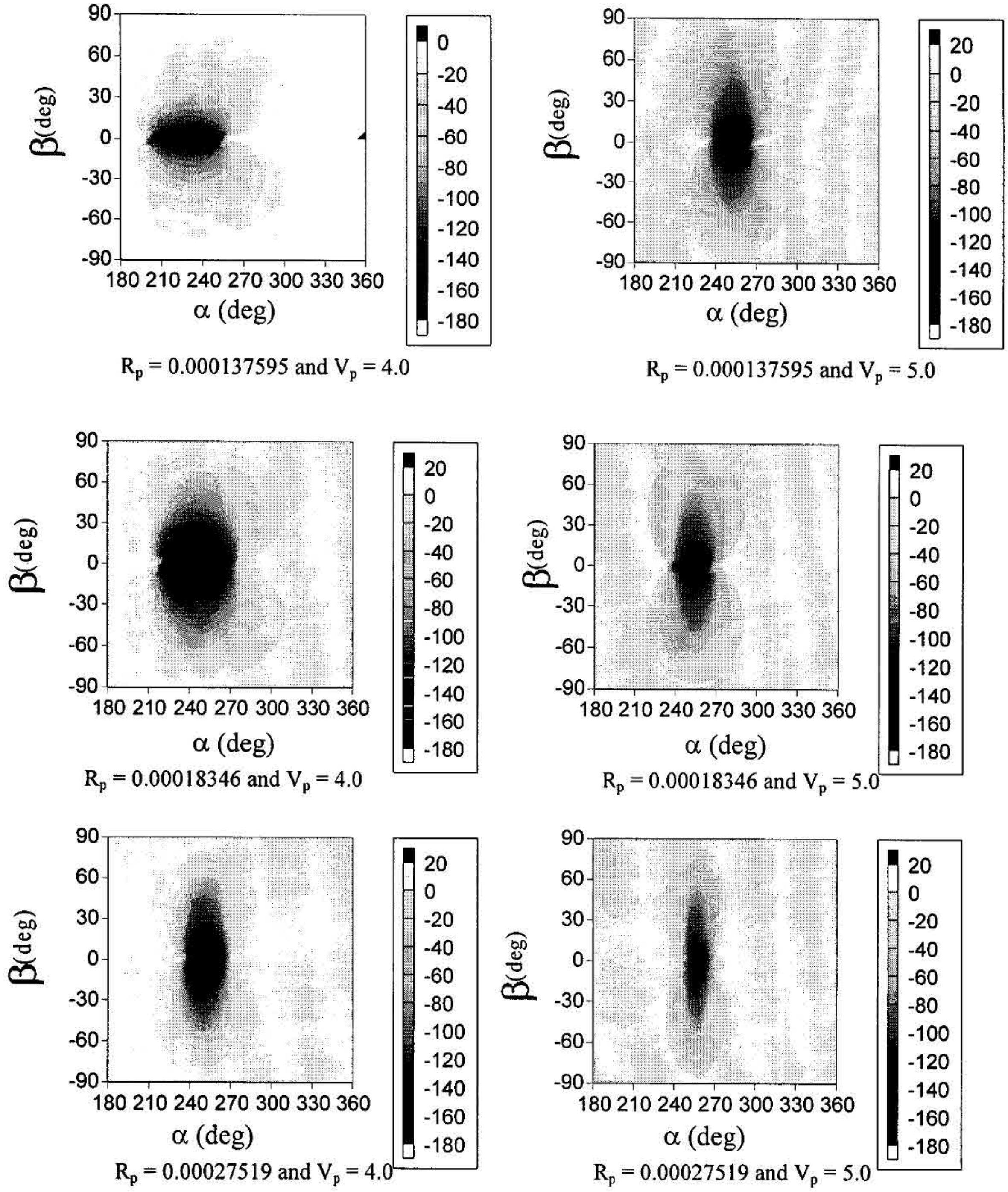


Figure 7 – Inclination chance resulted from a close approach in the Sun-Jupiter system.

5. THE GRAVITATIONAL CAPTURE

Then, attention is given to the problem of gravitational capture. For gravitational capture it is understood a phenomenon where a massless particle changes its two-body energy around one of the primaries from positive to negative. This capture is always temporary and, after some time, the two-body energy changes back to positive and the massless spacecraft leaves the neighborhood of the primary. Studies related to this problem are available in Yamakawa et. al. [12]; Yamakawa [5]; Vieira-Neto and Prado [13][14][15] and Vieira-Neto [16]. The importance of this temporary capture is that it can be used to decrease the fuel expenditure for a mission going from one of the primaries to the other, like an Earth-Moon mission. The goal is to apply an impulse to the spacecraft during this temporary capture to accomplish a permanent one. Since the impulse decrease the two-body energy of the spacecraft, its magnitude will be smaller if applied during this temporary capture. An important application of this technique can be found in trajectories to the Moon with fuel consumption smaller than the fuel required by the Hohmann [17] transfer (Belbruno, [18][19][20]; Krish [21]; Yamakawa [5]).

In this paper the main concern is to study the forces involved in this maneuver. The main force is the gravitational force due to the central body, in this case, the Moon. The others forces are perturbations on the movement of the massless particle. So, to understand the behavior of the perturbing forces, an analysis was made by measuring the components of each force. The chosen components are in the radial, transversal and in the direction of motion of the massless particle. In the radial direction, the positive sign means that the force is acting opposite to the direction of the body. In the transversal direction, the positive sign indicates that the force is acting in the counter-clockwise direction. In the direction of motion, the force is positive when it is being applied in the direction of the movement of the particle. Fig. 8 shows one of those trajectories and the forces acting in the spacecraft in every moment of time. The curves are: 1: Gravitational radial force; 2: Gravitational transversal force; 3: Centripetal radial force; 4: Centripetal transversal force; 5: Resultant radial force; 6: Resultant transversal force; 7: Gravitational force in the direction of motion; 8: Centripetal force in the direction of motion; 9: Resultant force in the direction of motion. In the radial direction the force due to the Earth has a negative sign. This means that the force is pushing the spacecraft to an opposite direction to the Moon. So, it is slowing down the object. In the transversal direction, this force is also negative, which means that it is accelerating the spacecraft in the clock-wise direction. In the direction of motion the sign is also negative, breaking the spacecraft all the time. The centripetal force acts in an opposite direction from the gravity force due to the Earth, but with smaller absolute values. So the net result is due to the gravity force of the Earth, which makes the vehicle to reduce its velocity. The resultant on the transversal direction accelerates the spacecraft in the clock-wise direction. It is visible that the components radial and in the direction of motion of the forces are very close to each other. This is a consequence of the fact that the trajectory is close to radial in most of the trajectory.

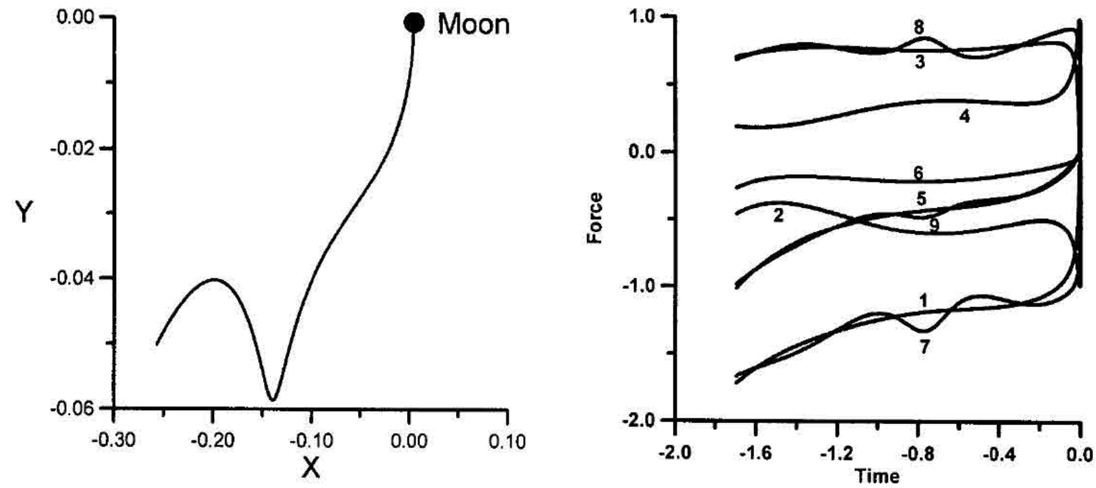


Figure 8 – Trajectory with C3 = –0.2 e $\alpha = 0^{o}$ at perilune and the forces.

The most relevant component is in the direction of motion. This component of the resulting forces show that, independent of where the particle came, it is breaking the particle all the time. The transversal component ever tries to pull the particle to the Earth-Moon axis. The radial component of the resultant force has the same effect of the component in the direction of motion. It also shows that the particle is loosing radial velocity when approaching the central body. These forces slow down the spacecraft working opposite to the movement of the spacecraft. This is equivalent of applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to the Earth and the centripetal force tends to equilibrium, but ever rest some work against the movement. This is also true for the component in the direction of motion. In the transversal direction the forces pull the particle to the Earth-Moon axis. Understanding these behaviors explain why a particle with a velocity slower than the escape velocity can escape from the Moon. It is the opposite case for the capture and it happens for all the cases studied. Some analytical results with respect to this problem is available in Prado [22].

6. CONCLUSIONS

In this paper, trajectories in the planar restricted three-body problem with near-zero ΔV to move a spacecraft between any two points in the group formed by the Earth and the Lagrangian points L_3, L_4, L_5 in the Earth-Moon and Earth-Sun systems are found. It is shown how to apply these results to build a cycler transportation system to link all the points in this group. For the Sun-Earth system, it is also shown how to use one or more "swing-by" with the Earth to build a cycler transportation system between the Earth and the Lagrangian points L_4 and L_5, with small ΔV required for maneuvers in nominal operation.

Looking at the results for the Earth-Moon system, we can conclude that: i) There are trajectories with ΔV near the escape velocity to move a spacecraft from the Moon back to the Moon in the Earth-Moon system, using the restricted three-body problem as a model; ii) There is a new type of trajectory for this transfer that requires a ΔV under the escape velocity; iii) There are trajectories connecting the Moon and the Lagrangian points L_3, L_4, and L_5; iv) There are trajectories that make consecutive close approaches with the Earth and the Moon. Those orbits are shown in this paper and they can be used in three situations: a) To transfer a spacecraft from the Moon back to the Moon; b) To transfer a spacecraft from the Moon to the respective Lagrangian points L_3, L_4 and L_5; c) To transfer a spacecraft to an orbit that passes close to the Moon and to the Earth several times, with the goal of building a transportation system between these two celestial bodies.

The three-dimensional restricted three-body problem is also used to study the swing-by maneuver. The effects of the close approach in the inclination of the spacecraft is studied and the results show several particularities, like: $\beta = 0°$ allows only $\pm 180°$ and $0°$ for Δi, $\beta = \pm 90°$ or $\alpha = 0°$ or $180°$ implies in $\Delta i = 0°$, etc.

Some particularities of the gravitational capture problem are also studied and the forces acting are also shown. This study answers some questions about the phenomena. Two of the forces are relatively weak, and they act as disturbing forces: the gravitational force due to Earth and the centrifugal force. These forces working together slow down the spacecraft with a force in the opposite direction of the spacecraft's motion. This is equivalent of applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to Earth and the centripetal force tends to equilibrium, but ever rest some work against the gravitation of the central body. It is also true for the components in the direction of motion for these forces. In the transversal direction, the forces pull the particle to the Earth-Moon axis. The understandings of these behaviors explain why a particle with a velocity slower than the escape velocity can escape from the Moon. It is the opposite case for the capture and it happens for all the cases studied.

Acknowledgment

The author is grateful to the Foundation to Support Research in the São Paulo State (FAPESP) for the research grant received under Contract 1995/9290-1, to CNPq (National Council for Scientific and Technological Development) for the Grant 1995/9290-1 and to the Foundation for the Improvement of the Superior Teaching Faculty (CAPES) for the support to present this paper.

REFERENCES

1. Broucke, R.A., The Celestial Mechanics of Gravity Assist, AIAA Paper 88-4220, Aug. (1988).

2. Broucke, R.A. and Prado, A.F.B.A., Jupiter Swing-By Trajectories Passing Near the Earth, Advances in the Astronautical Sciences, Vol. 82, No 2, (1993), pp. 1159-1176.

3. Prado, A.F.B.A., Optimal Transfer and Swing-By Orbits in the Two- and Three-Body Problems, Ph.D. Dissertation, University of Texas at Austin, Austin, TX, 1993.

4. Prado, A.F.B.A., Close-Approach Trajectories in the Elliptic Restricted Problem, Journal of Guidance, Control and Dynamics, Vol. 20, No. 4 (1997), pp. 797-802.

5. Yamakawa, H., On Earth-Moon Transfer Trajectory with Gravitational Capture, Ph.D. Dissertation, University of Tokyo, Tokyo, Japan, 1992.

6. Szebehely, V.G., Theory of Orbits, Academic Press, New York, 1967.

7. Prado, A.F.B.A. and Broucke, R.A., Transfer Orbits in the Earth-Moon System Using a Regularized Model, Journal of Guidance, Control and Dynamics, Vol. 19, No. 4 (1996), pp. 929-933.

8. Prado, A.F.B.A. and Broucke, R.A., Transfer Orbits in Restricted Problem, Journal of Guidance, Control and Dynamics, Vol. 18, No. 3, (1995), pp. 593-599.

9. Miele, A., Theorem of Image Trajectories in the Earth-Moon Space, Astronautica Acta, Vol. 6, (1960), pp. 225-232.

10. Felipe, G. e Prado, A.F.B.A., Study of the Inclination Change in Three-Dimensional Swing-By Trajectories, Book of Abstracts of the 22nd International Symposium on Space Technology and Science, Morioka, Japan, 28-May/04-June/2000, pg. 60 (Paper 00-j-02).

11. Prado, A.F.B.A., An Analytical Description of the Close Approach Maneuver in Three Dimensions, 51th International Astronautical Congress (Paper IAF-00-A.5.05), Rio de Janeiro, Brazil, 1-7/October/2000.

12. Yamakawa, H., Kawaguchi, J., Ishii, N., Matsuo, H., A Numerical Study of Gravitational Capture Orbit in Earth-Moon System, AAS paper 92-186, AAS/AIAA Spaceflight Mechanics Meeting, Colorado Springs, Colorado, 1992.

13. Vieira Neto, E. and Prado, A.F.B.A., A Study of the Gravitational Capture in the Restricted-Problem, Proceedings of the "International Symposium on Space Dynamics" pg. 613-622. Toulouse, France, 19-23/June/1995.

14. Vieira Neto, E. and Prado, A.F.B.A., Time-of-Flight Analyses for the Gravitational Capture Maneuver, Journal of Guidance, Control and Dynamics, Vol. 21, No. 1, (1998), pp. 122-126.

15. Vieira Neto, E. and Prado, A.F.B.A., Forças que Atuam na Captura Gravitacional Temporária. "IX Colóquio Brasileiro de Dinâmica Orbital", pg. 30. Águas de Lindóia, Brazil, 16-20/November/1998.

16. Vieira Neto, E., Estudo numérico da captura gravitacional temporária utilizando o problema restrito de três corpos. PhD Dissertation, Instituto Nacional de Pesquisas Espaciais, São José dos Campos, Brazil, 1999.

17. W. Hohmann, Die Erreichbarheit der Himmelskorper, Oldenbourg, Munich, 1925.

18. E.A. Belbruno, Lunar Capture Orbits, a Method of Constructing Earth Moon Trajectories and the Lunar Gas Mission, AIAA-87-1054, 19th AIAA/DGLR/JSASS International Electric Propulsion Conference, Colorado Springs, Colorado, May/1987.

19. E.A. Belbruno, Examples of the Nonlinear Dynamics of Ballistic Capture and Escape in the Earth-Moon System, AIAA-90-2896, AIAA Astrodynamics Conference, Portland, Oregon, Aug. 1990.

20. E.A. Belbruno, Ballistic Lunar Capture Transfer Using the Fuzzi Boundary and Solar Perturbations: a Survey, Proceedings for the International Congress of SETI Sail and Astrodynamics, Turin, Italy, 1992.

21. Krish, V., An Investigation Into Critical Aspects of a New Form of Low Energy Lunar Transfer, the Belbruno-Miller Trajectories, Master's Dissertation, Massachusetts Inst. of Technology, Cambridge, MA, 1991.

22. Prado, A.F.B.A., A Semi-Analytical Study of the Gravitational Capture, Proceedings of the International Symposium Space Dynamics, Biarritz-France, 26-30/June/2000.