ANALYSIS OF NAVIGATIONAL ALGORITHMS FOR A REAL TIME DIFFERENTIAL GPS SYSTEM

Leandro Baroni
National Institute for Space Research
Av. dos Astronautas, 1758 - CEP 12227-010
São José dos Campos - SP - Brazil
leandrobaroni@yahoo.com.br

Hélio Koiti Kuga
National Institute for Space Research
Av. dos Astronautas, 1758 - CEP 12227-010
São José dos Campos - SP - Brazil
hkk@dem.inpe.br

Abstract. *The Global Positioning System (GPS) is a navigation system based on satellites that allows the user to determine position and time with high precision. The GPS signal is subject to several error sources. The combined effects of these errors at signal propagation cause degradation in the positioning accuracy. However, there are methods to improve the positioning accuracy, like differential GPS (DGPS) and double difference positioning. This work proposes to compare results and accuracies obtained through Kalman filter and least squares for two implementations: differential GPS and double difference. These comparisons include data obtained from static and mobile receivers. These data were collected by two real time Ashtech Z12 geodetic quality receivers.*

Keywords: *GPS, differential GPS, double difference, navigation, real time.*

1. Introduction

In Global Positioning System (GPS), the pseudorange, which is the distance measurement between satellite and receiver, plus signal propagation error effects, is frequently used for estimating position coordinates and clock bias. Unfortunately, there are several errors in measurements which do not allow the GPS to reach the high precision demanded by certain applications (Misra and Enge, 2001). Therefore, the precision to which signal propagation errors are compensated for are extremely important (Parkinson and Spilker, 1996). There are several techniques for treating these errors, like differential GPS (DGPS) and double difference positioning.

This work proposes to compare results obtained through Kalman filter and least squares for two implementations: differential GPS and double difference. These algorithms were tested in two cases: (*i*) static, in which both receivers are kept fixed; (*ii*) dynamic, in which the user receiver is mounted on an aircraft. Both data sets (static and dynamic) were collected by two double frequency and geodetic quality Ashtech Z12 GPS receivers. These data were processed through Kalman filter and least squares algorithms, and results were compared with landmarks, previously surveyed by IBGE, and with the aircraft reference trajectory. Hence, these results allow to assess a position estimation method and to verify the accuracies obtained with these techniques.

2. Mathematical Modelling

Pseudorange is a distance measurement between satellite, at signal departure epoch, and receiver antenna at signal arrival epoch, obtained through signal time travel measuring, and it is affected by some error sources. The mathematical model for pseudorange ρ_u^i between satellite i and receiver u has the form (Parkinson and Spilker, 1996):

$$\rho_u^i = D_u^i + c \cdot (b_u - B^i) + T_u^i + I_u^i + \epsilon_u^i \tag{1}$$

where $D_u^i = |\mathbf{R}^i - \mathbf{r}_u|$ is geometric distance, $\mathbf{R}^i$ is satellite position, $\mathbf{r}_u$ is receiver antenna position, b_u is receiver clock bias, B^i is satellite clock bias, T_u^i and I_u^i are tropospheric and ionospheric errors, ϵ_u^i represents other unmodelled errors and c is light speed, 299792458 m/s. The positioning is made by linearization about the best estimate available for Eq. (1), from which:

$$\Delta\rho_u^i = \begin{bmatrix} -\frac{X^i - \hat{x}_u}{\hat{\rho}_u^i} & -\frac{Y^i - \hat{y}_u}{\hat{\rho}_u^i} & -\frac{Z^i - \hat{z}_u}{\hat{\rho}_u^i} & 1 \end{bmatrix} \Delta\mathbf{x} + \Delta\epsilon_u^i \tag{2}$$

The observable called *single difference* is formed taking the difference of pseudorange measurements between two receivers at a given epoch. Single difference for pseudorange measurement is given by (Misra and Enge, 2001):

$$\begin{aligned} \rho^i_{ub} &= \rho^i_u - \rho^i_b = (D^i_u - D^i_b) + (b_u - b_b) + (T^i_u - T^i_b) + (I^i_u - I^i_b) + (\epsilon^i_u - \epsilon^i_b) \\ &= D^i_{ub} + b_{ub} + T^i_{ub} + I^i_{ub} + \epsilon^i_{ub} \end{aligned} \tag{3}$$

where $(\cdot)_{ub} = (\cdot)_u - (\cdot)_b$.

The satellite clock bias term B_i, which is common for both measurements, is cancelled. The tropospheric and ionospheric terms are differences from corresponding errors at two receivers. The magnitude of these terms depends mainly on separation distance between receivers (baseline). When this distance is small, the tropospheric and ionospheric effects are almost the same and the residuals become negligible, in comparing with errors due to multipath and receiver internal noise. Thus, for a short baseline, the single difference for code pseudorange is simplified to:

$$\rho^i_{ub} = D^i_{ub} + b_{ub} + \epsilon^i_{ub} \tag{4}$$

The relative receiver clock bias term b_{ub} is common to all single difference measurements at each epoch. This term may be cancelled through *double difference* measurements, which are formed by subtracting two single differences referred to two distinct satellites i and j:

$$\rho^{ij}_{ub} = \rho^i_{ub} - \rho^j_{ub} \tag{5}$$

which may be rewrited in:

$$\rho^{ij}_{ub} = (D^i_{ub} - D^j_{ub}) + (\epsilon^i_{\rho,ub} - \epsilon^j_{\rho,ub}) = D^{ij}_{ub} + \epsilon^{ij}_{\rho,ub} \tag{6}$$

In particular, ρ^{ij}_{ub} may be formed by:

$$\rho^{ij}_{ub} = (\rho^i_u - \rho^i_b) - (\rho^j_u - \rho^j_b) \tag{7}$$

2.1 Positioning via DGPS on Position

Essentially this technique applies corrections directly in position coordinates. Two steps are required: first of all, using the raw pseudorange measurements, each receiver (base and user) computes their positions by the same algorithm, in this case, a conventional least squares method. Then the position correction on base receiver is applied to the user receiver

The base receiver is placed at a known landmark, with coordinates $\mathbf{r}_{ref} = \begin{bmatrix} x_{ref} & y_{ref} & z_{ref} \end{bmatrix}^T$ given in WGS84 coordinates. The base position coordinates $\mathbf{r}_b = \begin{bmatrix} x_b & y_b & z_b \end{bmatrix}^T$ is computed by using the raw pseudorange measurements, through the least squares method. Then the correction to the base position are inferred directly by comparison:

$$\delta\mathbf{r} = \mathbf{r}_{ref} - \mathbf{r}_b \tag{8}$$

Therefore the correction $\delta\mathbf{r}$ computed for the base receiver can be applied straightforwardly to the user receiver:

$$\hat{\mathbf{r}}_u = \mathbf{r}_u + \delta\mathbf{r} \tag{9}$$

where $\hat{\mathbf{r}}_u$ is the differentially corrected position estimate of the user receiver, and $\mathbf{r}_u$ is the position computed using raw pseudorange measurements of the user receiver.

2.2 Positioning via DGPS on Pseudorange

The correction generated on base $\delta\rho^i$ is simply the difference between the calculated pseudorange with reference coordinates $\hat{\rho}^i_b$ and measured pseudorange ρ^i_b:

$$\delta\rho^i = \hat{\rho}^i_b - \rho^i_b \tag{10}$$

This correction is used for correcting the corresponding pseudorange measurement of the user.

The user receiver obeys a dynamic model, given by (Maybeck, 1979; Brown and Hwang, 1996):

$$\dot{\mathbf{x}}_u = \mathbf{F}\mathbf{x}_u + \mathbf{G}\boldsymbol{\omega}_u \tag{11}$$

where $\mathbf{F}$ is a matrix that relates state with its derivative, $\boldsymbol{\omega}_u$ is a modelling noise and $\mathbf{G}$ is an addition noise matrix. $\boldsymbol{\omega}_u$ is assumed white noise.

The measurements model, at instant k, for this method can be obtained from linearization given by Eq. (2). By performing a Taylor expansion about current state, the measurements residuals can be written as a linear function of the estimated state error (Baroni *et al.*, 2003):

$$\Delta\rho^i_u = \mathbf{H}_k \Delta\mathbf{x}_u + \Delta\epsilon_u \tag{12}$$

onde $\mathbf{H}_k = \left[\frac{\partial D^i_u}{\partial \mathbf{x}_u}\right]_{\mathbf{x}=\hat{\mathbf{x}}}$.

2.3 Positioning via Double Difference

This positioning method combines pseudorange measurements from both receivers for generating double difference observables. The double difference observations are formed by choosing a master satellite M, generally by the maximum elevation criteria, and then forming double difference measurements. This procedure guarantees a linearly independent set of double differences (Saalfeld, 1999). Thus,

$$\rho_{ub}^{Mi} = (\rho_u^M - \rho_b^M) - (\rho_u^i - \rho_b^i),\ i = 1 \dots m,\ i \neq M \tag{13}$$

where m is the number of visible satellites. This equation also may be written, according to Eq. (6), in function of geometric distances:

$$\rho_{ub}^{Mi} = (D_u^M - D_b^M) - (D_u^i - D_b^i) + \epsilon_{ub}^{Mi} \tag{14}$$

Considering that the baseline is much shorter than distances between receivers and satellites by orders of magnitude, we can define the relation:

$$D_{ub}^i = D_u^i - D_b^i = \mathbf{1}_b^i \cdot \mathbf{x}_{ub} \tag{15}$$

where $\mathbf{1}_b^i = \begin{bmatrix} -\frac{X^i - x_b}{\rho_b^i} & -\frac{Y^i - y_b}{\rho_b^i} & -\frac{Z^i - z_b}{\rho_b^i} \end{bmatrix}$ is the unity vector pointing from base to satellite i and $\mathbf{x}_{ub}$ represents the baseline between receivers.

Using the described approximation, Eq. (14) becomes linear with respect to the baseline $\mathbf{x}_{ub}$:

$$\rho_{ub}^{Mi} = (\mathbf{1}_b^M - \mathbf{1}_b^i)\mathbf{x}_{ub} + \epsilon_{ub}^{Mi} \tag{16}$$

This equation constitutes the measurement model to be considered for solving the problem. It should be remarked that the double difference measurement covariance matrix has a non-diagonal form (Strang and Borre, 1997):

$$\mathbf{R}_{DD} = 2\sigma_0^2 \begin{bmatrix} 2 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 2 \end{bmatrix} \tag{17}$$

where σ_0^2 is pseudorange measurement error variance. It is recommended that $\mathbf{R}_{DD}$ be diagonal for sequential processing by Kalman filter. This diagonalization process is called whitening. The whitening process is described in Bierman, 1977 and Baroni and Kuga, 2004.

3. Static Tests

In this test both receivers remain static, at positions precisely known, for verifying the quality of proposed algorithm. The data were collected by two Ashtech Z12 receivers, in a campaign of about one hour, and 1 Hz of sampling rate. The base receiver was placed on a reference landmark with coordinates 23° 12' 40.40928" S, 45° 51' 38.38152" W and 612.0274 m, measured by IBGE. The graph in Fig. 1 shows the GDOP, PDOP, TDOP and number of visible GPS satellites during the observation campaign. The results were analyzed through comparing position error related to baseline of 5.20 m between receivers.

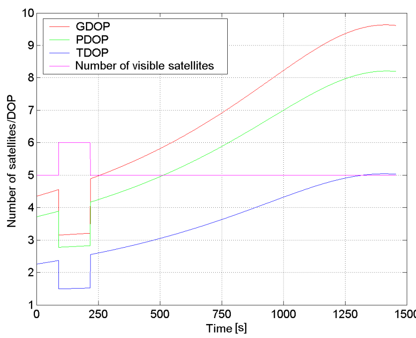


Figure 1. GDOP, PDOP, TDOP and number of visible satellites during experiment.

3.1 DGPS on Position

Firstly, for each receiver, position and clock bias are obtained for both receivers independently from measured data. So, base receiver computes the differential correction by comparison between calculated and known position. Then, this correction is added to user position. Baseline is obtained from user corrected position and base reference position.

Figure 2 shows the user position error related to 5.20 m baseline. The error remains most of time less than 3 m, however with some ripples reaching 7 m, due to high GDOP. This result is within level of accuracy expected for this DGPS technique (Parkinson and Spilker, 1996). This experiment shows a mean error of 0.53 m and standard deviation of 1.34 m.

3.2 DGPS on Pseudorange

The position is obtained by a Kalman filter in this method. The state to be considered for this experiment $\mathbf{x}_u$ is composed by position coordinates $\mathbf{r}_u = \begin{bmatrix} x_u & y_u & z_u \end{bmatrix}^T$, clock bias b_u and drift d_u, with $\dot{b}_u = d_u$. So,

$$\mathbf{x}_u = \begin{bmatrix} \mathbf{r}_u & b_u & d_u \end{bmatrix}^T \tag{18}$$

Therefore, the dynamical model is given by:

$$\dot{\mathbf{x}}_u = \left[\begin{array}{c:cc} \mathbf{0}_{3\times3} & \multicolumn{2}{c}{\mathbf{0}_{3\times2}} \\ \hdashline \mathbf{0}_{2\times3} & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \end{array}\right] \mathbf{x}_u + \mathbf{G}\boldsymbol{\omega}_u \tag{19}$$

whose transition matrix is:

$$\mathbf{\Phi}_{k,k-1} = \left[\begin{array}{c:c} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times2} \\ \hdashline \mathbf{0}_{2\times3} & \begin{matrix} 1 & \delta t \\ 0 & 1 \end{matrix} \end{array}\right] \tag{20}$$

The measurements model for use in the filter is given by Eq. (2). The pseudorange is measured with variance $\sigma_0^2 = (3\,\mathrm{m})^2$, so the $\mathbf{R}$ and $\mathbf{Q}$ matrix for this filter are given, in S.I. units, by:

$$\mathbf{R} = \begin{bmatrix} 3^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 3^2 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \left[\begin{array}{c:c} 0.009^2 \cdot \mathbf{I}_{3\times3} & \mathbf{0}_{3\times1} \\ \hdashline \mathbf{0}_{1\times3} & 0.9^2 \end{array}\right] \tag{21}$$

The graph of Fig. 3 shows the position error for this method, together with estimation error of ± 1-σ, given by the trace of the covariance matrix. A good estimate for user initial position is the base position, which gives an initial error of 4 m. After the filter convergence, the estimated curve remains with mean error of 0.11 m and standard deviation of 0.29 m. This method has better precision than DGPS on position, because of smoothing characteristics of Kalman filter, due to the adopted dynamical model.

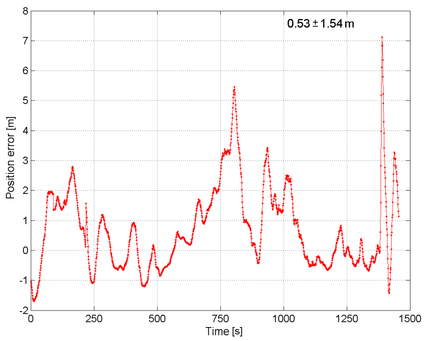


Figure 2. DGPS on position error.

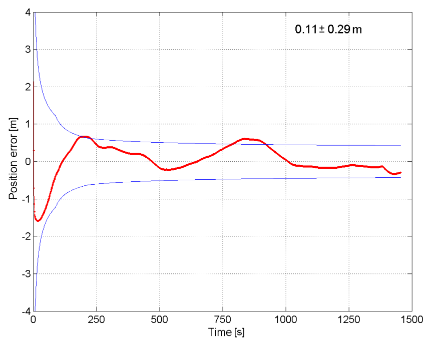


Figure 3. DGPS on pseudorange error.

3.3 Double Difference

The state vector $\mathbf{x}_{ub}$ for double difference positioning consists only of the baseline offset coordinates, such that:

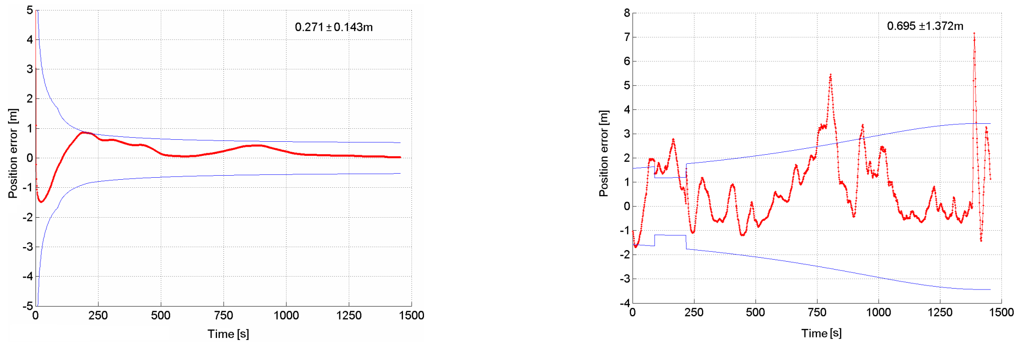
$$\mathbf{x}_{ub} = \begin{bmatrix} \delta x & \delta y & \delta z \end{bmatrix}^T \tag{22}$$

The dynamic model is given by:

$$\dot{\mathbf{x}}_{ub} = \mathbf{0} + \mathbf{G}\omega_r \tag{23}$$

because the receivers are static. So, transition matrix is reduced to the identity matrix. The observation matrix $\mathbf{H}$ is given by Eq. (16).

The graphs of Fig. 4 show the positioning solution related to the 5.20 m baseline, together with the ± 1-σ estimation error for Kalman filter and least squares solution. The precision in both methods is similar, except by the fact of least squares solution is done point-to-point, that is, the solution is obtained for each instant and without dynamic model. The Kalman filter solution is recursive, obtained through the previous epochs improvement using current epoch measurements. This generates more stable results than point-to-point solution.

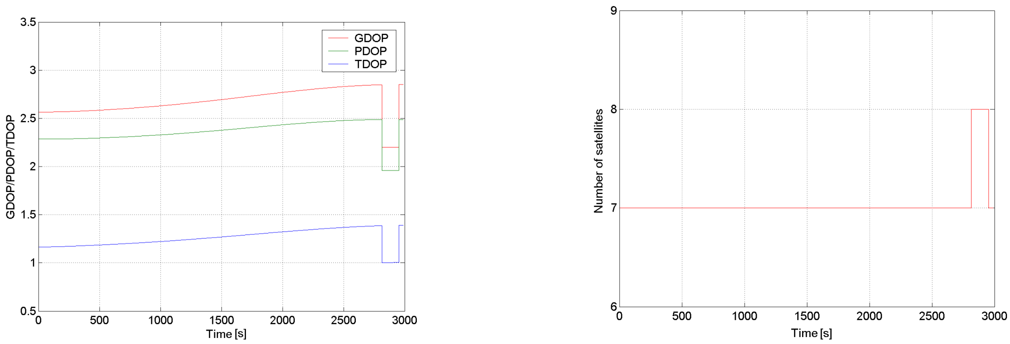


(a) (b)

Figure 4. Double difference error using (a) Kalman filter and (b) least squares.

4. Dynamic Tests

The positioning algorithms described in previous sections were applied to flight data from a mobile user and a fixed base. These data were collected by a receiver installed on an aircraft and a fixed receiver. The base position coordinate are given by 23° 13' 42.9859" S, 45° 51' 23.4615" W and 686.227 m. The GDOP, PDOP and TDOP and number of visible satellites along the time are shown on graphs of Fig. 5.



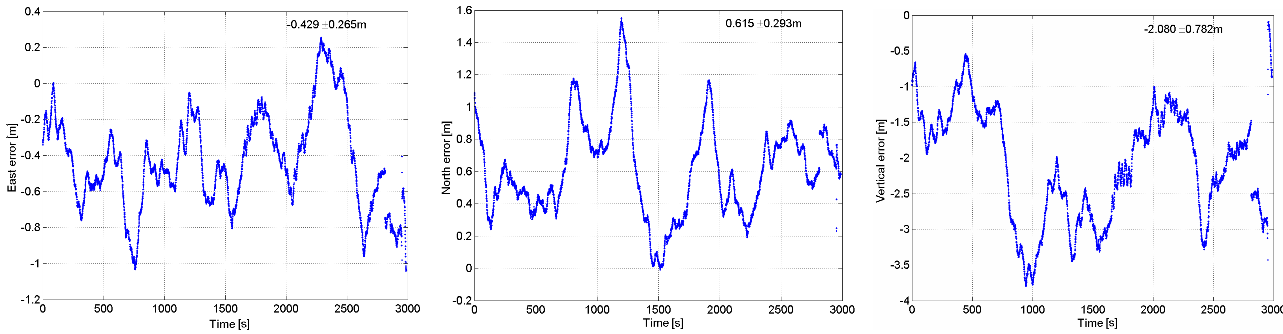
(a) (b)

Figure 5. (a) GDOP, PDOP and TDOP and (b) Number of visible satellites during experiment

4.1 DGPS on Position

The solution of this method consists only of the aircraft position. The graphs of Fig. 6 show the error components on directions east, north and vertical. The horizontal errors have the same precision (about 0.2 m), while vertical error is less precise (0.7 m), and biased about 2 m.

One can notice that this positioning method has a precise solution, but the accuracy is not so good, because the horizontal error is not distributed about zero. The result of this method has precision of about 0.25 m, using all visible satellites. For comparison, Martínez *et al.*, 2000 obtains solution in two dimension with a similar method, but using only three satellites, chosen by DOP criteria. His solution has a precision of about 5 m.



(a) (b) (c)

Figure 6. DGPS on position: error components (a) east, (b) north and (c) vertical.

4.2 DGPS on Pseudorange

The model used for describing the aircraft trajectory consists on estimating position, velocity and clock terms. The state vector to be considered consists of 8 elements:

$$\mathbf{x} = \begin{bmatrix} \mathbf{r}_u & \mathbf{v}_u & b_u & d_u \end{bmatrix}^T \tag{24}$$

where $\mathbf{r}_u = \begin{bmatrix} x_u & y_u & z_u \end{bmatrix}^T$ are user position coordinates, $\mathbf{v}_u = \begin{bmatrix} \dot{x}_u & \dot{y}_u & \dot{z}_u \end{bmatrix}^T$ are velocity coordinates and (b_u, d_u) are clock bias and drift, respectively. Thus, the aircraft dynamics is represented by the following state equation:

$$\dot{\mathbf{x}} = \left[\begin{array}{cc:cc} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \multicolumn{2}{c}{\mathbf{0}_{3\times2}} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \multicolumn{2}{c}{\mathbf{0}_{3\times2}} \\ \hdashline \mathbf{0}_{2\times3} & \mathbf{0}_{2\times3} & \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \end{array}\right] \mathbf{x} + \mathbf{G} \begin{bmatrix} \boldsymbol{\omega}_r \\ \omega_b \end{bmatrix} \tag{25}$$

where $\mathbf{G} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & 0 & 0 \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0 & 1 \end{bmatrix}^T$.

Hence, state transition matrix assumes the form:

$$\boldsymbol{\Phi}_{k,k-1} = \left[\begin{array}{cc:c} \mathbf{I}_{3\times3} & \delta t \cdot \mathbf{I}_{3\times3} & \mathbf{0}_{3\times2} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times2} \\ \hdashline \mathbf{0}_{2\times3} & \mathbf{0}_{2\times3} & \begin{matrix} 1 & \delta t \\ 0 & 1 \end{matrix} \end{array}\right] \tag{26}$$

where δt is time interval between measurements.

The $\mathbf{H}_k$ matrix, described in Eq. (12), is given by:

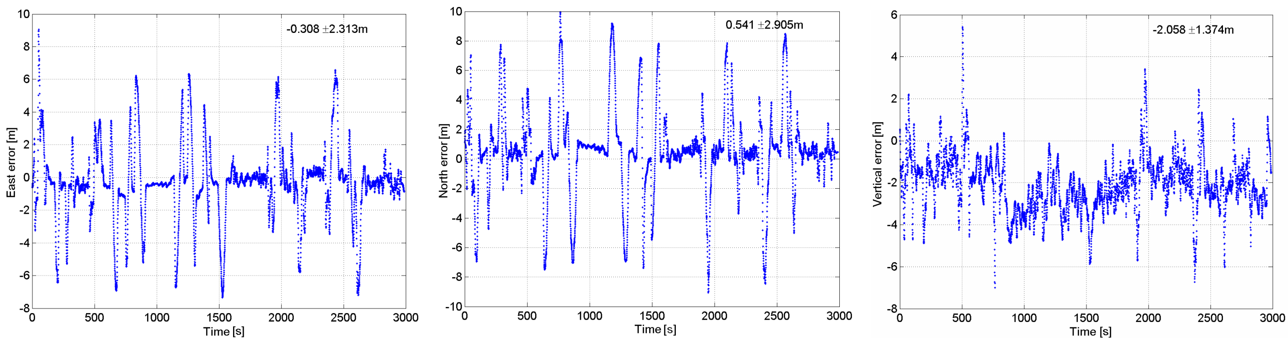
$$\mathbf{H}_k = \begin{bmatrix} -\frac{X^i - \hat{x}_u}{\hat{\rho}_u^i} & -\frac{Y^i - \hat{y}_u}{\hat{\rho}_u^i} & -\frac{Z^i - \hat{z}_u}{\hat{\rho}_u^i} & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{27}$$

The filter was initialized with base position and null velocity. The pseudorange was measured with variance $\sigma_0^2 = (3\,\mathrm{m})^2$. The $\mathbf{Q}$ matrix was tuned as:

$$\mathbf{Q} = \begin{bmatrix} q_r \cdot \mathbf{I}_{3\times3} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & q_b \end{bmatrix} \tag{28}$$

with $q_r = 0.8^2\,\mathrm{m}^2/\mathrm{s}^5$ and $q_b = 0.1^2\,\mathrm{m}^2/\mathrm{s}^5$.

The graphs of Fig. 7 show the position errors, in terms of east, north and vertical components, using this method. These errors have peaks, of about 6 m for east component and 10 m for north component. These peaks occur at moment that aircraft changes its trajectory. This fact shows the dynamic model is not precise enough to follow fast changes in aircraft trajectory. The standard deviations of horizontal errors (east and north) are bigger than ones of vertical error due to mentioned peaks. The vertical error does not have such peaks, probably because the vertical component does not have severe variations like horizontal components.



(a) (b) (c)

Figure 7. DGPS on pseudorange: error components (a) east, (b) north and (c) vertical.

4.3 Double Difference Positioning

The implementation of this method by Kalman filter assumes as state to be estimated the baseline $(\delta x, \delta y, \delta z)$ and its variation $(\dot{\delta x}, \dot{\delta y}, \dot{\delta z})$, so

$$\mathbf{x}_{ub} = \begin{bmatrix} \delta x & \delta y & \delta z & \dot{\delta x} & \dot{\delta y} & \dot{\delta z} \end{bmatrix}^T \tag{29}$$

The equation which describes the aircraft dynamic, considering the state given, is written by:

$$\dot{\mathbf{x}}_{ub} = \left[\begin{array}{c:c} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ \hdashline \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{array}\right] \mathbf{x}_{ub} + \mathbf{G}\omega_r \tag{30}$$

where $\boldsymbol{\omega}_r$ is the noise on $(\ddot{\delta x}, \ddot{\delta y}, \ddot{\delta z})$, with power spectral density $\mathbf{Q}$ and $\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}^T$ is the noise addition matrix. Thus, the state transition matrix is given by:

$$\mathbf{\Phi}_{k,k-1} = \left[\begin{array}{c:c} \mathbf{I}_{3\times3} & \delta t \cdot \mathbf{I}_{3\times3} \\ \hdashline \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{array}\right] \tag{31}$$

The matrix which relates the state vector and double difference measurements is given by:

$$\mathbf{H} = \left[\begin{array}{c:ccc} \mathbf{1}_b^M - \mathbf{1}_b^i & 0 & 0 & 0 \end{array}\right] \tag{32}$$

where $\mathbf{1}_b^i$ is the line of sight between receiver b and satellite $i \neq M$ and M indicates the master satellite.

The least squares solution was obtained by batch measurement processing at each epoch, by the equations:

$$\begin{aligned} \mathbf{P} &= (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1} \\ \Delta\mathbf{x}_{ub,k} &= \mathbf{P}\mathbf{H}^T\mathbf{W}\Delta\mathbf{z}_{DD} \end{aligned} \tag{33}$$

where $\mathbf{W}$ is a weighting matrix, given by $\mathbf{W} = \mathbf{R}_{DD}^{-1}$, $\Delta\mathbf{z}_{DD} = \mathbf{z}_{DD} - \hat{\mathbf{z}}_{DD}$ is the double difference residual and $\Delta\mathbf{x}_{ub,k}$ is the baseline increment. The value of estimated baseline $\hat{\mathbf{x}}_{ub}$, at instant k, is calculated by:

$$\hat{\mathbf{x}}_{ub,k} = \mathbf{x}_{ub,k-1} + \Delta\mathbf{x}_{ub,k} \tag{34}$$

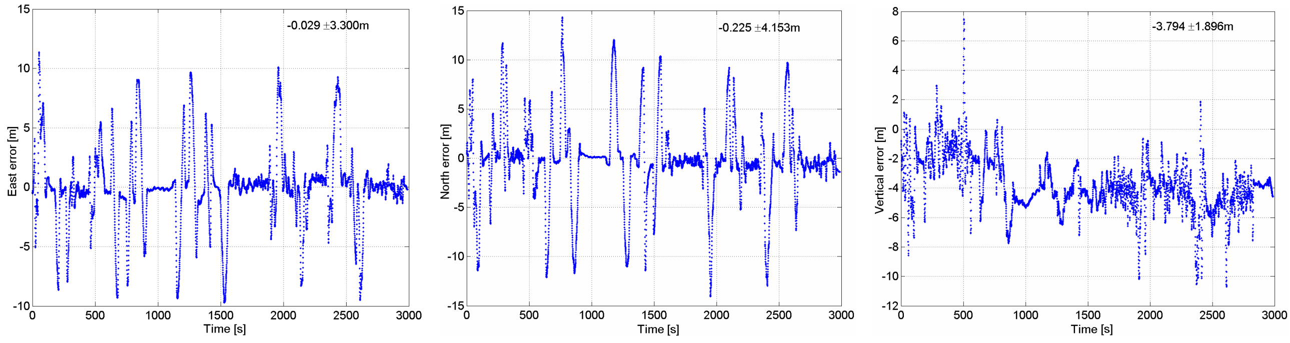
The initial values for state vector $\hat{\mathbf{x}}_{ub}$ are 0 m for baseline and 0 m/s for baseline variation, while its covariance $\hat{\mathbf{P}}_{ub}$ is a diagonal matrix, where the values corresponding to baseline are $(100\,\text{m})^2$ and corresponding to baseline variation are $(10\,\text{m/s})^2$. The measurements noise covariance matrix $\mathbf{R}_{DD}$ is, according to Eq. (17):

$$\mathbf{R}_{DD} = 18 \begin{bmatrix} 2 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 2 \end{bmatrix} (\text{m})^2 \tag{35}$$

The filter was tuned with the value of $\mathbf{Q} = 1.12 \cdot \mathbf{I}_{3\times3}\,\text{m}^2/\text{s}^5$, constant during the whole experiment period.

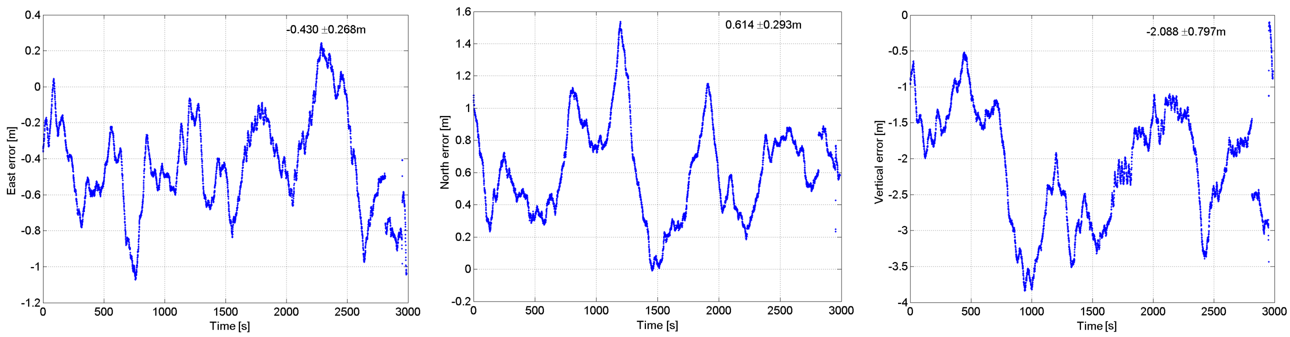
The graphs of Fig. 8 show aircraft position errors for east-north-vertical coordinates using Kalman filter. The east and north components mean remain about zero, but the standard deviation are 3.3 m and 4.1 m respectively. These errors also present peaks due to system dynamic model.

The graphs of Fig. 9 show position errors using least squares method. This method reached accuracy practically the same of DGPS on position method.



(a) (b) (c)

Figure 8. Double difference: error using Kalman filter, in components (a) east, (b) north and (c) vertical.



(a) (b) (c)

Figure 9. Double difference: error using least squares, in components (a) east, (b) north and (c) vertical.

5. Conclusion

For a static case, the DGPS on position method, using least squares, reached an accuracy of 0.53±1.34 m (1 σ). The DGPS on pseudorange method, using Kalman filter, reached an accuracy of 0.11±0.29 m (1 σ) and it is smoother than previous method, due to filter recursive characteristics. The accuracies obtained for double difference positioning were 0.27±0.14 m (1 σ) for Kalman filter and 0.69±1.37 m (1 σ) for least squares. The Kalman filter presented better accuracies than least squares in the mean error and standard deviation.

The cited algorithms were applied on aircraft navigation data. The modelled dynamics for all methods that uses Kalman filtering presented enough accuracy to describe aircraft position with errors of about 5 m. In high maneuvers situations (changes in direction or altitude), it occurred peaks of 10 m, compared to a reference trajectory. In general, the error for DGPS on pseudorange method, which used Kalman filter as estimator, was -0.308±2.313 m for east direction, 0.541±2.905 m for north direction and -2.058±1.374 m for vertical (1 σ). For double difference, the accuracies were -0.029±3.300 m, -0.225±4.153 m and -3.794±1.896 m for east, north and vertical errors respectively (1 σ).

On the other hand, the accuracies obtained with least squares methods. Where the position is calculated point-to-point, and the dynamic modelling is not necessary were better than Kalman filtering. This fact reinforces local observability characteristics. In this case, the user depends only on the sampling rate offered by receiver, here 2 Hz. However, the dynamics must be considered in cases of interpolation between sample intervals. The DGPS on position method had accuracies of -0.429±0.265 m for east error, 0.615±0.293 m for north error and -2.080±0.782 m for vertical error (1 σ), while in double difference, the errors were -0.430±0.268 m, 0.614±0.293 m and -2.088±0.797 m for east, north and vertical components, respectively. An analysis in a rectilinear part of aircraft trajectory shows that the mentioned peaks are due to curve maneuver on horizontal plane. In this part it can be shown that accuracies obtained by Kalman filter are comparable to those obtained by least squares processing.

6. References

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