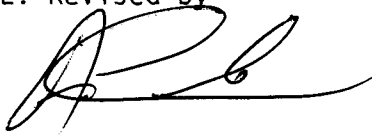
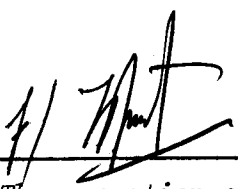



1. Publication Nº INPE-3492-PRE/729	2. Version	3. Date April, 1985	5. Distribution <input type="checkbox"/> Internal <input checked="" type="checkbox"/> External <input type="checkbox"/> Restricted
4. Origin DME/DAM	Program ANGRA (AR4/5)		
6. Key words - selected by the author(s) DIFFUSION MODEL COASTAL WATERS			
7. U.D.C.: 551.465.15:519.6			
8. Title COMPARISON OF A DIFFUSION MODEL WITH DYE DISPERSION MEASUREMENTS TO STUDY TURBULENCE IN COASTAL WATERS		10. Nº of pages: 25	
		11. Last page: 24	
9. Authorship Merritt R. Stevenson Héctor M. Inostroza V.		12. Revised by  Alejandro Camerlengo	
Responsible author 		13. Authorized by  Marco Antônio Raupp Director General	
14. Abstract/Notes The operation of a coastal power plant provides the opportunity to conduct studies, which deals with circulation and horizontal mixing in coastal waters near a power plant. This study was initiated with field experiments in which small quantities of rhodamine dye solution were dispersed in the inlet and outlet bays adjacent to the power station. Sequential aerial photographs permitted the estimation of the magnitude of horizontal mixing (diffusion) coefficients. The high cost of the dye, however, makes the utilization of diffusion models an attractive supplement to such studies, since the model can be run a number of times with varied parameters. The simulated concentration fields can then be compared with the field experiments. The simple diffusion model selected for this study is based on a point discharge of a dye solution. The model assumes an isotropic field with a diffusion coefficient constant over the period of the study. The relation between dye concentration (S) and position (R) as time (t) changes is: $S = M/4\pi Kt \exp(-R^2/4Kt)$. This equation is modified to allow for simple horizontal advection and becomes: $S = M/4\pi Kt \exp(-(X-Ut)^2 - (Y-Vt)^2)/4Kt)$. To simulate the two-dimensional dye patches, an equispaced grid (441 points) was used with 5m between grid points. The time step was set for 10 minute intervals. The results of the comparison between the numerical simulation and a dye experiment are discussed.			
15. Remarks This work was presented at the I Seminário de Modelagem Numérica do Mar, December 12-14, 1984. Instituto de Pesquisas Espaciais, INPE, São José dos Campos, SP, Brasil.			

COMPARISON OF A DIFFUSION MODEL WITH DYE DISPERSION MEASUREMENTS
TO STUDY TURBULENCE IN COASTAL WATERS

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ABSTRACT

The operation of a coastal power plant provides the opportunity to conduct various studies, including the present one, which deals with circulation and horizontal mixing in coastal waters near a power plant. This study was initiated with field experiments in which small quantities of rhodamine dye solution were dispersed in the inlet and outlet bays adjacent to the power station. Sequential aerial photographs permitted the estimation of the magnitude of horizontal mixing (diffusion) coefficients. The high cost of the dye, however, makes the utilization of diffusion models an attractive supplement to such studies, since the model can be run a number of times with varied parameters. The simulated concentration fields can then be compared with the field experiments. The simple diffusion model selected for this study is based on a point discharge of a dye solution. The model, previously discussed in the literature, assumes an isotropic field with a diffusion coefficient constant over the period of the study. The relation between dye concentration (S) and position (R) as time (t) changes is:

$$S = \frac{M}{4\pi Kt} \exp\left(-\frac{R^2}{4Kt}\right) .$$

This equation is modified to allow for simple horizontal advection and becomes:

$$S = \frac{M}{4\pi Kt} \exp\left(\frac{-(X-Ut)^2 - (Y-Vt)^2}{4Kt}\right) .$$

To simulate the two-dimensional dye patches, an equispaced grid consisting of 21 X 21 points (441 total) was used with 5m between grid points. The time step was set for 10 minute intervals, although other specific times were also determined. The results of the comparison between the numerical simulation and a dye experiment are discussed.

RESUMO

A operação de uma usina costeira geradora de energia oferece a oportunidade de fazer vários estudos, incluindo o presente, que tratam de circulação e mistura horizontal em águas costeiras adjacentes a ela. Este estudo foi iniciado com experimentos de campo nos quais pequenas quantidades de rodamina foram dispersas nas baías adjacentes à usina. Fotografias aéreas em série possibilitaram estimar a magnitude dos coeficientes de mistura horizontal (difusão). Porém, o alto custo do corante faz com que a utilização de modelos de difusão seja somente um suplemento atrativo de tais estudos, porque o modelo pode ser desenvolvido inúmeras vezes com vários parâmetros. Os campos de concentração simulada podem então ser comparados com os dados experimentais. O simples modelo de difusão selecionado para este estudo está baseado numa descarga pontual de uma solução de corante. O modelo, previamente discutido na literatura, considera um campo isotrópico com um coeficiente de difusão constante no período de estudo. A relação entre a concentração de corante (S) e a posição (R) que variam com o tempo (t) é:

$$S = \frac{M}{4\pi Kt} \exp\left(-\frac{R^2}{4Kt}\right)$$

Esta equação é modificada para incluir a advecção horizontal ficando:

$$S = \frac{M}{4\pi Kt} \exp\left(\frac{-(X-Ut)^2 - (Y-Vt)^2}{4Kt}\right)$$

Para simular numericamente as manchas bidimensionais de corante, uma grade de 21 X 21 pontos igualmente espaçada (441 total) foi utilizada com 5m entre os pontos da grade. O intervalo de tempo foi de 10 minutos, embora outros intervalos também tivessem sido utilizados. Discutem-se os resultados de comparação entre a simulação numérica e um experimento de campo.

1. INTRODUCTION

The operation of a coastal nuclear power plant offers the opportunity to make various studies, including the present one, that deals with circulation and horizontal mixing in coastal water adjacent to the plant. This study was initiated with field experiments in which small quantities (~100g) of rhodamine-B dye were dispersed in the two bays adjacent to the power plant (Figure 1). A series of aerial photographs made it possible to estimate the magnitude of the coefficients of horizontal mixing. Because of the high cost of the dye and the relatively complex methods required to conduct the measurements, the use of numerical simulation to improve our understanding of mixing processes in the coastal bays appeared highly desirable.

During the execution of a dye dispersal study to obtain information on advection of local surface water and the estimation of the magnitude of horizontal eddy diffusion, it became apparent that it would be beneficial to further extend the study through the use of numerical simulation of dye patches, using a linear diffusion model. The objective of this numerical simulation was to generate several two-dimensional "images" of an expanding dye patch and to compare these simulations with dye patch data from a field experiment.

The simple diffusion model selected for this study is based on a point source discharge of dye. The model assumes a constant and radially symmetric coefficient of diffusion, during the period of the study, but is related to more elaborate models, discussed in the literature (e.g., Carter and Okubo, 1965; Okubo, 1968). It is based upon the equations of continuity of momentum and of conservation of mass.

The derivation (Proudman, 1953) for our model begins with:

$$\frac{\partial S}{\partial t} + U \frac{\partial S}{\partial x} + V \frac{\partial S}{\partial y} + W \frac{\partial S}{\partial z} = \frac{\partial}{\partial x} \left(K_x \frac{\partial S}{\partial y} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial S}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial S}{\partial z} \right), \quad (1)$$

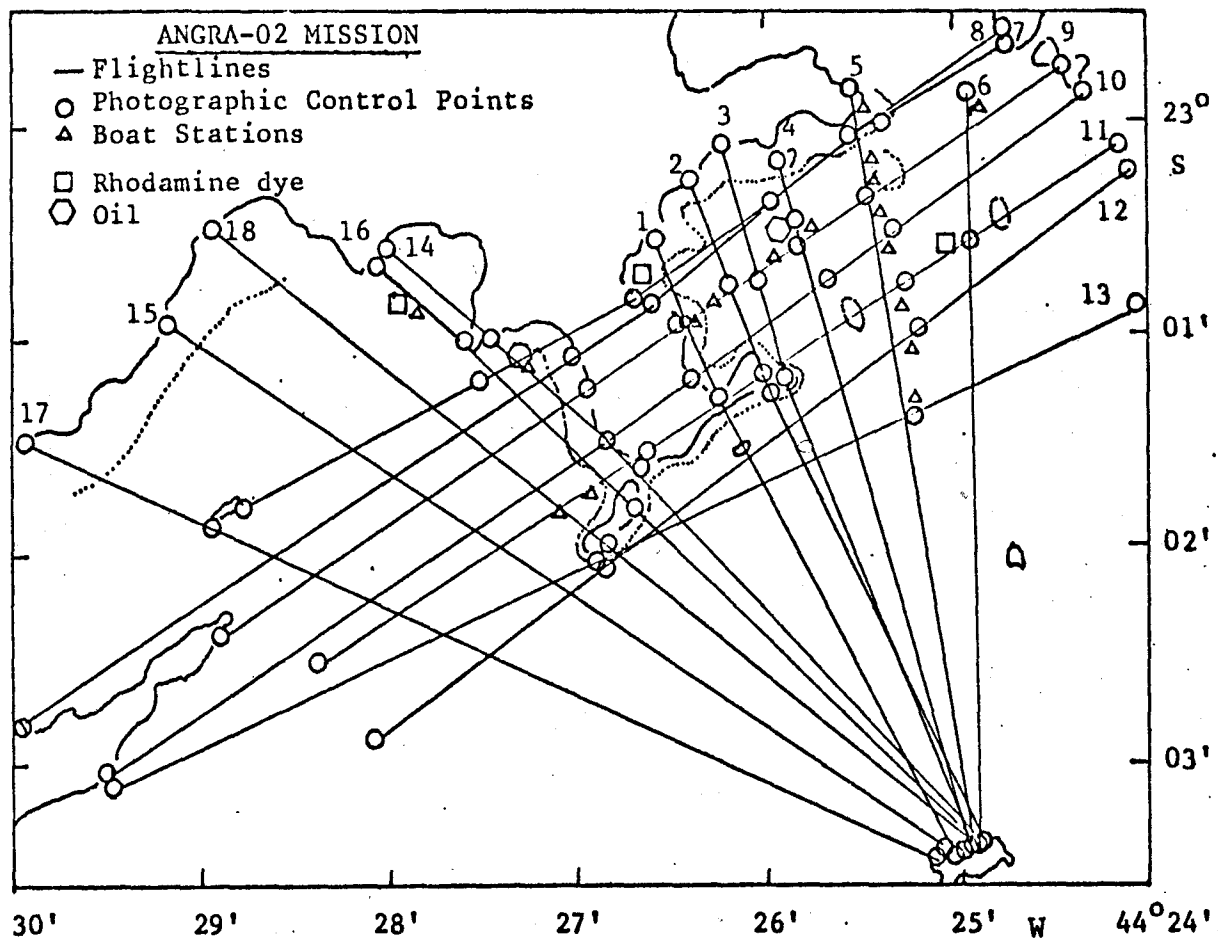


Fig. 1 - Locations of boat stations, dye dispersals and photographic navigation points for flight lines during the ANGRA-02 Mission on 22 October 1981.

SOURCE: Stevenson and Inostroza (in press).

where K_x , K_y and K_z are the eddy diffusion coefficients in X, Y and Z directions, and the coordinate origin corresponds to the initial location of the dye patch. Considering only horizontal (two-dimensional) diffusion, we assume that $K_x = K_y = K$, which is constant during the experiment. Equation 1 then simplifies to:

$$\frac{\partial S}{\partial t} + U \frac{\partial S}{\partial x} + V \frac{\partial S}{\partial y} = K \frac{\partial^2 S}{\partial x^2} + K \frac{\partial^2 S}{\partial y^2} . \quad (2)$$

Since the mean velocity components do not affect the actual diffusion of the dye patch, Equation 2 can be linearized to:

$$\frac{\partial S}{\partial t} = K \frac{\partial^2 S}{\partial x^2} + K \frac{\partial^2 S}{\partial y^2} = K \nabla^2 S . \quad (3)$$

Considering the expansion of the dye patch to be radially symmetric, we can say that:

$$\frac{\partial S}{\partial t} = K \frac{\partial^2 S}{\partial R^2} , \quad (4)$$

which upon integration gives:

$$S = \frac{M}{4\pi Kt} \exp \left\{ - \frac{R^2}{4Kt} \right\} . \quad (5)$$

In practice, however, it is more convenient to use X,Y coordinates for locating points within the patch, so we can use:

$$S = \frac{M}{4\pi Kt} \exp \left\{ - \frac{X^2 + Y^2}{4Kt} \right\} . \quad (6)$$

As previously noted, the inclusion of mean values of U and V affects the advection of the patch, not the actual diffusion within the patch.

Insertion of U and V in Equation 6, provides:

$$S = \frac{M}{4\pi Kt} \exp \left\{ -\frac{(X-UT)^2 - (Y-Vt)^2}{4Kt} \right\}, \quad (7)$$

where X and Y are distances measured from the original point of dispersal and U and V, the mean advection present during the simulation. Of course, if two or three estimates of advection are available, they can be used as input to the model.

From Equation 7 we can readily obtain the unit concentration of dye at time t, for a given U and V at a location X,Y, and for a constant diffusion coefficient K. Our numerical simulation was made using the diffusion model in the form of Equation 7. The method used for dye dispersal produces an initial patch area of about 1 m². Since the basic assumption of the model is based on a point source that expands with time, we note that t=0 (Equation 7) yields an undefined value for peak concentration in the center of the patch. To avoid this unrealistic situation we consider t=0 to correspond to an initial area of 1 m². Since the patch expands rapidly with time, the model is valid except for the first few seconds, when the model predicts that the patch size will be less than 1 m².

2. DATA AND METHODOLOGY

2.1 - FIELD EXPERIMENT

Data from the 22 October 1981, ANGRA-02 Mission, as used in this study, consisted of a series of 23cm X 23cm film images, taken with a RC-10 metric camera, during flights made at 917m altitude over the adjacent bays. Considering the camera and flight altitude, the film scale was 6000:1; that is, 1 mm on the film represents 6 m at sea level. Those films containing images of dye patches (Figure 2) were processed to: a) determine the area (m²) of each patch and the geographic center of each patch; b) calculate U and V velocity components from successive positions and elapsed times of the patch images; and c) construct dispersion diagrams using patch areas and time to obtain a mean coefficient value for

eddy diffusion. The dispersion diagram for the third dye patch (Figure 3) was used to obtain the coefficient $K = 7.8 \times 10^3 \text{ cm}^2 \text{ s}^{-1}$, used in our simulation (see Stevenson et al., 1984 for details).

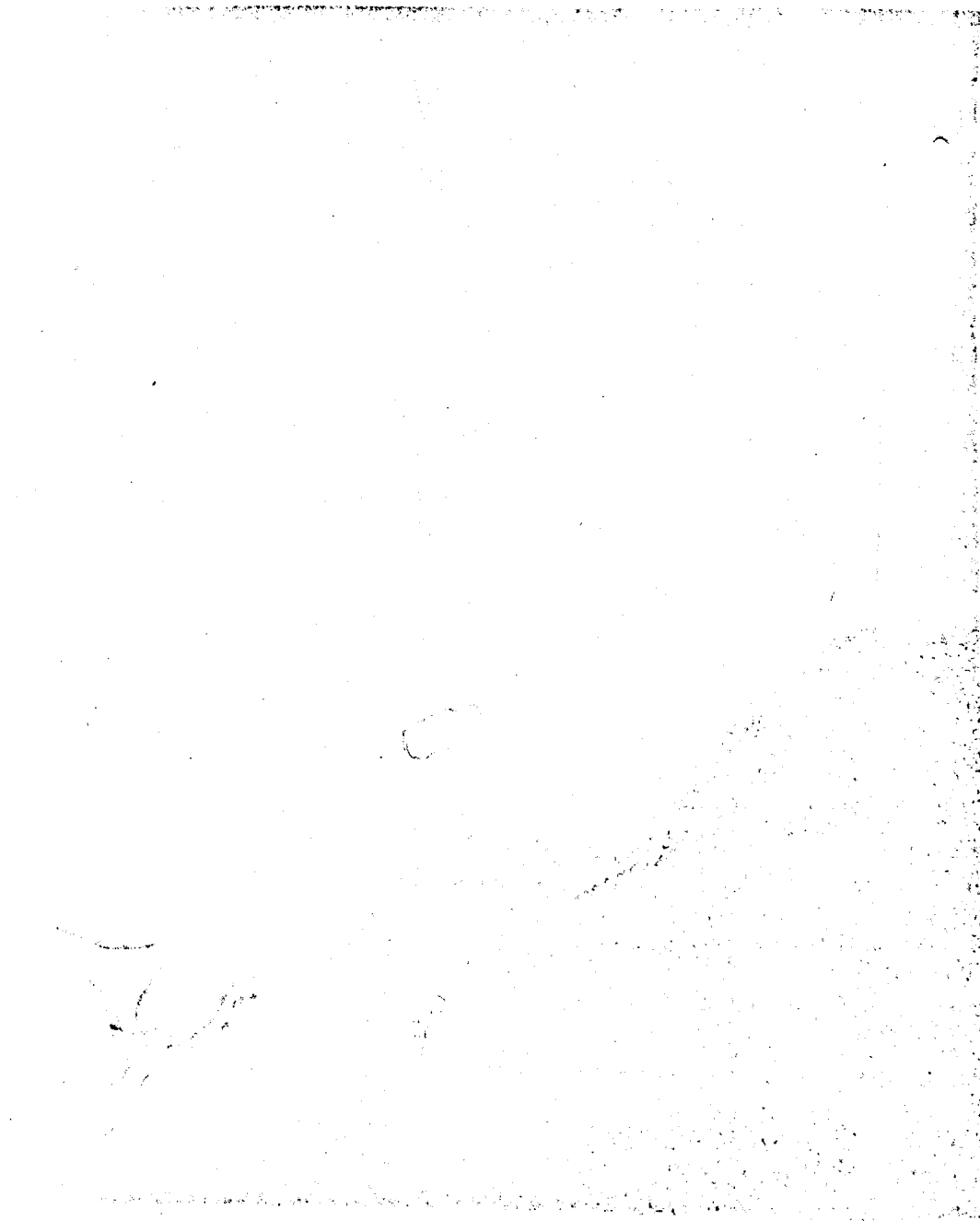


Fig. 2 - Aerial image of dye patch (reddish in color) from field experiment on 22 October, 1981.

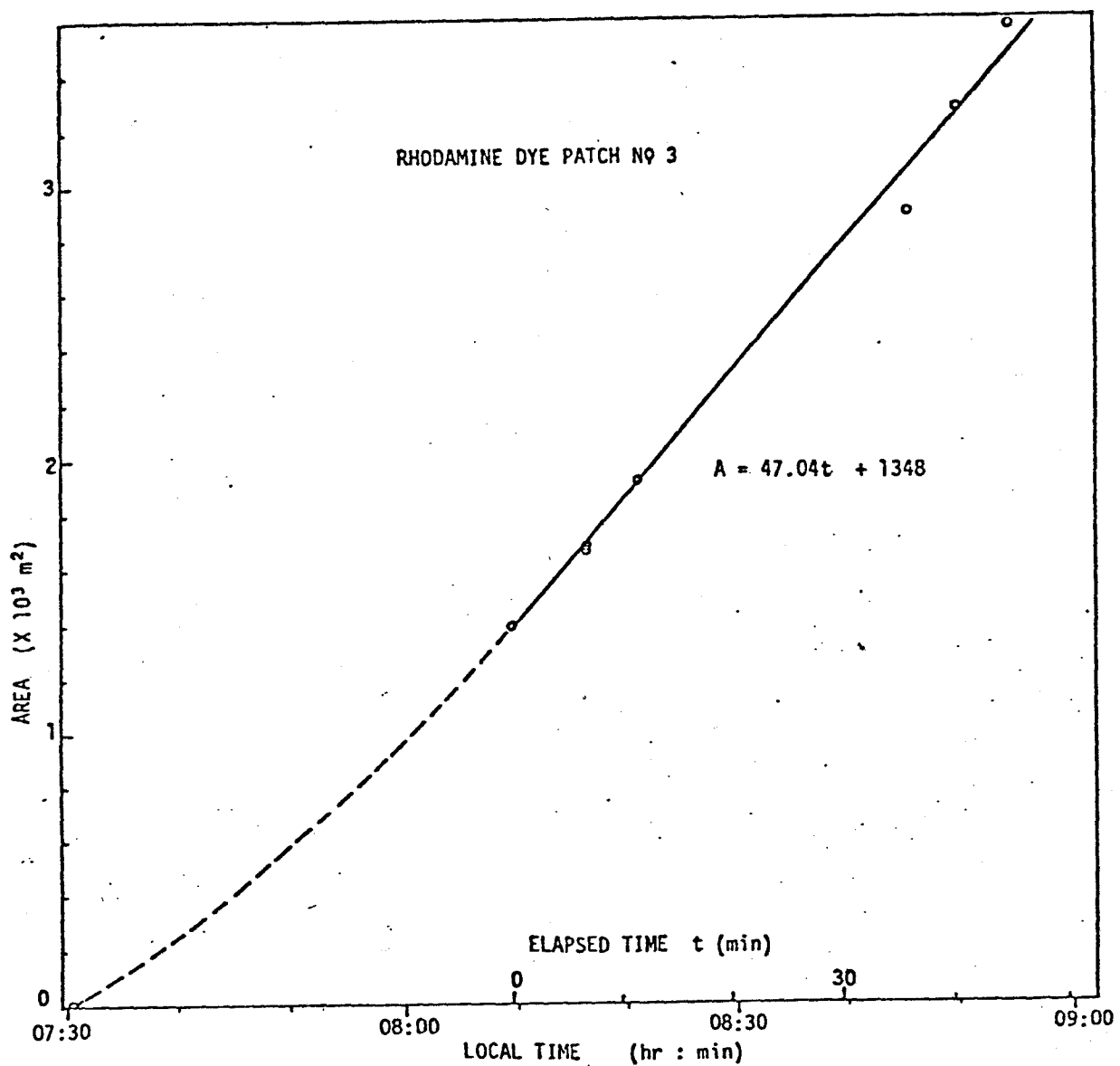


Fig. 3 - Dispersion diagram of dye patch inside Piraquara de Fora Bay.

SOURCE: Stevenson and Inostroza (in press).

In order to compare the actual dye patch data with output from the model, it was necessary to obtain horizontal cross-sectional profiles of concentration (optical density) from the film images. Two orthogonal cross-sectional profiles were considered adequate for comparison. The cross-sectional profiles were made using the automatic interactive imaging system (IMAGE-100) located at INPE (SJC). A vidicon camera first

scanned each film placed on a viewing table beneath the camera. The system then digitized (8 bits resolution equals 256 gray levels) the image into an equivalent image consisting of 512 scanlines, with each scanline composed of 512 picture elements (pixels). The digital image was then stored in one of the interactive channels of the IMAGE-100. Considering the optical characteristics of the vidicon camera, together with the film, one pixel is equivalent to 0.25m^2 at sea level. A digital printout was then obtained for each film image from which the orthogonal cross-sectional profiles were extracted.

The digital values appeared to contain considerable variation between pixels: a combination of electronic noise, nonuniformities in the film and small scale natural turbulence within the dye patch. The numerical model does not consider a variable scale length, so the profile data were smoothed using a 5-point cosine filter on interior points and a 3-point filter for the end points.

2.2 - NUMERICAL SIMULATION

Equation 7 was used to simulate two-dimensional distributions of a dye patch. First a 21×21 equispaced grid (441 points) was specified with a gridpoint separation of 5m. Next values of U and V, estimated from the time displacements of the dye patch images, were determined and entered into the program of a microcomputer along with the diffusion constant K and the dye mass M. Values of concentration (S) were then computed for the X,Y gridpoints using a specified value for time (t). If not otherwise specified, the program initially set time at 10 minutes after dispersal and after computing all of the gridpoints, the time was incremented by 10 minutes, and so on. Because the simulated dye patch is radially symmetric, the resulting isolines of concentration form circles of equal concentration.

General boundary conditions for our numerical model included:

- 1) An initial dye mass of 95g, the same as that used in the field experiment.

- 2) An initial time of 37 minutes (2220 seconds), which corresponded to the first aerial film. A final time of 83 minutes was used, which corresponded to our final film image.
- 3) A constant diffusion coefficient of $7.8 \times 10^3 \text{ cm}^2 \text{ s}^{-1}$, determined by independent evaluation of film images of the dye patch.
- 4) Mean U and V components, determined from sequential aerial images of the dye patch.
- 5) X,Y origin taken as the location of dye release.
- 6) Value of X,Y at t=37 minutes, determined from aerial images of dye patch.
- 7) The numerical simulation was limited to an area of 100m X 100m because the dye patch area, in the first 2-3 hours, was known to be of this order of magnitude.

In practice, the program computed the dye concentration point-by-point, starting with the upper righthand corner of the spatial array and moving to the left, along the uppermost line. Upon completion of a line, the program shifted down to the next line and commenced from right to left, until the end of the array. The program printed a list of gridpoints identified by X,Y position along with the dye concentration. These data were then manually plotted and contoured at intervals of $0.5 \times 10^{-7} \text{ g cm}^{-2}$.

3. RESULTS

3.1 - CROSS-SECTIONAL PROFILES

After digitization of film images of the dye patch, horizontal cross-sections were constructed for t=37 minutes (Figures 4 and 5) and t=83 minutes (Figures 6 and 7). Both raw and smoothed profile data are included to give the reader an indication of noise in the data.

The abscissa axis shows either range in pixel number or scanline number across the profile. The image intensity (ordinate axis) is given as the complement of the maximum value, so that the resulting range increases with increase in dye concentration. For the pixel scanline scale, 20 units equal 5m, the gridpoint spacing in the model. The increase in size of the dye patch with time is readily seen when comparing the profile widths of Figures 4 and 5 with those of 6 and 7.

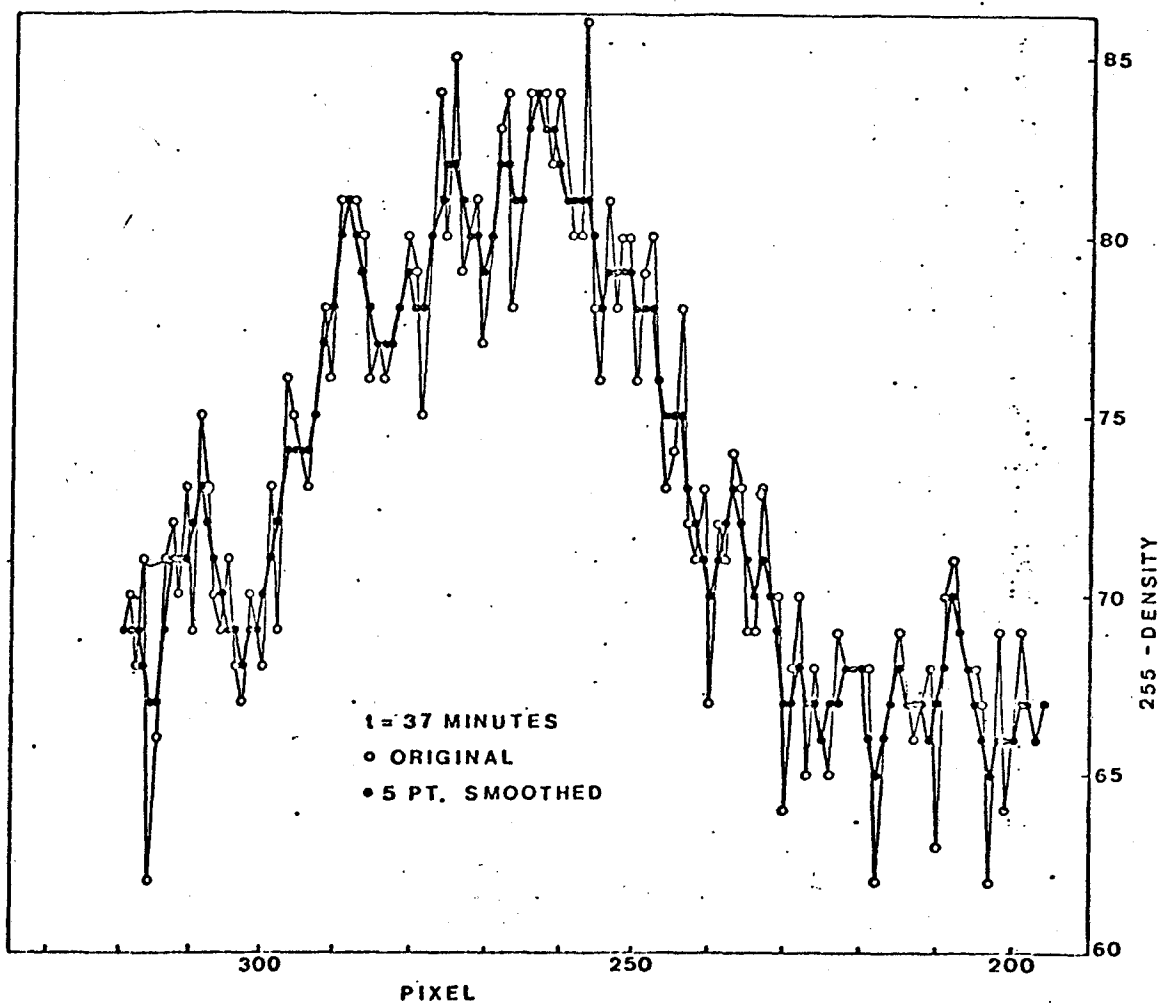


Fig. 4 - Cross-sectional profile of relative dye concentration, photo-digitized from film image taken 37 minutes after dye dispersal.

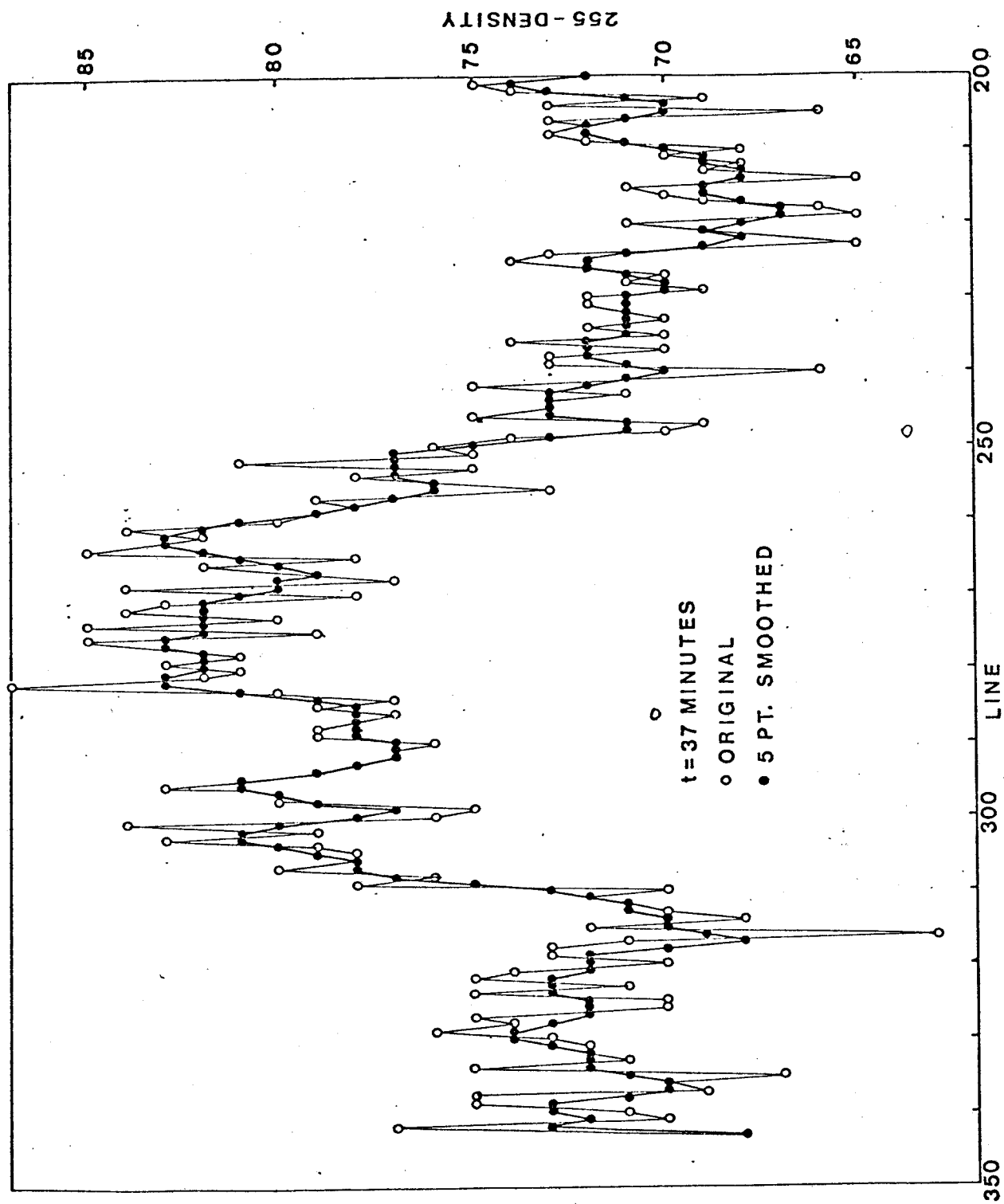


Fig. 5 - Cross-sectional orthogonal profile of relative dye concentration, photodigitized from film image taken 37 minutes after dye dispersal.

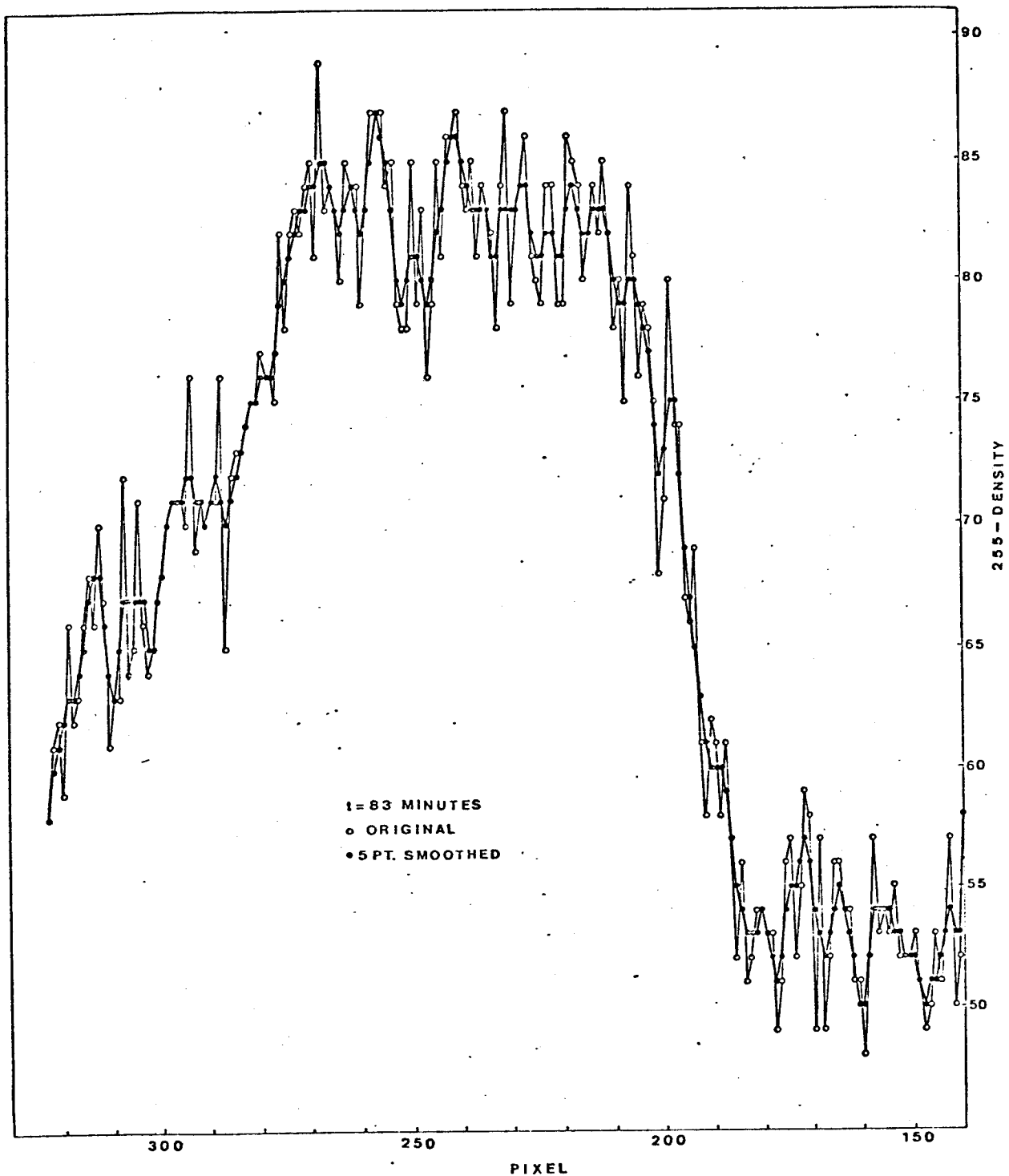


Fig. 6 - Cross-sectional profile of relative dye concentration, photo-digitized from film image taken 83 minutes after dye dispersal.

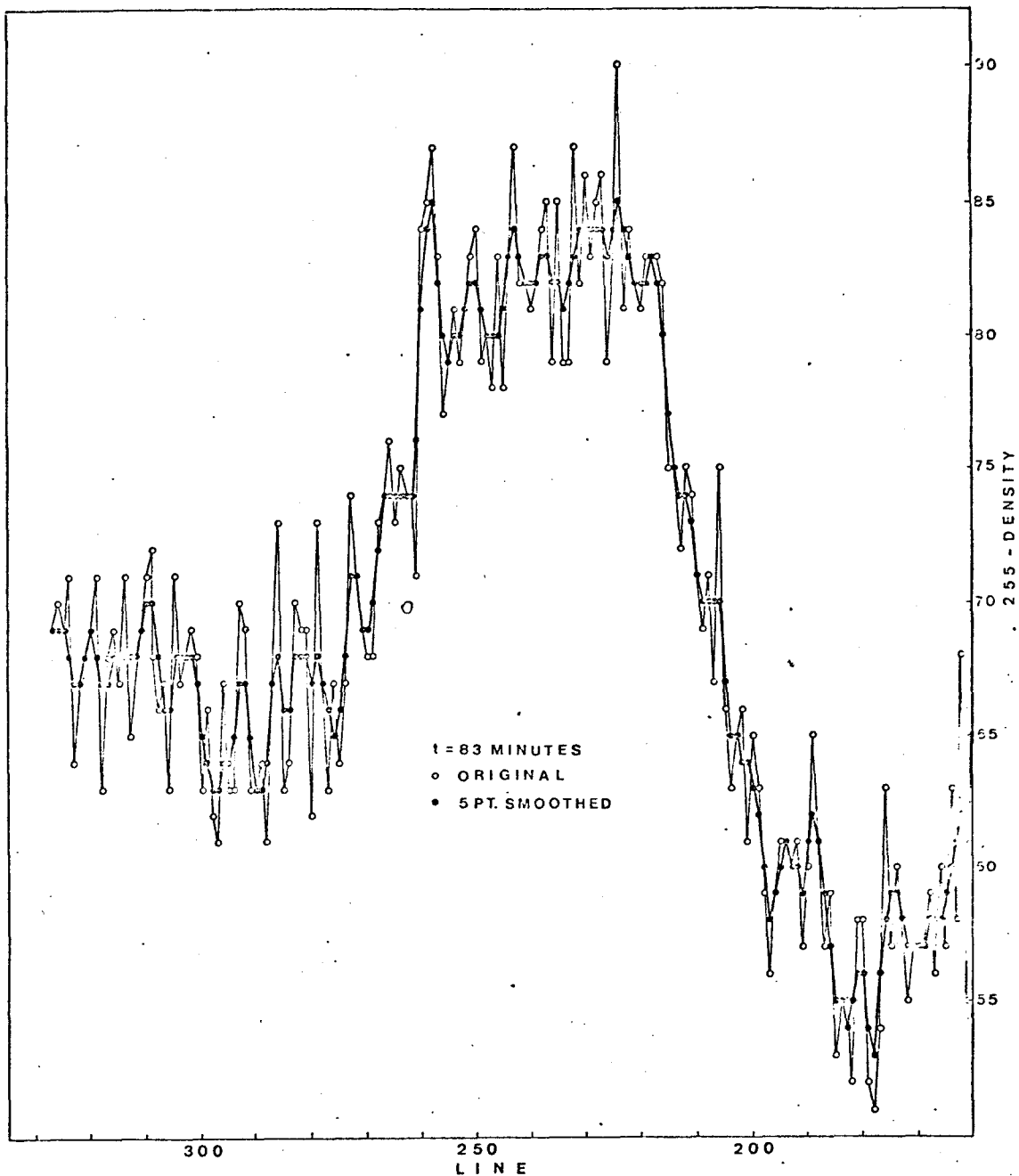


Fig. 7 - Cross sectional orthogonal profile of relative dye concentration, photodigitized from film image taken 83 minutes after dye dispersal.

3.2 - NUMERICAL SIMULATION

Numerical simulations were made for three times: $t=37$ minutes (Figure 8), $t=44$ minutes (Figure 9) and $t=83$ minutes (Figure 10). The exponent of the dye concentration is 10^{-7} , with units of gm cm^{-2} . For

convenience, unit thickness of the dye patch is used, since the film images vertically integrate the dye concentration. The lower parts of Figures 7-10 provide the horizontal cross-sectional profiles passing through the center of the patch. The three model figures are to the same scale; it is informative to note the change in concentration for $t=37 \rightarrow 83$ minutes.

3.3 - COMPARISON OF FIELD AND MODEL DATA

The final part of the study was the comparison of the diffusion model with data from the field experiment (Figures 11-14). The dashed lines represent the cross-sectional profiles determined by the model (Figures 8 and 10). The dye concentration determined by the model (lefthand scale) is for unit depth. In reality the dye was distributed in a surface layer of at least 10cm, but lack of profiling capability during the field work limited our ability to determine the vertical extent of the mixing. The model concentrations can be divided by the depth of mixing, when this depth is known. At any rate, the dye patch, as shown in the film images, is not sensitive to the vertical scale because the film integrates light from the different depths.

The field data for $t=37$ and 83 minutes shown in Figures 11-14 are the same as that shown in Figures 4-7, respectively. Some asymmetry in the field data profiles are seen when they are compared with the model profile. The adjustment of the two vertical scales was empirically made with consideration given to match the peak concentrations from the paired profiles. Another factor, that affects how closely the two profiles fit, is where the cross-sectional profiles were taken across the experimental patch images. Because the dye patch images are oval in shape, while the model patches circular, we expect some error when fitting the two sets of curves.

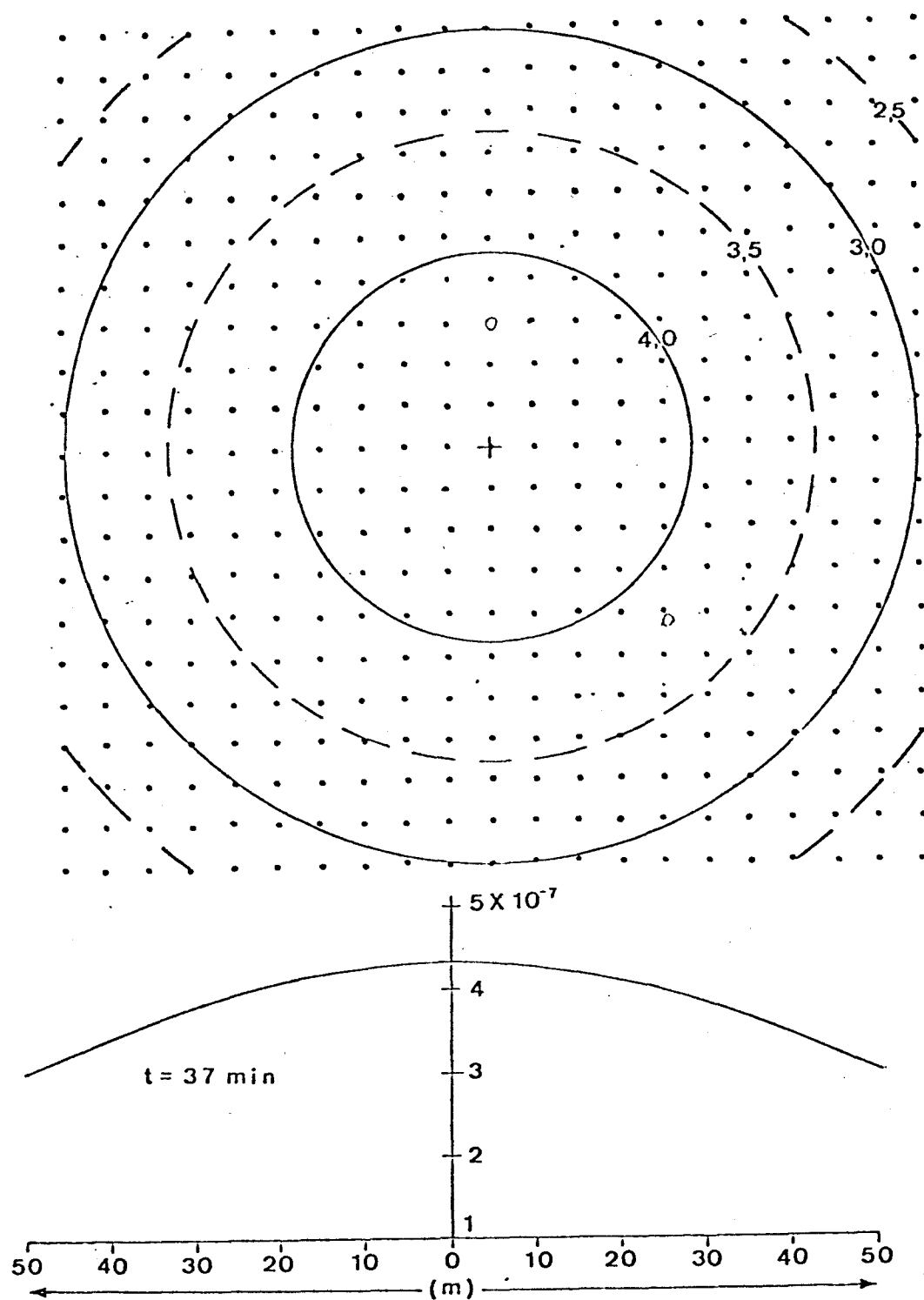


Fig. 8 - Numerical simulation of dye concentration in X,Y plane, using equation 7 of the text, 37 minutes after dispersal. Upper Panel: concentration is in units of $\times 10^{-7} \text{ g cm}^{-2}$. Lower Panel: Cross-sectional profile through center of upper panel.

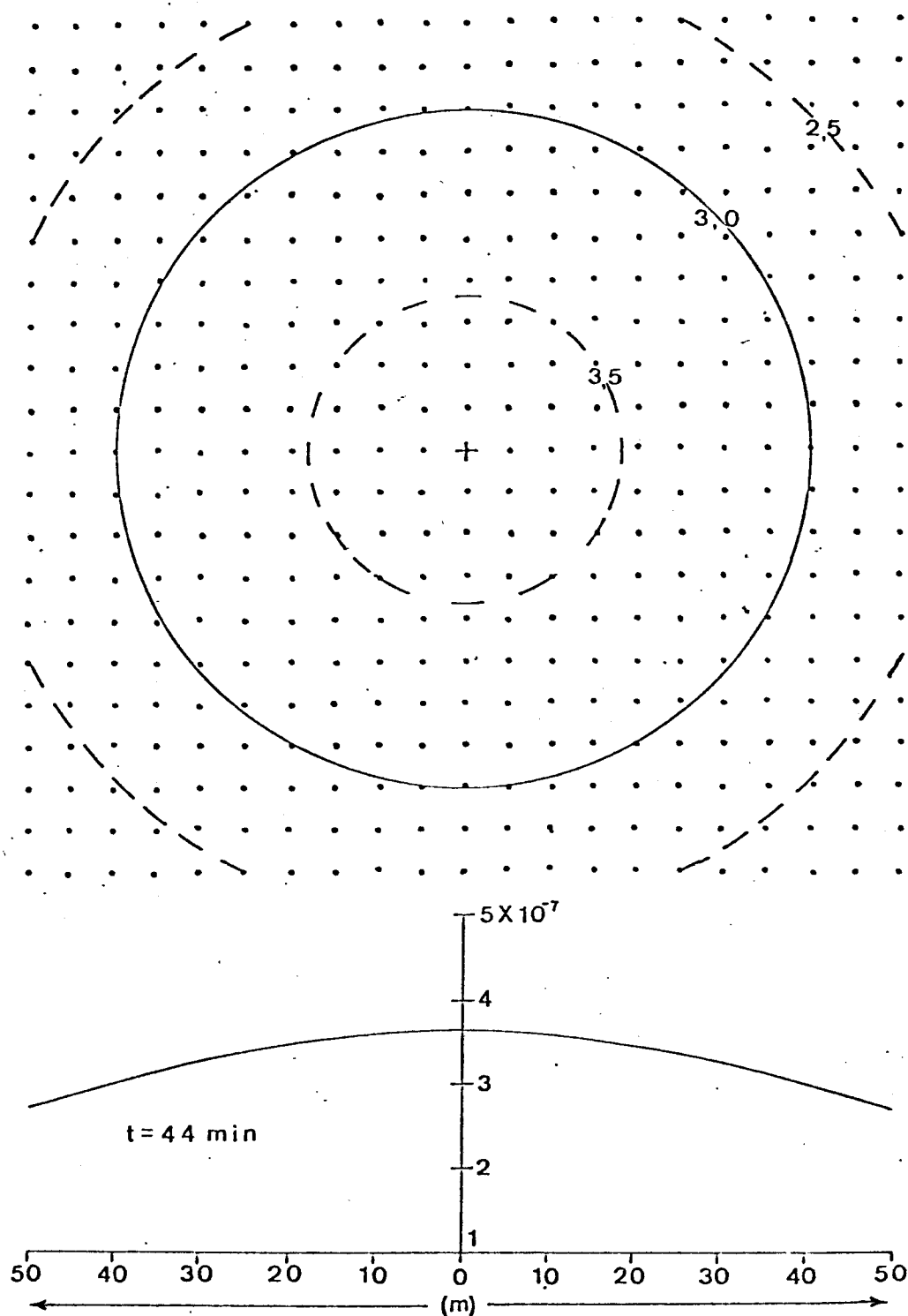


Fig. 9 - Numerical simulation of dye concentration in X,Y plane, using equation 7 of the text, 44 minutes after dispersal. Upper Panel: concentration is in units of $\times 10^{-7} \text{ g cm}^{-2}$. Lower Panel: Cross-sectional profile through center of upper panel.

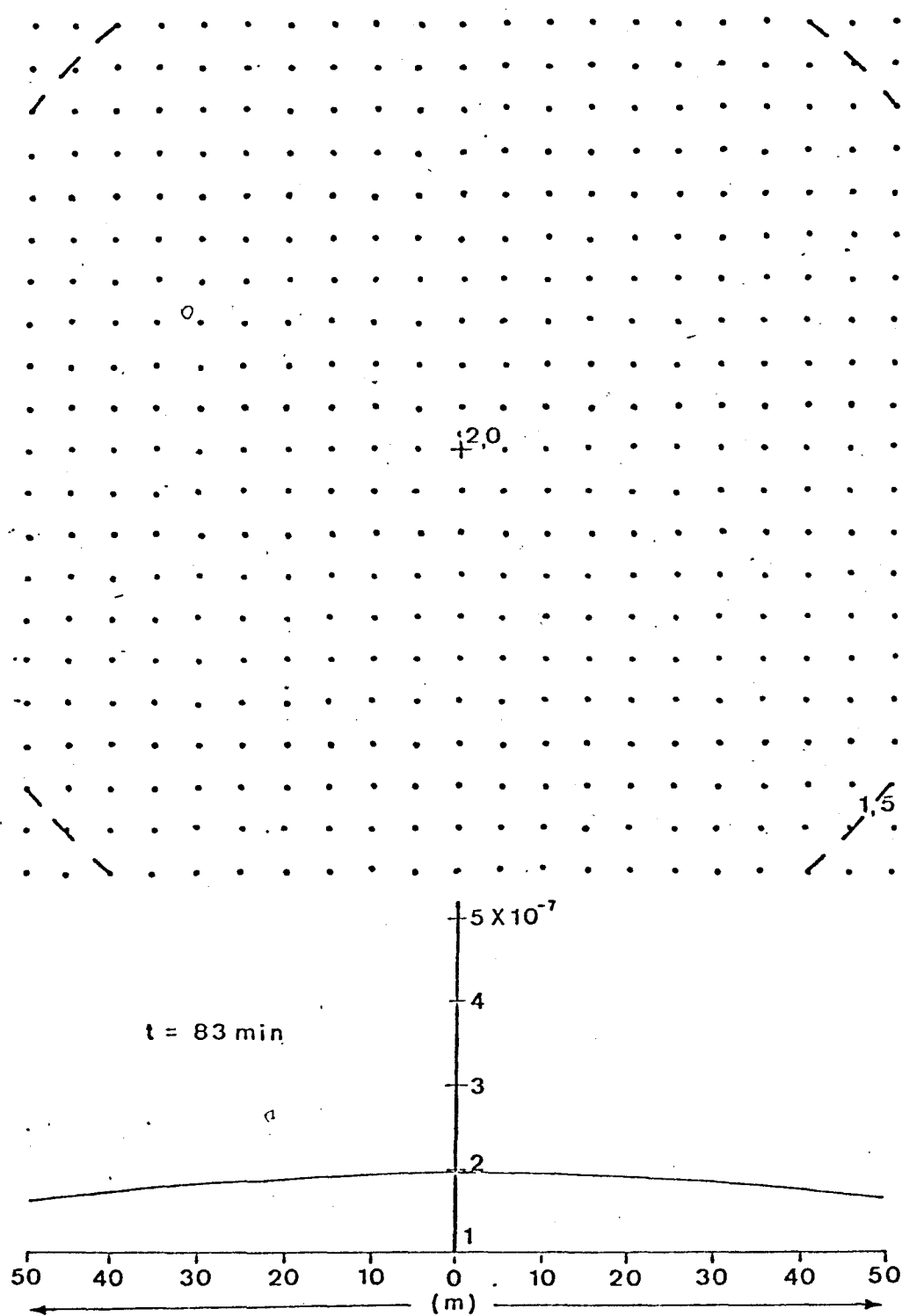


Fig. 10 - Numerical simulation of dye concentration in X,Y plane, using equation 7 of the text, 83 minutes after dispersal. Upper Panel: concentration is in units of $10^{-7} \text{ g cm}^{-2}$. Lower Panel: Cross-sectional profile through center of upper panel.

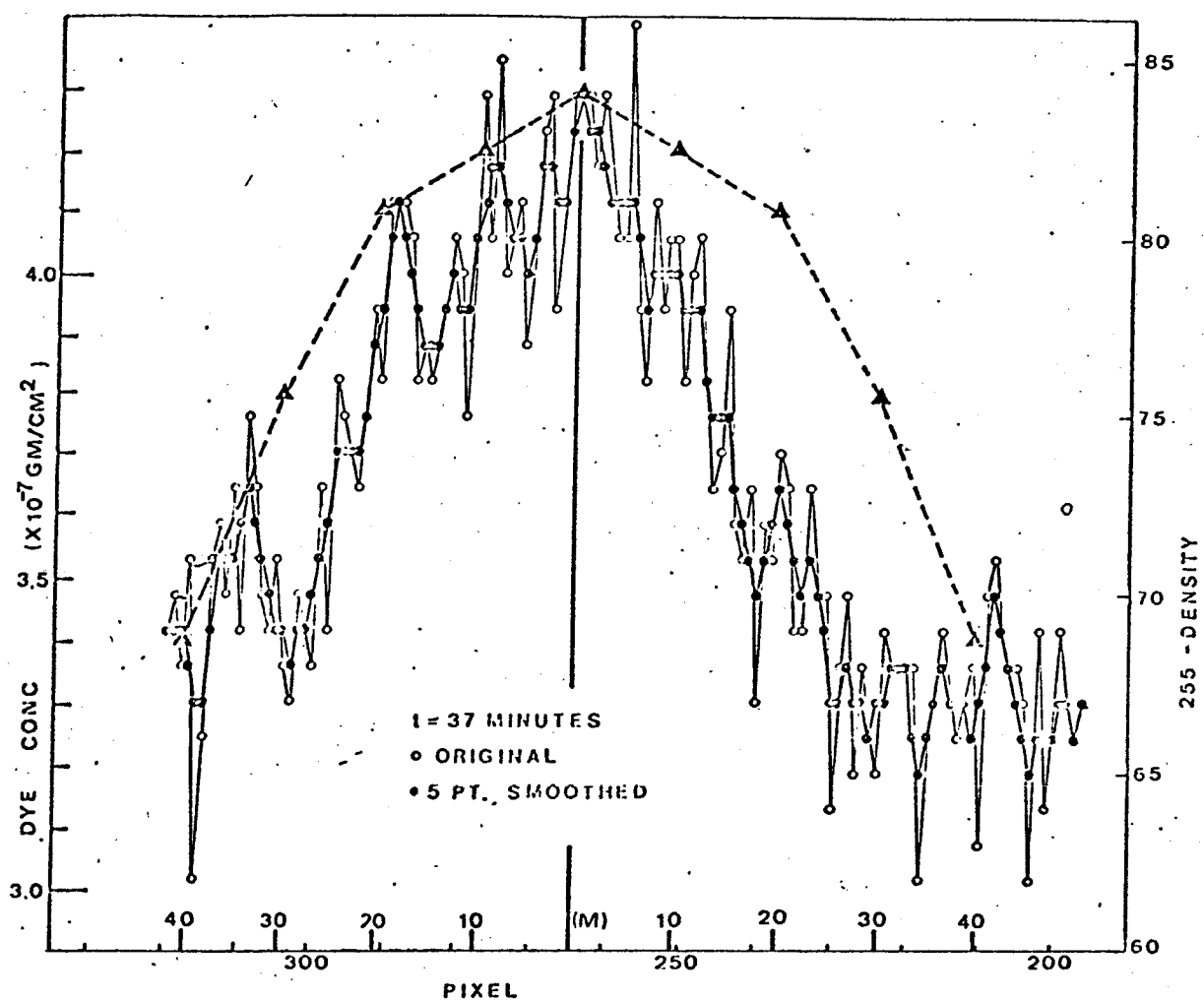


Fig. 11 - Superposition of cross-sectional profiles from field data (Figure 4) and model (Figure 8), shown here by dashed line.

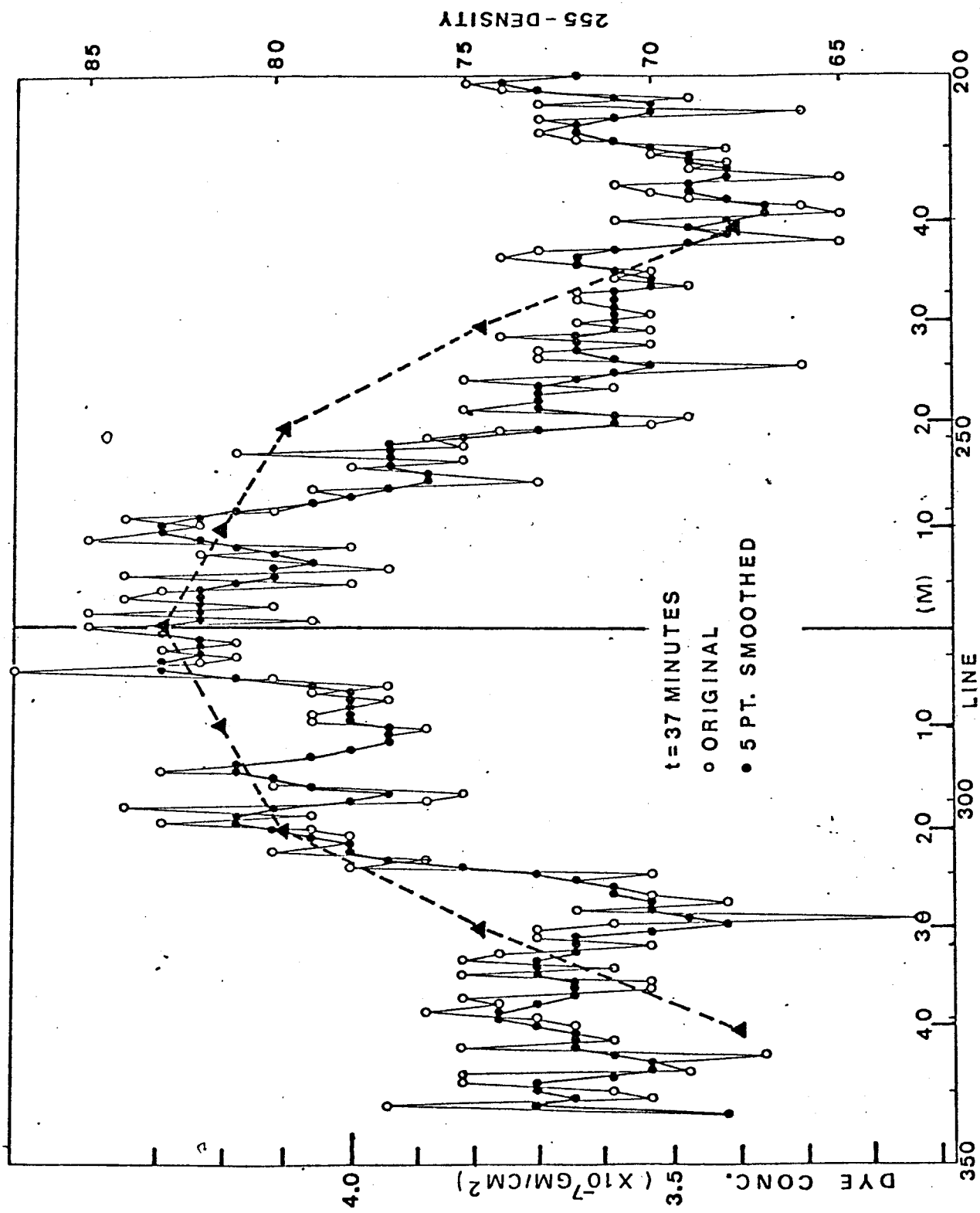


Fig. 12 - Same comparison as for Figure 11, except for orthogonal profiles of Figures 5 and 8.

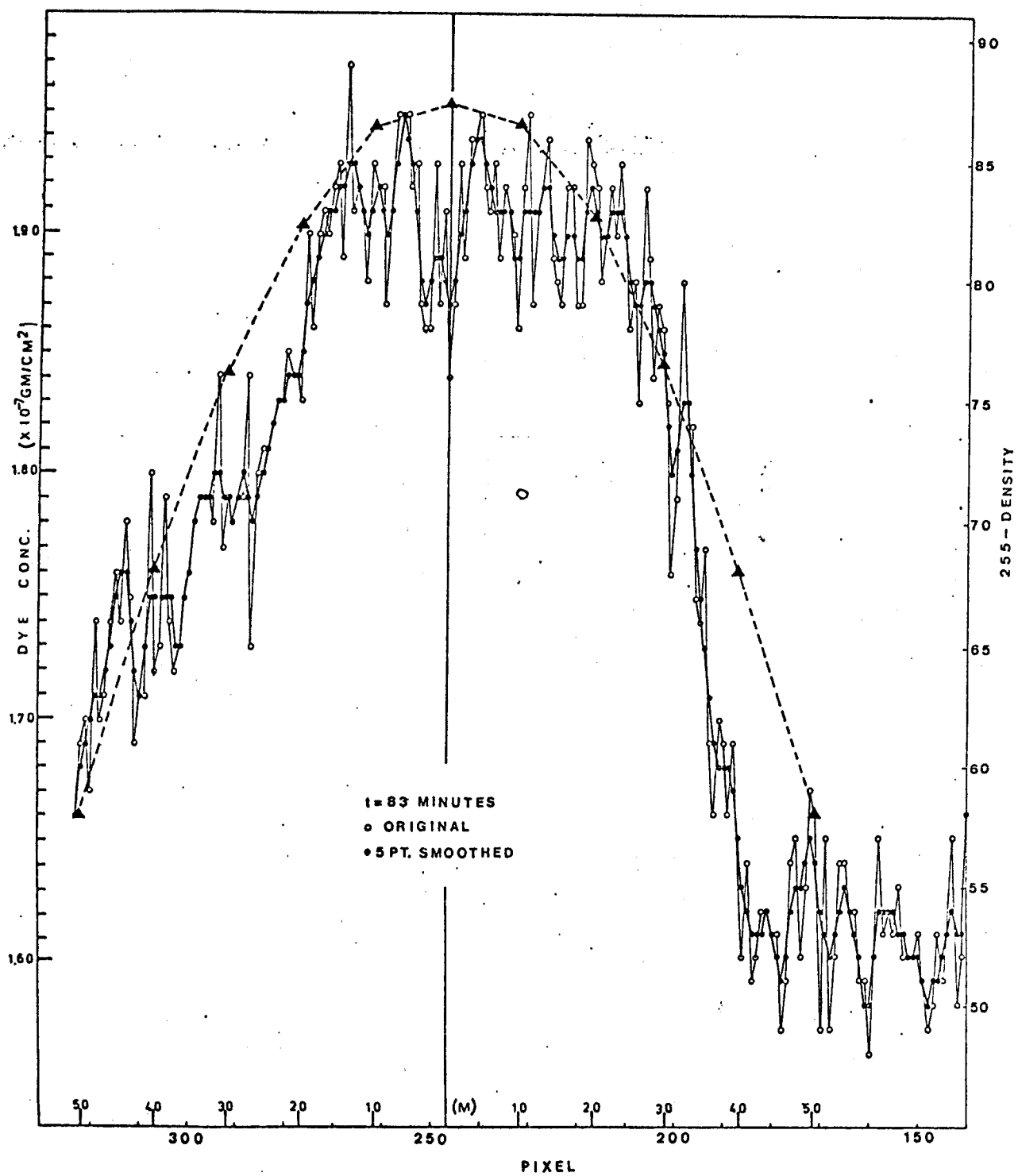


Fig. 13 - Superposition of cross-sectional profiles from field data (Figure 6) and model (Figure 10).

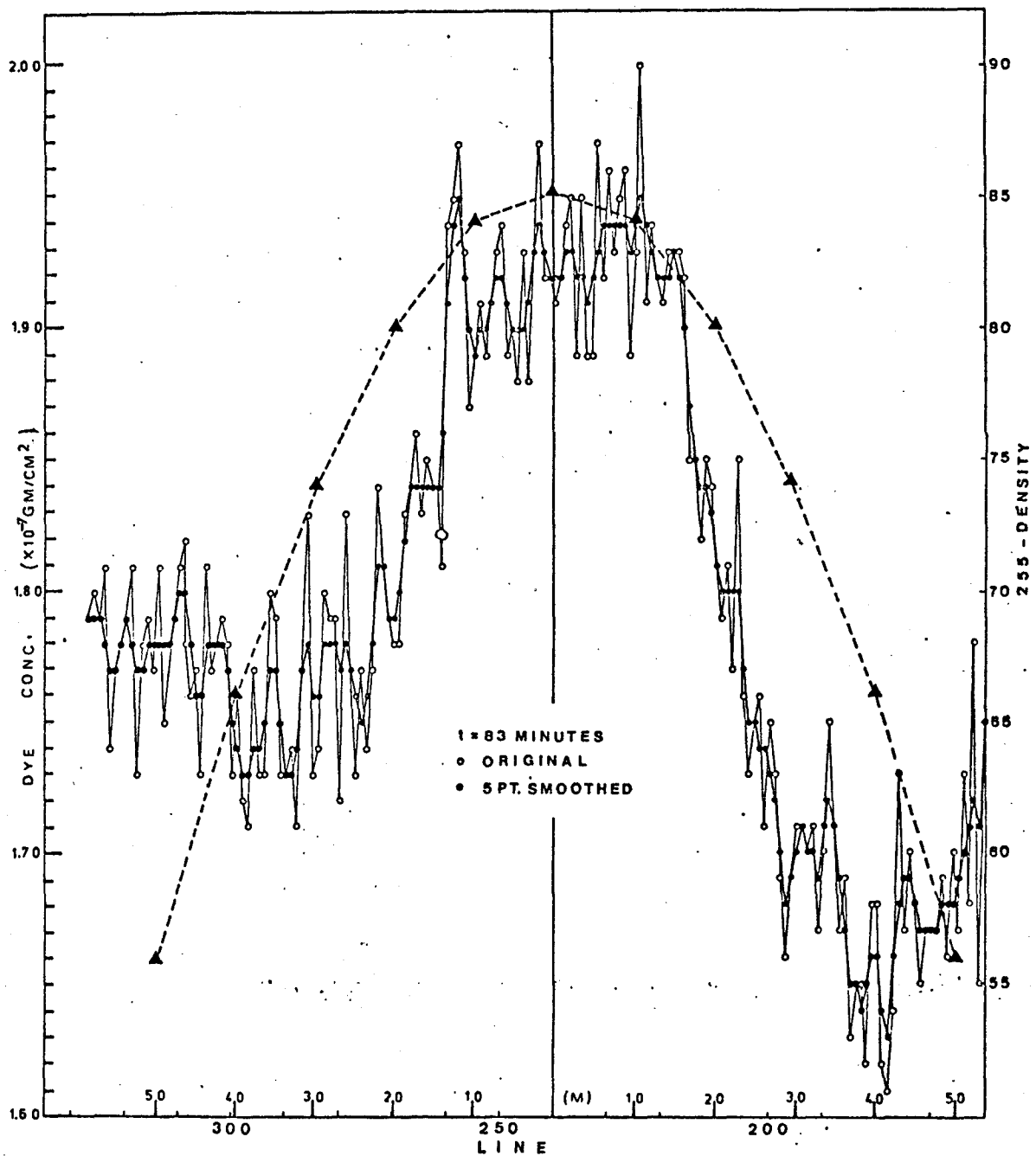


Fig. 14 - Same comparison as for Figure 13, except for orthogonal profiles of Figures 7 and 10.

When the approximations and simplifications of the model are considered, however, the comparison shows a reasonable superposition. One modification of the model, not attempted in this study, is to use different values of K , that is $K_x \neq K_y$. Subsequent rotation of the model distribution, with the larger K value aligned with the greater dimension of the experimental dye patch, would produce dye distribution ellipses even more similar to the observed dye geometry.

4. CONCLUSIONS

- 1) We conclude from this study that it is both practical and very desirable to use numerical simulation of dye patches along with actual field experiments in order to more effectively understand mixing processes and to determine distributions of certain properties of water.
- 2) A better comparison between model and field data would have resulted if the model had considered separate diffusion coefficients: $K_x \neq K_y$, rather than an overall value for K . Separate values for K_x and K_z should be attempted in future work.
- 3) A more realistic comparison would have also resulted if current shears were included in the diffusion model. Because current shear data were not available from the field experiment, it did not seem realistic to consider this refinement at the initial stage of model evaluation. The authors plan to include current shear in a revised model in the near future, using data from a different dye diffusion experiment.
- 4) The present study supports the idea that, while film images of dye patches taken from aircraft provide excellent horizontal distribution for dye concentration, *in situ* measurements of dye concentration are also required to better define absolute concentrations, and more generally the physical processes present in the vicinity of the diffusing patch.

5. ACKNOWLEDGMENTS

The field experiment was supported through a contract between CNPq/INPE and CNEN via Contract Nº 170.1.092.12/84. Support for the elaboration of the model and preparation of this report was kindly provided through CNPq/INPE CIRM. Valuable assistance was given by Sr. Nelson Hideo Yokoyama, who operated the vidicon scanner/IMAGE-100 System to convert film images into equivalent digital images.

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