MORPHOLOGICAL DECOMPOSITION OF EXTREME GRAY–LEVEL LOCATION OPERATORS

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CONTENT

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(1/1)

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R. Rector & G. Alexy
The 8085 Book
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(1980)

INTRODUCTION
(1/1)

Extreme gray–level ≡ maximum/minimum gray–level.

Extreme gray–level location is an important issue in image pattern matching.

Mathematical morphology is the appropriate framework to study the construction of operators for extreme gray–level location.

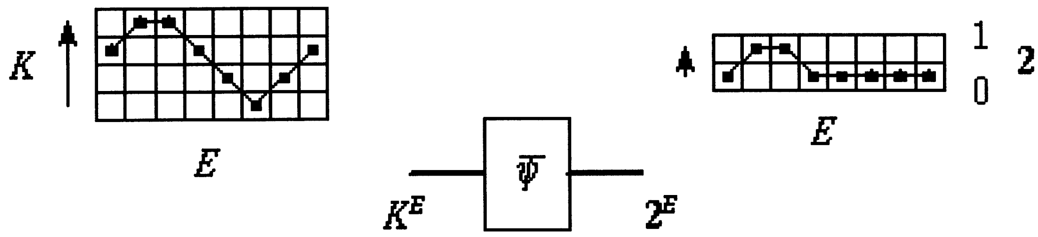
Some implementation aspects are important in terms of computation time.

EXTREME GRAY–LEVEL LOCATION

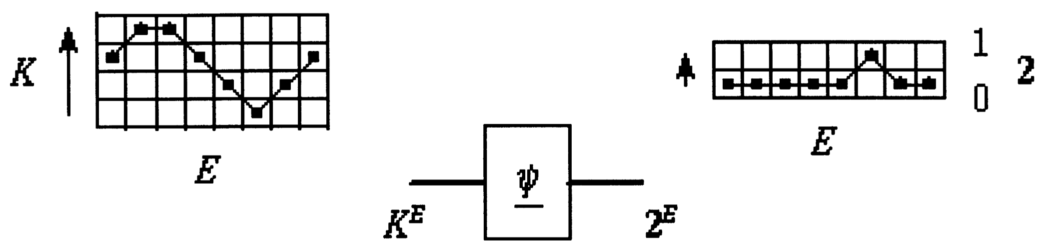
(1/1)

Let E be a nonempty set and
let $(K, \leq)$ be a finite chain of gray–levels.

Let $\overline{\psi}$ be the *maximum gray–level location operator.*



Let $\underline{\psi}$ be the *minimum gray–level location operator.*

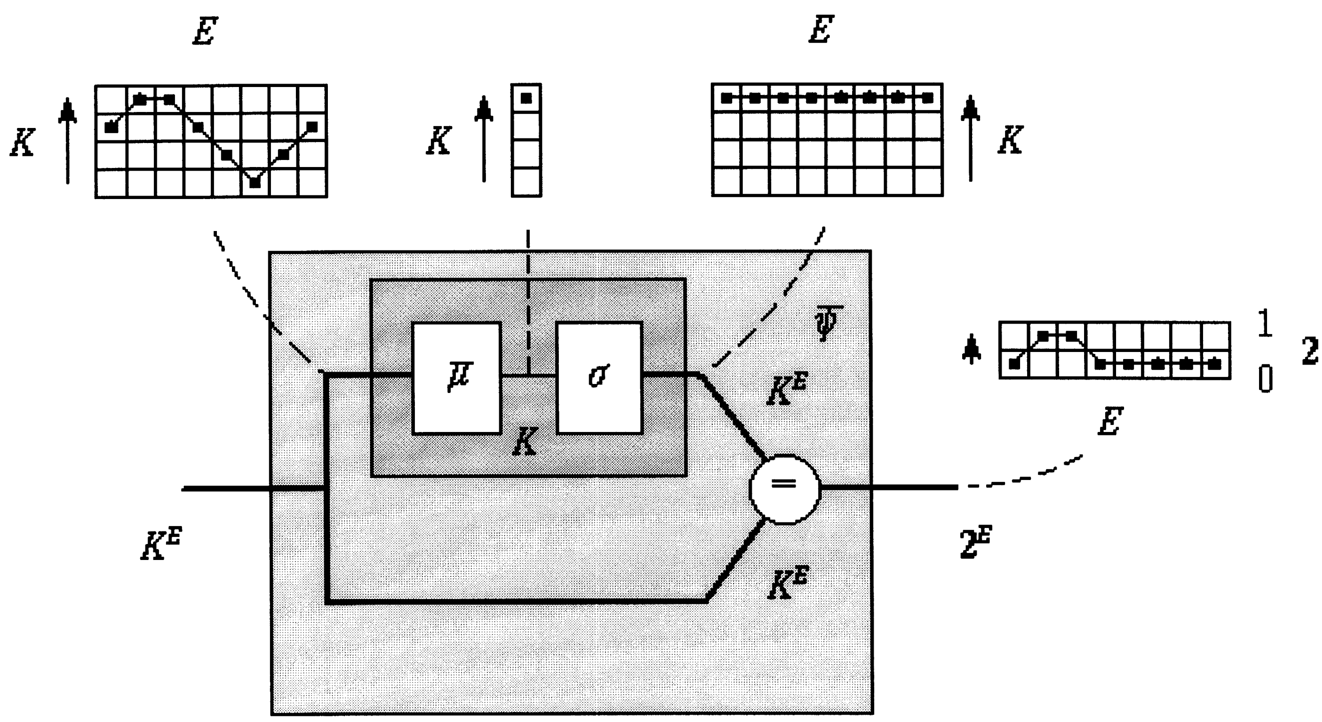


How to construct the operators $\overline{\psi}$ and $\underline{\psi}$?

OPERATOR CONSTRUCTION
(intuitive approach)

(1/2)

Let $\overline{\mu}$ be the *maximum gray–level evaluation measure* and let σ be its Galois compagnon.
Let $=$ be the identity relation between images.

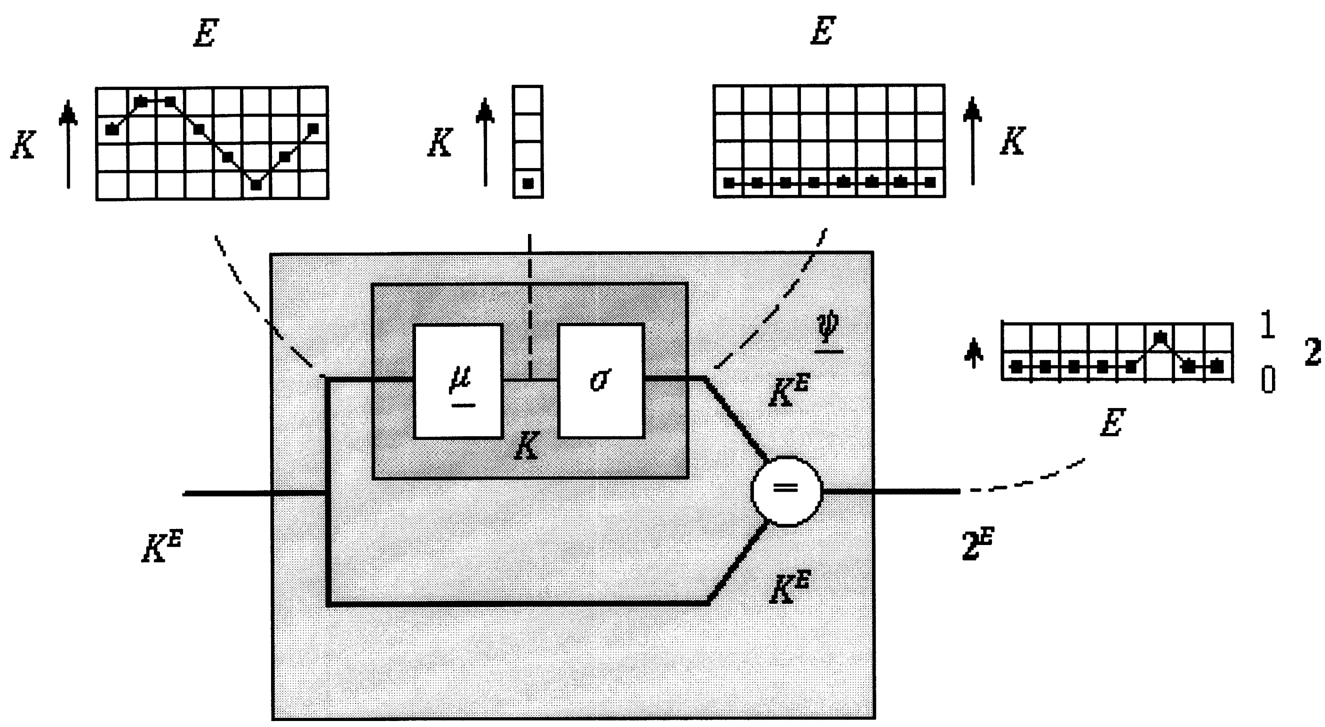


The measure $\overline{\mu}$ is a dilation and
the operator $\sigma \circ \overline{\mu}$ is a morphological closing

OPERATOR CONSTRUCTION
(intuitive approach)

(2/2)

Let $\underline{\mu}$ be the *minimum gray–level evaluation measure* and let σ be its Galois compagnon.
Let $=$ be the identity relation between images.



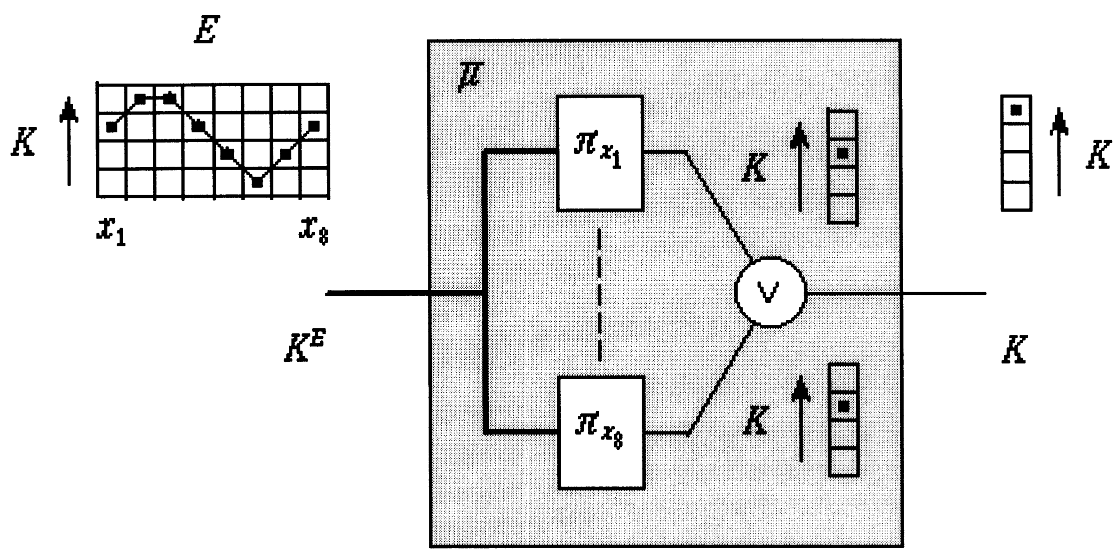
The measure $\underline{\mu}$ is an erosion and
the operator $\sigma \circ \underline{\mu}$ is a morphological opening

How to construct the measures $\overline{\mu}$ and $\underline{\mu}$?

MEASURE CONSTRUCTION (decomposition approach)

(1/2)

Let π_x be the *projection w.r.t x* and let $\vee$ be the union on the chain $(K, \leq)$

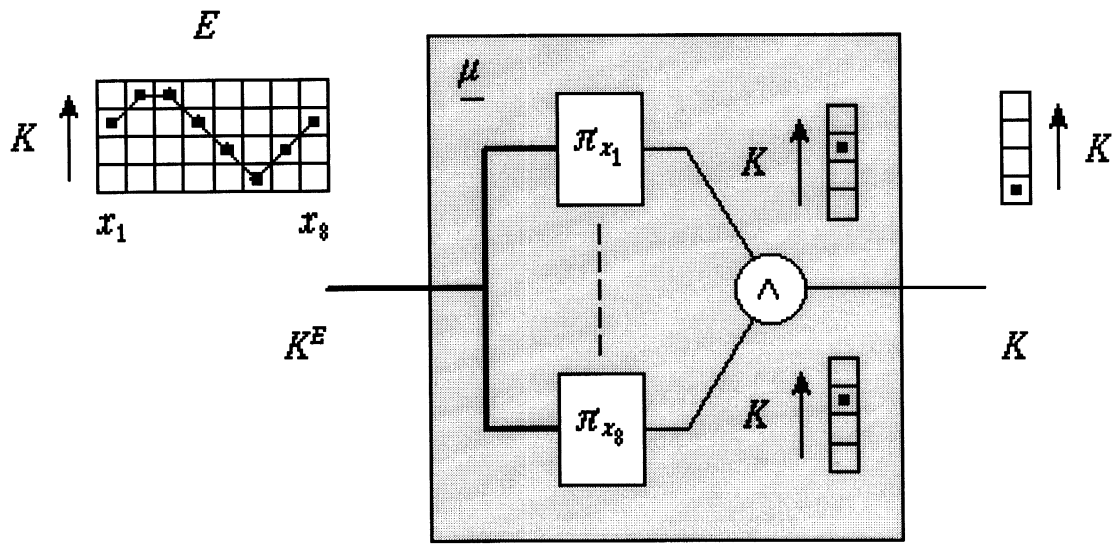


The measures π_x are dilations and erosions and the measure $\overline{\mu}$ is a dilation.

MEASURE CONSTRUCTION
(decomposition approach)

(2/2)

Let π_x be the *projection w.r.t x* and
let $\wedge$ be the intersection on the chain $(K, \leq)$



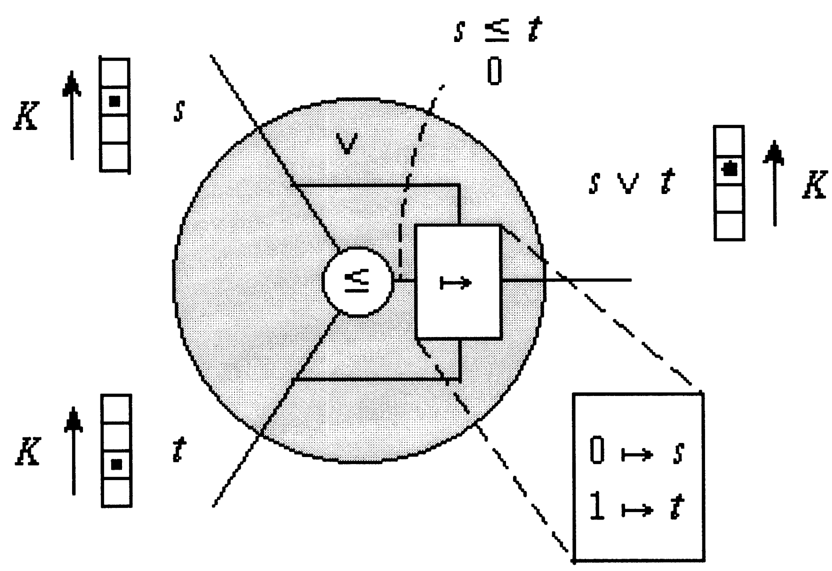
The measures π_x are dilations and erosions and
the measure $\underline{\mu}$ is an erosion.

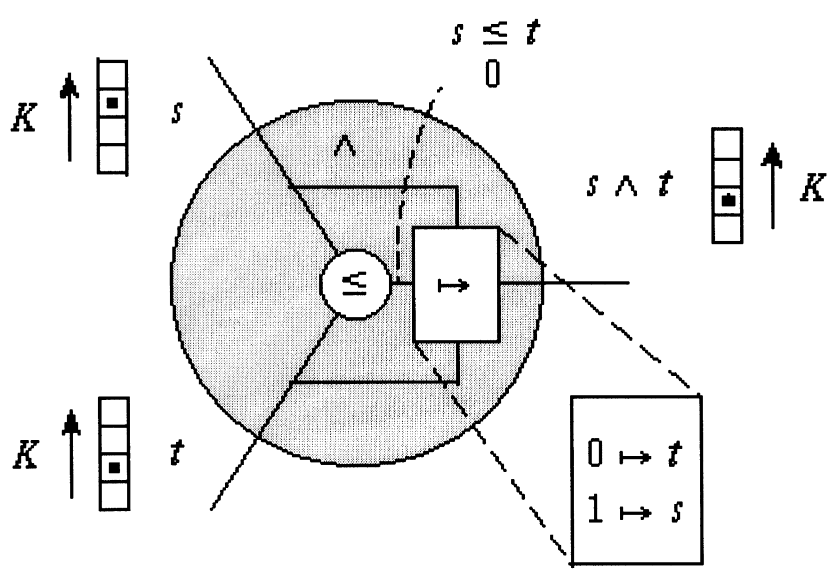
How to construct the operations $\vee$ and $\wedge$?

UNION AND INTERSECTION CONSTRUCTION

(1/7)

Let $(K, \leq)$ be a finite chain of gray–levels and let the order relation $\leq$ assume value 1 when it is true and value 0 otherwise.





UNION AND INTERSECTION CONSTRUCTION

(2/7)

Let $(2^n, \preccurlyeq)$ be the usual computer chain codification of the first integers.
Let $\sqcup$ and $\sqcap$ be the union and intersection on $(2^n, \preccurlyeq)$.

The assembler program for $\sqcup$ on the 8086 is:

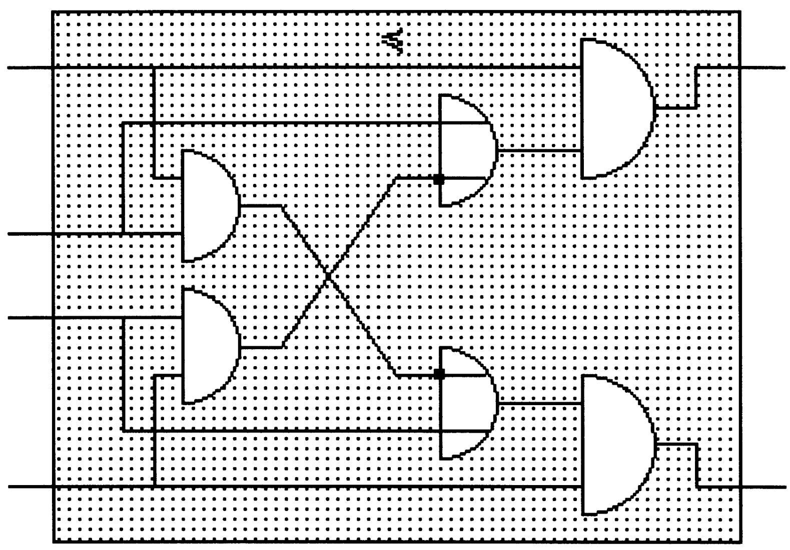
	XOR	AH,AH	
J1	MOV	AL,[BX+SI]	8+EA
	CMP	AH,AL	3
	JAE	J2	4/16
	MOV	AH,AL	2
J2	INC	SI	2
	DEC	CX	3
	JNZ	J1	16
			49 (EA=6)

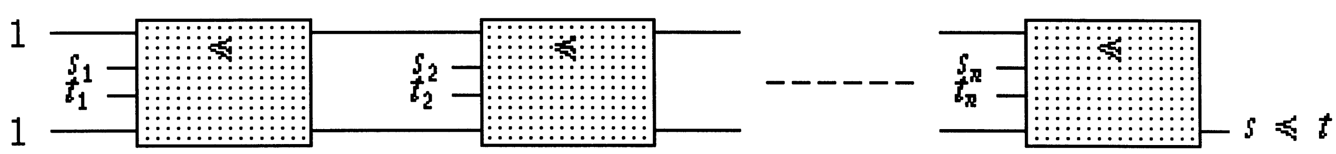
UNION AND INTERSECTION CONSTRUCTION

(3/7)

Construction of $\preccurlyeq$

Let $s = (s_1, ..., s_n)$ and $t = (t_1, ..., t_n)$ be two elements of $\mathbf{2}^n$.



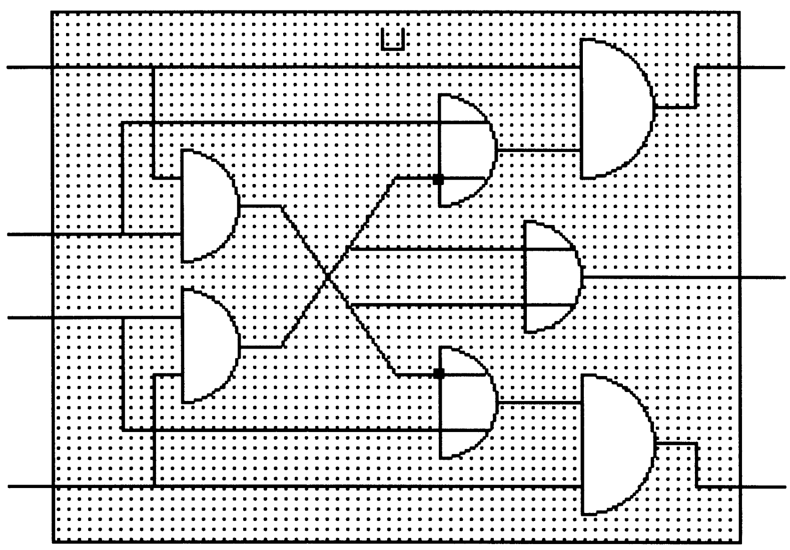


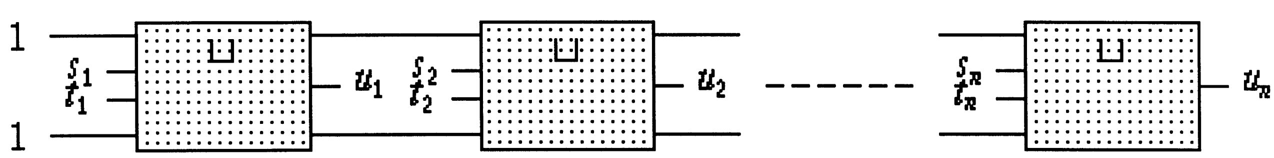
UNION AND INTERSECTION CONSTRUCTION

(4/7)

Improvement for the union $\sqcup$

Let $s = (s_1,...,s_n)$, $t = (t_1,...,t_n)$, and $u = (u_1,...,u_n)$ be three elements of $\mathbf{2}^n$ such that $u = s \sqcup t$.





Let call `IOR` the corresponding computer instruction.

UNION AND INTERSECTION CONSTRUCTION

(5/7)

Improvement for the union $\sqcup$

The assembler program for $\sqcup$ on the 8086 like becomes:

```
          XOR    AH,AH         |
----------------------------------------------
J1        IOR    AL,[BX+SI]    |  9+EA (OR)
----------------------------------------------
          INC    SI            |   2
          DEC    CX            |   3
          JNZ    J1            |  16
                               |  ----
                               |  36    (EA=6)
```

Time cut is:

$$(1 - \frac{36}{49}).100 = 26.5\%$$

(6 hours per day)

UNION AND INTERSECTION CONSTRUCTION

(6/7)

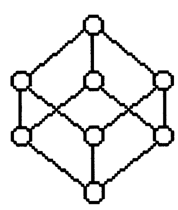
Chain codification (threshold decomposition)

Let $D(\mathbf{2}^n)$ be the set of decreasing elements of $\mathbf{2}^n$.

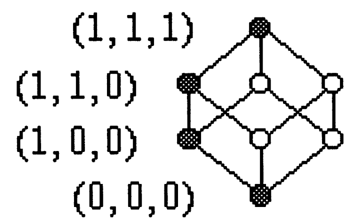
Example:

$$D(\mathbf{2}^3) = \{(0,0,0),(1,0,0),(1,1,0),(1,1,1)\}$$

The subposet $(D(\mathbf{2}^n), \leq)$ of $(\mathbf{2}^n, \leq)$ is a chain.



$(\mathbf{2}^3, \leq)$



$(D(\mathbf{2}^3), \leq)$

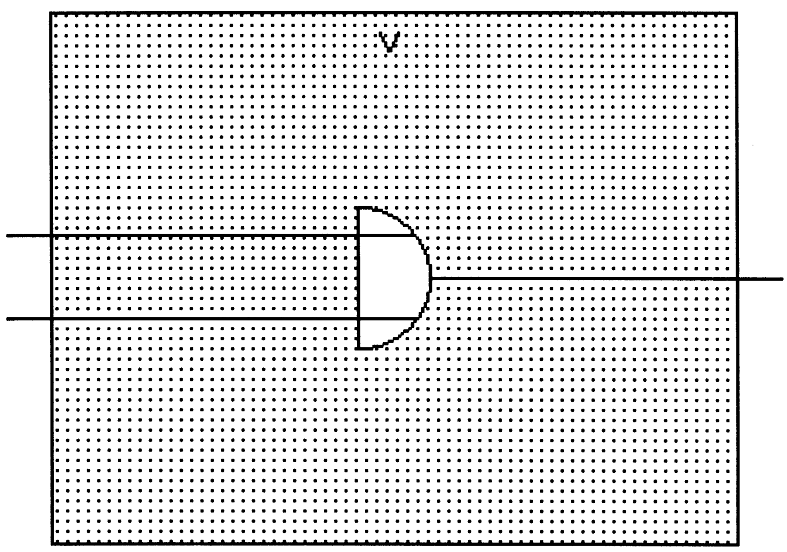
UNION AND INTERSECTION CONSTRUCTION

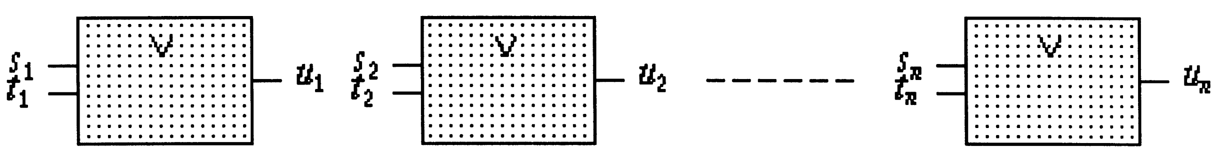
(7/7)

Let $\vee$ and $\wedge$ be the union and intersection on $(\mathrm{D}(\mathbf{2}^n), \leq)$.

Construction of the union $\vee$

Let $s = (s_1,...,s_n)$, $t = (t_1,...,t_n)$, and $u = (u_1,...,u_n)$ be three elements of $\mathrm{D}(\mathbf{2}^n)$ such that $u = s \vee t$.





There is no carry.

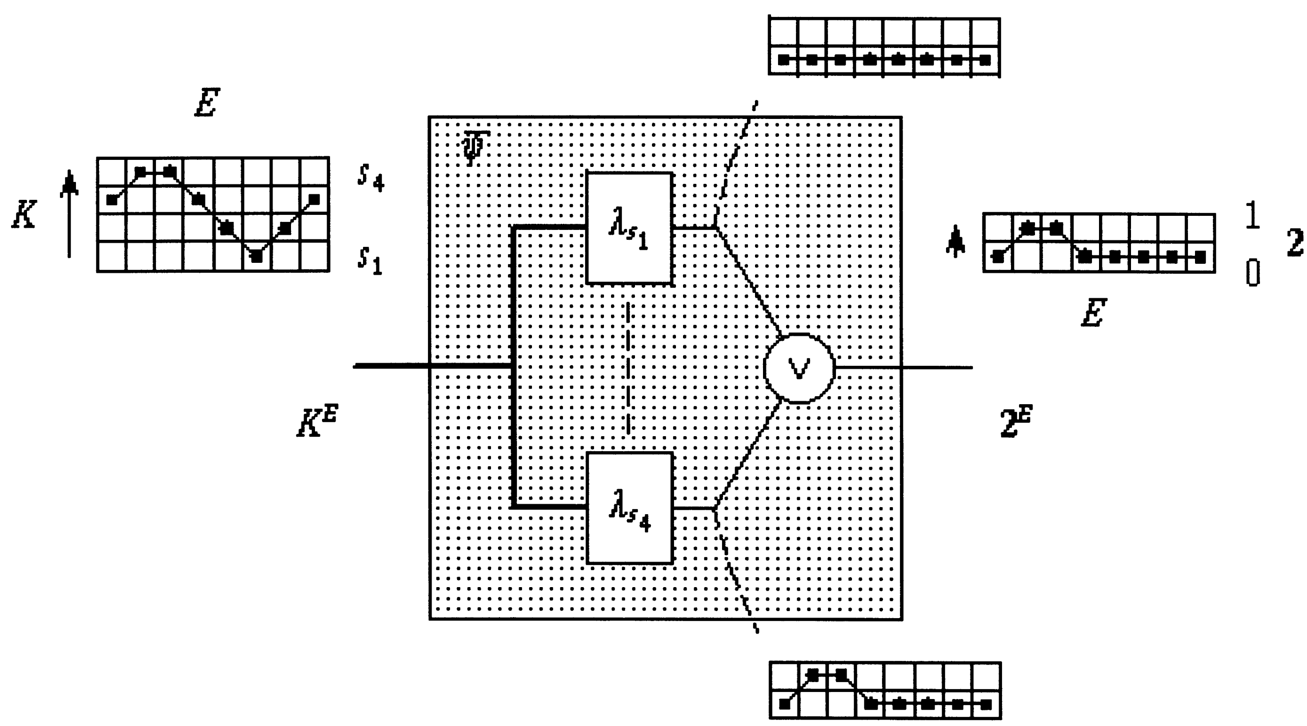
OPERATOR CONSTRUCTION
(decomposition approach)

(1/4)

The *maximum gray–level location operator* $\overline{\psi}$

can be decomposed as a union of sup–generating operators

(Banon & Barrera, 1993).



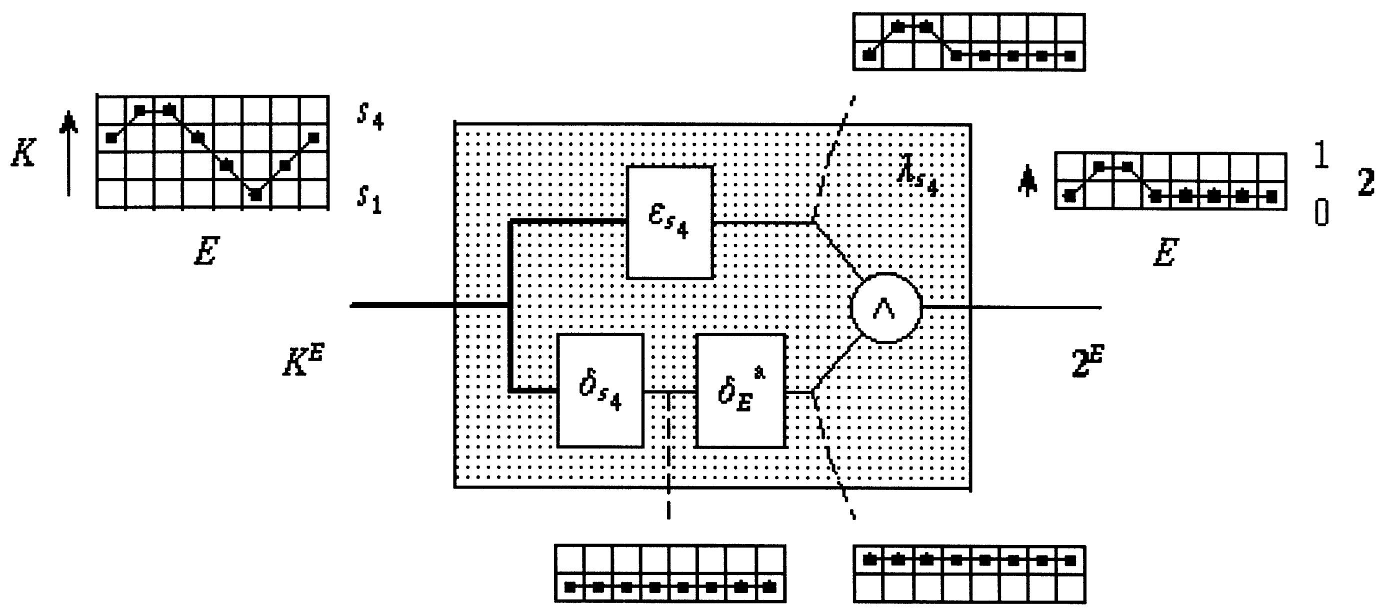
OPERATOR CONSTRUCTION
(decomposition approach)

(2/4)

Let $K = \{s_1, s_2, s_3, s_4\}$ and

let $s_1 \leq s_2 \leq s_3 \leq s_4$.

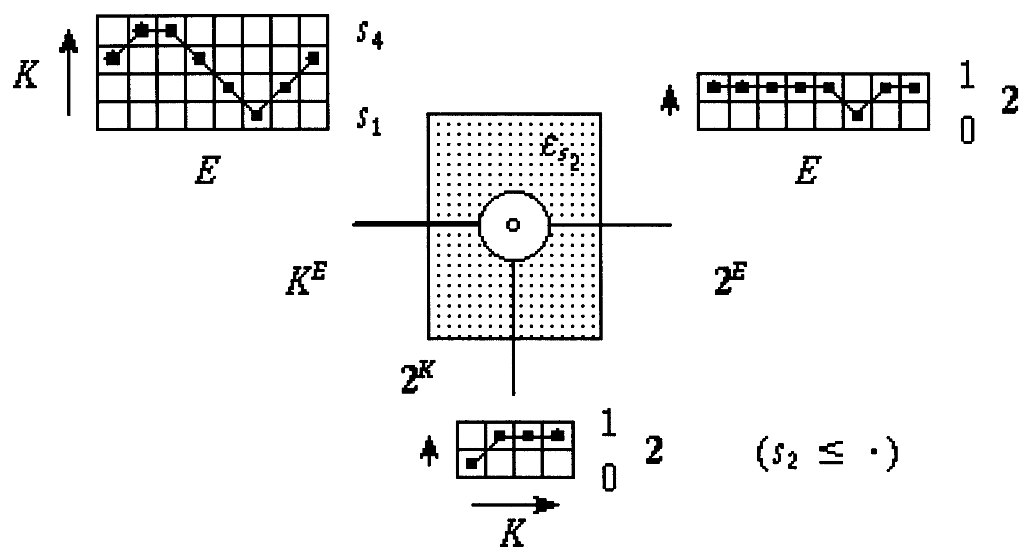
Each of the four sup–generating operators is an intersection of an erosion and an anti–dilation (Banon & Barrera, 1993).

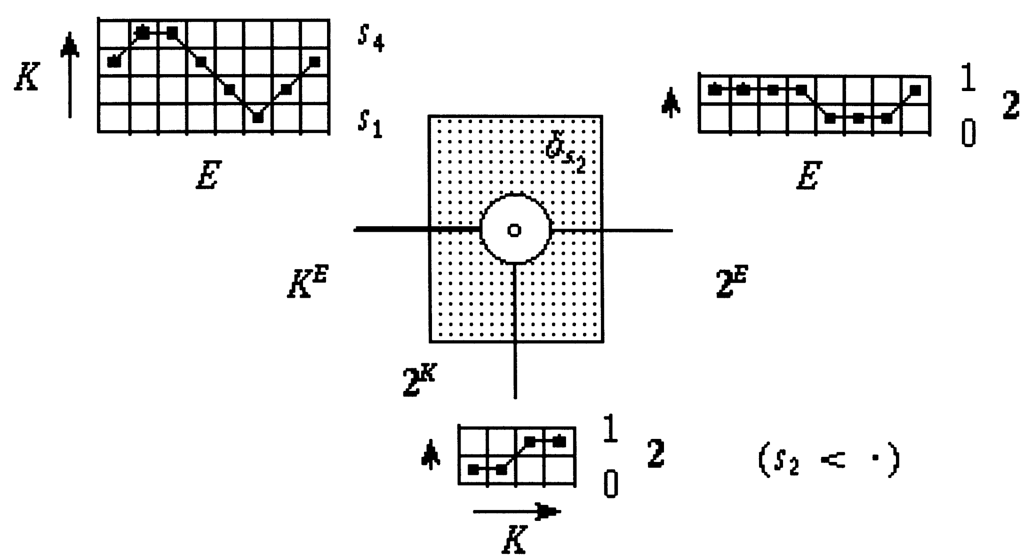


OPERATOR CONSTRUCTION
(decomposition approach)

(3/4)

The erosion and dilation are threshold operators.

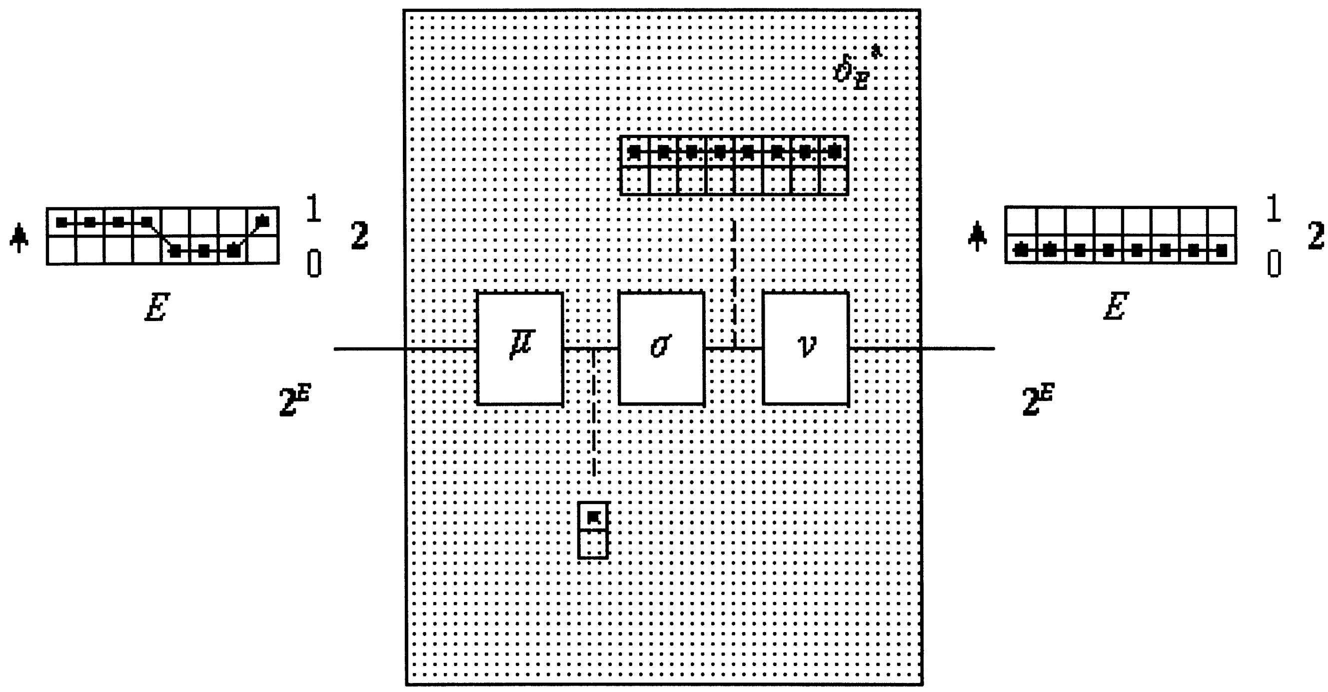




OPERATOR CONSTRUCTION
(decomposition approach)

(4/4)

The anti–dilation is a closing followed by a negation.



CONCLUSION

(1/1)

Some implantation aspects are important in terms of computation time.

$(2^n, \preccurlyeq)$	$(D(2^n), \leq)$	time
CMP		1
IOR IAN	OR AND	.73

The decomposition approach uses implicitly the Threshold Decomposition which may allow a 26.5% time cut on sequential machines as the 8086.