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ESTIMATION OF INTERNAL SOURCES IN NATURAL WATERS USING REMOTE SENSING DATA

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ABSTRACT

An inverse analysis for the estimation of internal sources in natural waters, using remote sensing data, is presented. The analysis involves a forward model that utilizes an analytical discrete-ordinates method for solving the radiative transfer equation and an inverse model which contains an algorithm for least-squares estimation that is iteratively solved for retrieving the internal source profile by using the Levenberg-Marquardt optimizer. The experimental data are simulated with synthetic data (exit radiances) that are calculated at the surface of the water and corrupted with noise. The results show that the internal sources can be recovered with good accuracy, even for high noisy data.

Keywords: Inverse hydrologic optics, internal sources, remote sensing, analytical discrete-ordinatesm ethods.

NOMENCLATURE

Following isa list of the most important symbolsus ed in thisw ork:

I	intensity (radiance) of the radiation field
I_0	incident beam strength
L	order of the anisotropy
M	number of optimization functions
N	quadrature order
p	phase function
P	Legendre polynomial
P^m	associated Legendre function
R	number of spatial regions
S	inhomogeneouss ource term
S_0	internal source of radiation
z	geometrical thicknesso f the medium
β	expansion coefficient
ζ	optical thicknesso f the medium
Θ	direction of propagation of the radiation in the medium
λ	photon wavelength
μ	cosine of the polar angle (measured from the *positive* τ axis)
μ_0	cosine of polar angle of incidence
ϖ	albedo for single scattering
τ	optical variable
φ	azimuthal angle
φ_0	azimuthal angle of incidence

Subscripts/Superscripts

g	wavelength interval index
l	expansion order of the phase function
m	Fourier component index
r	spatial region index

INTRODUCTION

The inverse analysis of radiation in a participating medium has a broad range of applications, including, among others,r emote sensing of the atmosphere and the determination of radiative properties in natural waters. McCormick in his articles [1–3] presents reviews of methods for solving the inverse radiation problems, such as the estimation of optical properties, the thickness of a medium and the presence of a spatially distributed source. In hydrologic optics, the estimation of the radiative properties can be performed by using either data from *in situ* or remote sensing data.

One of the most important challenges in inverse hydrologic optics is to retrieve the properties of the physical system from remote sensed data. There are only few worksin the literature on this subject, most of them are based on the Gershun's equation and the estimation of the apparent optical property named irradiance attenuation coefficient [4] or diffuse attenuation coefficient [5]. Another simplification for remote sensing estimates of bio-optical properties is to consider an homogeneouso cean [6–8].

Differently from our early estimations with *in situ* radiometric measurements [9–15], the source term estimation is performed using remote sensing data, that are represented by the exit radiances. The method presented in this paper is completely different from the one presented in Ref. [16], which is based on the reciprocity principle. However, there isn o testsin thisr eference, not even a numerical one, for the validation of that methodology.

The inverse analysis involves the following two basic steps: (a) a forward problem solution and (b) an inverse problem solution. In the first step, the radiative transfer equation is solved by an analytical discrete-ordinates method [17–19] to determine the exact (synthetic) exit radiances, and, in the second, an algorithm [20,21] for least-squares estimation is iteratively utilized to retrieve the spatially distributed sources. In the analysis, the experimental data are simulated with synthetic data corrupted with noise from 1 to 5%.

PRELIMINARY ANALYSIS

For a multispectral problem, we consider the equation of transfer

$$\mu\frac{\partial}{\partial\tau}I(\tau,\mu,\varphi,\lambda)+I(\tau,\mu,\varphi,\lambda)=\varpi(\lambda)\times\int_{-1}^{1}\int_{0}^{2\pi}\int_{\lambda}p(\cos\Theta,\lambda)I(\tau,\mu',\varphi',\lambda')\mathrm{d}\lambda'\mathrm{d}\varphi'\mathrm{d}\mu'+S_0(\tau,\lambda),\quad(1)$$

subject to the boundary conditions

$$I(0,\mu,\varphi,\lambda)=L(\mu,\varphi,\lambda)=I_0(\lambda)\delta(\mu-\mu_0)\delta(\varphi-\varphi_0)\quad(2a)$$

and

$$I(\zeta,-\mu,\varphi,\lambda)=R(\mu,\varphi,\lambda)=0,\quad(2b)$$

where $I(\tau,\mu,\varphi,\lambda)$ denotes the intensity (radiance) of the radiation field, $\tau\in(0,\zeta)$ the optical variable, with ζ representing the optical thickness of the medium, $\mu\in[-1,1]$ and $\varphi\in[0,2\pi]$, respectively, the cosine of the polar angle (measured from the *positive* τ axis) and the azimuthal angle, that specify the direction of propagation Θ of the radiation in the medium, and λ the photon wavelength. In addition, $\varpi(\lambda)\in[0,1]$ is the albedo for single scattering, where $\varpi=b/(a+b)$ with a and b representing the absorption and scattering coefficients, $p(\cos\Theta,\lambda)$ is the phase function for scattering from $\{\mu',\varphi',\lambda'\}$ to $\{\mu,\varphi,\lambda\}$, $S_0(\tau,\lambda)$ an internal source of radiation, and $L(\mu,\varphi,\lambda)$ and $R(\mu,\varphi,\lambda)$ are the distributed incident radiances at the boundaries. The incident beam is characterized by a beam strength $I_0(\lambda)$ and a beam direction (μ_0,φ_0). An outline of the physical process isde picted in Fig. 1.

In this work, we discretize Eqs. (1) and (2) in the wavelength variable and then consider all wavelength-dependent values as being averages over a wavelength interval (band) $\Delta\lambda_g$. Thus, for a generic variable $F(\lambda)$, we have

$$F_g=F(\lambda_g)=\frac{1}{\Delta\lambda_g}\int_{\Delta\lambda_g}F(\lambda)\mathrm{d}\lambda,\quad(3)$$

where λ_g is an average wavelength in the interval g. To further simplify the calculations, we consider that a particle can only be scattered to within the same interval, and so we write our original equation of transfer, for a specific wavelength λ_g, as,

$$\mu\frac{\partial}{\partial\tau}I_g(\tau,\mu,\varphi)+I_g(\tau,\mu,\varphi)=\varpi_g\times\int_{-1}^{1}\int_{0}^{2\pi}p(\cos\Theta)I_g(\tau,\mu',\varphi')\mathrm{d}\varphi'\mathrm{d}\mu'+S_{0,g}(\tau),\quad(4)$$

subject to the boundary conditions

$$I_g(0,\mu,\varphi)=I_{0,g}\delta(\mu-\mu_0)\delta(\varphi-\varphi_0)\quad(5a)$$

and

$$I_g(\zeta,-\mu,\varphi)=0.\quad(5b)$$

Here, the phase function $p(\cos\Theta)$, for scattering from $\{\mu',\varphi'\}$ to $\{\mu,\varphi\}$, is represented by a finite Legendre polynomial expansion given in terms of the cosine of the scattering angle Θ,

$$p(\cos\Theta)=\frac{1}{4\pi}\sum_{l=0}^{L}\beta_l P_l(\cos\Theta),\quad\beta_0=1\quad\text{and}\quad|\beta_l|<2l+1\ \text{ for }\ 0<l\le L,\quad(6)$$

where β_l and P_l are, respectively, the coefficient and the Legendre polynomial in the L^{th}-order expansion of the phase function.

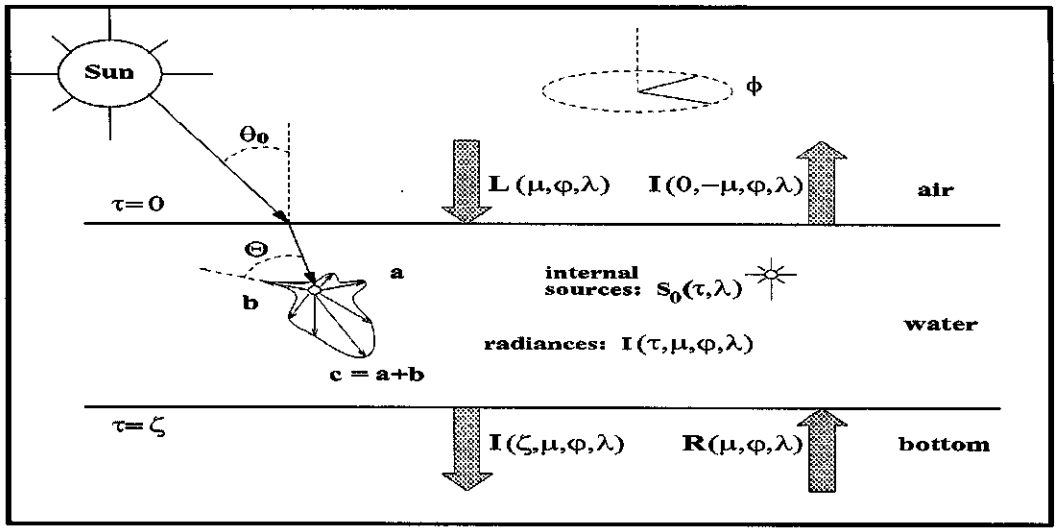

Figure 1: Pictorial representation of the radiative processin natural waters.

FORWARD PROBLEM SOLUTION

Following Chandrasekhar [22], we then write the intensity of the radiation field as

$$I_g(\tau,\mu,\varphi) = I_{u,g}(\tau,\mu,\varphi) + I_{s,g}(\tau,\mu,\varphi), \quad (7)$$

where $I_{u,g}(\tau,\mu,\varphi)$ is the unscattered component which satisfies a version of Eq. (4) with zero right-hand side and boundary conditions similar to Eqs. (5),a nd $I_{s,g}(\tau,\mu,\varphi)$ is the scattered component that must satisfy

$$\mu\frac{\partial}{\partial\tau}I_{s,g}(\tau,\mu,\varphi) + I_{s,g}(\tau,\mu,\varphi) = \varpi_g \times \int_{-1}^{1}\int_{0}^{2\pi} p(\cos\Theta)I_{s,g}(\tau,\mu',\varphi')\mathrm{d}\varphi'\mathrm{d}\mu' + S_g(\tau,\mu,\varphi), \quad (8)$$

for $\tau \in [0,\zeta]$, $\mu \in [-1,1]$ and $\varphi \in [0,2\pi]$, and the boundary conditions

$$I_{s,g}(0,\mu,\varphi) = I_{s,g}(\zeta,-\mu,\varphi) = 0, \quad (9)$$

for $\mu \in (0,1]$ and $\varphi \in [0,2\pi]$. The inhomogeneouss ource term $S_g(\tau,\mu,\varphi)$ isg iven by

$$S_g(\tau,\mu,\varphi) = S_{0,g}(\tau) + \varpi_g \times \int_{-1}^{1}\int_{0}^{2\pi} p(\cos\Theta)I_{u,g}(\tau,\mu',\varphi')\mathrm{d}\varphi'\mathrm{d}\mu'. \quad (10)$$

Continuing, we make use of the Fourier *cosine* decomposition [22]

$$I_{s,g}(\tau,\mu,\varphi) = \frac{1}{2}\sum_{m=0}^{L}(2-\delta_{0,m})I_g^m(\tau,\mu) \times \cos[m(\varphi-\varphi_0)] \quad (11)$$

along with the addition theorem [23] for the Legendre polynomials to deduce that the original problem can be reduced to the problem of solving, for $m = 0, 1, \ldots, L$,

$$\mu\frac{\partial}{\partial\tau}I_g^m(\tau,\mu) + I_g^m(\tau,\mu) = \frac{\varpi_g}{2}\sum_{l=m}^{L}\beta_l P_l^m(\mu) \times \int_{-1}^{1} P_l^m(\mu')I_g^m(\tau,\mu')\mathrm{d}\mu' + S_g^m(\tau,\mu), \quad (12)$$

where

$$P_l^m(\mu) = \left[\frac{(l-m)!}{(l+m)!}\right]^{1/2}(1-\mu^2)^{m/2}\frac{\mathrm{d}^m}{\mathrm{d}\mu^m}P_l(\mu) \quad (13)$$

denotes an associated Legendre function, and the inhomogeneouss ource term isg iven by

$$S_g^m(\tau,\mu) = 2S_{0,g}(\tau)\delta_{0,m} + \frac{\varpi_g}{2}\frac{I_{0,g}}{\pi}e^{-\tau/\mu_0}\sum_{l=m}^{L}\beta_l P_l^m(\mu_0)P_l^m(\mu), \quad (14)$$

subject to the boundary conditions,

$$I_g^m(0,\mu) = I_g^m(\zeta,-\mu) = 0, \quad (15)$$

for $\mu \in (0,1]$. It is clear that once we solve the problems formulated by Eqs. (12) and (15), for $m = 0, 1, \ldots, L$, we can compute the scattered component of the intensity with Eq. (11).

Thus far we have considered a problem formulated as a single region. The extension of this method to multi-region geometry is based on the work performed by Dias and Garcia [24,25]. So we use, as mentioned in Refs. [24] and [25], "an iterative approach that is based on solving the problem one region at a time and using spatial sweeps to connect these solutionsa nd guide them to convergence." For the treatment of the space variable, we consider a system of R regions, as shown in Fig. 2.

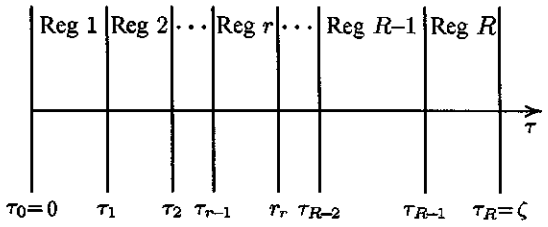

Figure 2. A system of R regions

The multi-region problem is then expressed, for $\mu \in (0,1]$ and $r = 1, 2, \ldots, R$, as

$$\mu \frac{\partial}{\partial \tau} I^m_{r,g}(\tau,\mu) + I^m_{r,g}(\tau,\mu) = \frac{\varpi_{r,g}}{2} \times \sum_{l=m}^{L} \beta_{l,r} P^m_l(\mu) \int_{-1}^{1} P^m_l(\mu') I^m_{r,g}(\tau,\mu') \mathrm{d}\mu' + S^m_{r,g}(\tau,\mu), \quad (16)$$

subject, for $\mu \in (0,1]$, to the boundary conditions,

$$I^m_{1,g}(\tau_0,\mu) = I^m_{R,g}(\tau_R,-\mu) = 0, \quad (17)$$

and to the interface conditions, for $r = 1, 2, \ldots, R-1$,

$$I^m_{r,g}(\tau_r,\pm\mu) = I^m_{r+1,g}(\tau_r,\pm\mu), \quad (18)$$

where $S^m_{r,g}(\tau,\mu)$ isg iven by

$$S^m_{r,g}(\tau,\mu) = 2S_{0,g}(\tau)\delta_{0,m} + \frac{\varpi_{r,g}}{2}\frac{I_{0,g}}{\pi} e^{-\tau/\mu_0} \times \sum_{l=m}^{L} \beta_{l,r} P^m_l(\mu_0) P^m_l(\mu). \quad (19)$$

To define our discrete-ordinatesv ersion of the problem posed by Eqs. (16) to (18), we utilize a quadrature of order N with nodes $\{\mu_j\}$ and weights $\{\eta_j\}$ to approximate the integral in Eq. (16). The selected quadrature scheme is the double quadrature of order $N = 2n$ obtained by applying a standard Gauss-Legendre scheme of order n to each of the half-intervals $[0,1]$ and $[-1,0]$. By using the elementary solutionso f the discrete-ordinates equations and their orthogonality property developed in Ref. 26, we can write the general discrete-ordinatess olution of order N as

$$I^m_{r,g}(\tau,\mu_j) = \sum_{k=1}^{n} \Big[A_{k,r,g}\Phi_{r,g}(\nu_k,\mu_j) e^{-(\tau-\tau_{r-1})/\nu_k} + B_{k,r,g}\Phi_{r,g}(-\nu_k,\mu_j) e^{-(\tau_r-\tau)/\nu_k} \Big] + \sum_{k=1}^{n} [\mathfrak{A}_{k,r,g}(\tau)\Phi_{r,g}(\nu_k,\mu_j) + \mathfrak{B}_{k,r,g}(\tau)\Phi_{r,g}(-\nu_k,\mu_j)]. \quad (20)$$

In Eq. (20), ν_k and $-\nu_k$, $k = 1, 2, \ldots, n$,de note, respectively, the inverses of the *positive* and the *negative* eigenvalues of the $N \times N$ matrix $\Xi^{-1}(\mathbf{I} - \mathbf{W_{r,g}})$, where $\Xi = \mathrm{diag}\{\mu_1, \mu_2, \ldots, \mu_N\}$, $\mathbf{I}$ is the identity matrix of order N, and $\mathbf{W_{r,g}}$ is an $N \times N$ matrix with elements

$$W_{i,j,r,g} = \frac{\varpi_{r,g}}{2}\eta_j \sum_{l=m}^{L} \beta_{l,r} P^m_l(\mu_i) P^m_l(\mu_j). \quad (21)$$

The elementary solutions $\Phi_{r,g}(\nu_k,\mu_j)$ and $\Phi_{r,g}(-\nu_k,\mu_j)$ present in Eq. (20) are, respectively, the jth components of the eigenvectors $\mathbf{\Phi_{r,g}}(\nu_k)$ and $\mathbf{\Phi_{r,g}}(-\nu_k)$, associated, respectively, with the eigenvalues $1/\nu_k$ and $-1/\nu_k$. Also the coefficients $\{\mathfrak{A}_{k,r,g}(\tau)\}$ and $\{\mathfrak{B}_{k,r,g}(\tau)\}$ of the particular solution can be expressed as[26]

$$\mathfrak{A}_{k,r,g}(\tau) = \frac{1}{N_{r,g}(\nu_k)} \sum_{i=1}^{N} \eta_i \Phi_{r,g}(\nu_k,\mu_i) \times \int_{\tau_{r-1}}^{\tau} S_{r,g}(x,\mu_i) e^{-(\tau-x)/\nu_k} \mathrm{d}x \quad (22a)$$

and

$$\mathfrak{B}_{k,r,g}(\tau) = -\frac{1}{N_{r,g}(-\nu_k)} \sum_{i=1}^{N} \eta_i \Phi_{r,g}(-\nu_k,\mu_i) \times \int_{\tau}^{\tau_r} S_{r,g}(x,\mu_i) e^{-(x-\tau)/\nu_k} \mathrm{d}x, \quad (22b)$$

with

$$N_{r,g}(\pm\nu_k) = \sum_{i=1}^{N} \eta_i \mu_i [\Phi_{r,g}(\pm\nu_k,\mu_i)]^2. \quad (23)$$

Note that the coefficientso f the homogeneouss olution $\{A_{k,r,g}\}$ and $\{B_{k,r,g}\}$ are the solutions to the linear system of N algebraic equations obtained by imposing that the general solution expressed by Eq. (20) satisfies the boundary and interface conditions of the problem. These calculationsa re reported in detail in Refs. 18 and 19.

INVERSE PROBLEM SOLUTION

The inverse problem is formulated as an optimization problem for the minimization of the norm of the differences between the measured data and the data obtained through the forward model presented in the previouss ection.

To simplify our solution, we split the internal source of radiation $S_{0,g}(\tau)$ in two functions

$$S_{0,g}(\tau) = S_0(\tau)\, Q(\lambda_g) = S_0(\tau)\, Q_g, \qquad (24)$$

and considered the function with spectral dependency Q_g a known function.

While the internal source of radiation $S_{0,g}(\tau)$ is unknown, the expansion coefficients $\{\beta_{l,r}\}$ of the phase function for anisotropic scattering, the single scattering albedo $\{\varpi_{r,g}\}$ and the optical thickness ζ, as well as the measured (exact) values of the exit radiances $I_g(0, -\mu, \varphi)$ are considered available. The inverse problem is iteratively solved by an algorithm for least-squares estimation where, in each iteration, values of the exit radiances $I_g(0, -\mu, \varphi)$ are computed from estimated values of the internal source profile $S_0(\tau)$. Here the calculated radiances are obtained from a modified and adapted package of subroutines taken from the forward model code, which we here refer to as subroutine *Peesna*,a nd the estimated parameters are obtained with the IMSL subroutine *Dbclsf* [27]. Basically, these calculations constitute our inverse model.

The IMSL subroutine *Dbclsf* uses a modified Levenberg-Marquardt method [20,21] of minimization and an active set strategy [28] to solve nonlinear least squares problems subject to simple bounds on the variables. The problem is stated as follows[27,29]:

$$\min_{x \in \Re^K} \frac{1}{2} Y^T(x) Y(x) = \min_{x \in \Re^K} \frac{1}{2} \sum_{i=1}^{M} [y_i(x)]^2,$$
$$d_j \le x_j \le u_j, \qquad (25)$$

where $M \ge K$, $Y : \Re^K \to \Re^M$, $y_i(x)$ is the i-th component function of $Y(x)$, and d_j and $u_j, j = 1, \ldots, K$, are the lower and upper bounds, respectively. The functions $y_i, i = 1, \ldots, M$, represent the differences between the experimental radiances and radiances that are calculated through each call to subroutine *Peesna*, and $x_j, j = 1, \ldots, K$, the unknown variables to be estimated by the IMSL subroutine *Dbclsf*. We implemented two techniques for retrieving the internal source profile. In the first one, referred to as the *average-value* technique, we consider within each region r, an average source value $\bar{S}_0$ represented by the variable x_j, and thus we estimate $K = R$ variables. And in the second one,r eferred to as the *three-coefficient* technique, the source $S_0(\tau)$ is determined by the quadratic expression

$$S_0(\tau) = x_1 + x_2(\tau/\zeta) + x_3(\tau/\zeta)^2, \qquad (26)$$

where the coefficients x_1, x_2 and x_3 are the only variables to be estimated. Note that, using the *average-value* technique for retrieving the source becomes very expensive, computationally speaking, as a large number of regions R is needed to generate a smooth profile.

NUMERICAL SOLUTION

Two problems were chosen to test the inverse model. The parameters that define the chosen problems were based on the simulation presented in Mobley's book [30], *Light and Water - Radiative Transfer in Natural Waters*, in Section 11.8 - "A Simulation of Case I Water". In addition, we considered an internal source generated by bioluminescent organisms.

The single scattering albedo $\varpi_{r,g}$, is calculated by the expression

$$\varpi_{r,g} = \frac{b_{r,g}}{c_{r,g}} = \frac{b_{r,g}}{b_{r,g} + a_{r,g}}, \qquad (27)$$

where $c_{r,g}, a_{r,g}$ and $b_{r,g}$ are the attenuation, absorption and scattering coefficients, respectively. The absorption and scattering coefficients, which are expressed in m^{-1}, are approximated by the equations[30]

$$a_{r,g} = \left[a_g^w + 0.06\, a_g^c\, C^{0.65}(z)\right] \times \left[1 + 0.2\, e^{-0.014(\lambda_g - 440)}\right] \qquad (28)$$

and

$$b_{r,g} = \left(\frac{550}{\lambda_g}\right) 0.30\, C^{0.62}(z), \qquad (29)$$

where a_g^w is the absorption coefficient of pure water, a_g^c a nondimensional, statistically derived

chlorophyll-specific absorption coefficient, and $C(z)$ the chlorophyll concentration, in $mg\ m^{-1}$, which is approximated as a background value plus a Gaussian [30]

$$C(z) = C_0 + \frac{h}{s\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{z-z_{max}}{s}\right)^2}, \qquad (30)$$

where the geometric depth z is expressed in meters. The values of the parameters a_g^w, a_g^c, for three chosen values of λ_g (500, 550 and 600), and those of C_0, h, s and z_{max}, obtained from Ref. 30, are shown in Table 1.

Table 1. Parameterso btained from Ref. [30]

Parameter	Value
$a^w(\lambda_g$=500; 550; 600)	0.026; 0.064; 0.245 m^{-1}
$a^c(\lambda_g$=500; 550; 600)	0.668; 0.357; 0.236
C_0	0.2 mg m^{-3}
h	144 mg m^{-2}
s	9 m
z_{max}	17 m

Note that the inherent optical properties, $c_{r,g}, a_{r,g}$ and $b_{r,g}$, are regarded as being averages within each region r, and that the correspondence between the optical depth and geometric depth is given by the expression $\tau(z) = \tau_{r-1} + (z - z_{r-1})c_{r,g}$, where

$$\tau_{r-1} = \sum_{i=1}^{r-1}(z_i - z_{i-1})c_{i,g}. \qquad (31)$$

For both problems, we consider $\varphi_0 = 0$, $\mu_0 = 1$, the Henyey-Greenstein parameter $f = 0.924$, the quadrature order $N = 130$, a water layer thickness of 40 meters, equidistantly divided into $R = 5$ regions. With the f factor, applied to all regions, the β_l values in Eq. (6) are obtained in two steps. First the code calculates the Henyey-Greenstein phase function [31]

$$\tilde{\beta}_{HG}(f;\Theta) \equiv \frac{1}{4\pi}\frac{1-f^2}{(1+f^2-2f\cos\Theta)^{3/2}}, \qquad (32)$$

then iteratively searches for a scattering order L that generates, through Eq. (6), a phase function whose graphic representation compares well with the graphic representation of the Henyey-Greenstein phase function given by Eq. (32), *i.e.*, the iteration process is stopped when corresponding points on the two graphs do not differ by more than $\pm 1\%$. The β_l values of Eq. (6) are determined through the expression [32]

$$\beta_l = (2l+1)\ f^l. \qquad (33)$$

Note that we consider $\beta_{l,1} = \beta_{l,2} = \beta_{l,...} = \beta_{l,R}$, for the chosen multi-region problems.

With these input parameters, the simulated measured exit radiances $I(\tau = 0, -\mu, \varphi = 0)$, were determined by the forward model code at five values of μ ($-.96, -.97, -.98, -.99$ and -1), for each one of the three chosen values of λ_g,t otalizing, in Eq. (25), $M = 15$ optimization functions. We used a constant source profile ($S_0(\tau) = 0.5$) and a sine source profile ($S_0(\tau) = \sin(\pi\tau/\zeta)$), in the first and second problems, respectively, considering in both problems$Q_g = 1.0$, in Eq. (24).

In order to simulate measured radiances Z_m containing measurement errors, the calculated data Z_e were corrupted with noise by using the IMSL subroutine *Drnnor* [33], which generates pseudo-random numbers from a standard normal distribution. Thusw e have

$$Z_m = Z_e(1 + \kappa\,\xi), \qquad (34)$$

where κ is the percent noise and ξ is a random variable calculated by subroutine *Drnnor*.

We used both techniques, the *average-value* technique and the *three-coefficient* technique, to retrieve the internal source profiles. Besides the excessive computational time involved to solve the problems,w hen using the *average-value* technique, we were not able to obtain results with a good degree of accuracy. In Figs. 3 and 4 we show the estimated source profiles, using the *three-coefficient* technique for four selected experimental errors, $\kappa = 0, 1, 2$ and 5%. The values in parentheses indicate the percentage deviations of the calculated areasu nder the estimated curves from those under the exact curves.

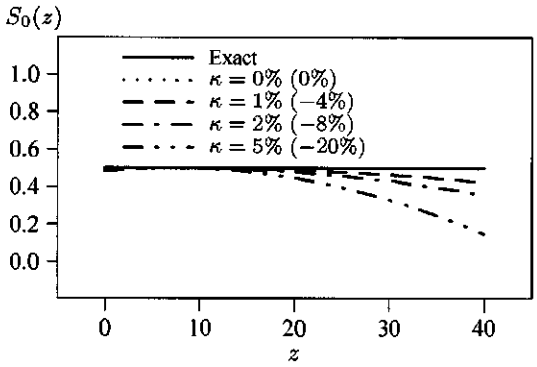

Figure 3: Estimation of the constant source profile.

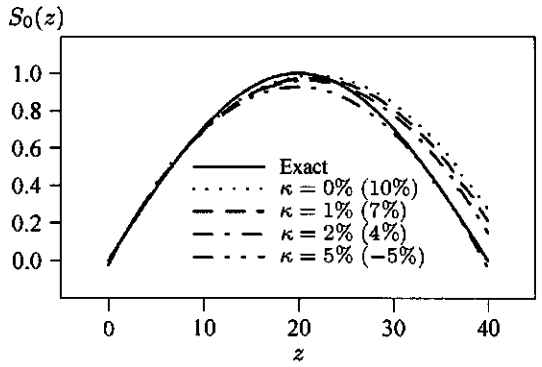

Figure 4: Estimation of the sine source profile.

FINAL COMMENTS

The inverse problem of estimating the internal sources in natural waters, using remote sensing data, is solved by an analytical discrete-ordinates method [17–19] and a modified Levenberg-Marquardt method [20,21,27] for the adopted forward model and inverse technique, respectively.

In the solution of the two problems chosen to test the implemented inversion technique, the analyses were performed by using simulated measurements containing random errors varying from 0 to 5%. We note that, as shown in Fig 3 and 4, the differences between the exact and estimated source profiles increase with depth. In the opinion of the authors, thisb iasisdue to the physical nature of the problem, where the radiances exponentially decay within water, so the signal to noise relation greatly degradesf or greater depths.

In order to solve our inverse problem, various modeling aspects had to be formulated, including the consideration of splitting the internal source in two functions, one carrying the spatial dependency and the other the spectral dependency, where the latter was considered as a known function. This simplification was applied in order to reduce the computational time involved in the solution.

Two techniques, the *average-value* technique and the *three-coefficient* technique, were implemented for retrieving the internal source profiles. The *average-value* technique was considered inadequate, due to its ineffectiveness and inaccuracy. With the *three-coefficient* technique, we were able to recover the desired profiles with an acceptable degree of accuracy, even with inputda ta containing significant measurement errors and with a small number of measurement points.

We note that other techniques can be used to solve this type of inverse problems, such as the Tikonov regularization techniques or the principle of maximum entropy (in its various forms), Kalman filtering technique and the variational methods, and that we consider applying them in future works.

The estimation of the optical properties by using remote sensing data is still a challenge to the scientific community. This work represents the first effective step forward for the multispectral inversion. It is important to point out that there is no need in our analysis to impose an homogeneous ocean or any other constrained assumptions. Although we only present preliminary results of our research, we are able to conclude that it is possible to develop techniques for estimating the desired properties, using remote sensing data.

To close this work, we finally note that we expect soon to be able to extend our analysis for the estimation of other optical properties, such as the absorption and scattering coefficients.

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