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INVERSE PHOTON TRANSPORT: SPACE SCIENCE APPLICATIONS

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ABSTRACT

This paper is focused on the application of inverse problem methodology for solving some problems that have emerged in space science. The inverse model is an *implicit* technique: a constrained non-linear optimization problem, in which the forward problem is iteratively solved for successive approximations of the unknown parameters. Iteration proceeds until an objective-function, representing the least-square fit of model results and experimental data added to a regularization term, converges to a specified small value.

1. FORMULATION OF THE INVERSE PROBLEM

A technique for property reconstruction from measurements can be described as a generalized least squares approximation. The standard least squares solution can be unstable in the presence of noise. In order to have a robust inverse model, assuring that parameter variation is bounded to become the final solution physically acceptable, some a priori information must be added to the quadratic difference term. In general, this additional information associated to the inverse solution means smoothness.

Denoting by $\boldsymbol{p} = \left[p_1, p_2, \ldots, p_{N_p}\right]^T$ the unknown vector to be determined by the inverse analysis, the inverse problem can be formulated as a nonlinear optimization problem,

$$\min\ J(\boldsymbol{p}) \quad , \quad l_q \leq p_q \leq u_q \quad , \quad q = 1, \ldots, N_p \quad , \tag{1}$$

where the lower and upper bounds l_q and u_q are chosen in order to allow the inversion to lie within some known physical limits, and the objective function is given by

$$J(\boldsymbol{p}) = \sum_{i=1}^{N_m} \left[\, \Phi_i^{\mathrm{Exp}} - \Phi_i^{\mathrm{Mod}}(\boldsymbol{p}) \right]^2 + \Omega[\boldsymbol{p}] \ , \tag{2}$$

where denotes the number of measurement points (or points for comparison), the regularization operator, and a quantity that can be measured and modeled in a mathematical formulation.

The optimization problem (1) has been solved by two deterministic schemes: quasi-Newton [1], and Levenberg-Marquard [2] methods; and a stochastic scheme: genetic algorithm [3]. Regularizations operators used are described below.

1.1. Tikhonov Regularization

A well-known regularization technique proposed by Tikhonov [4] can be expressed by

$$\Omega[\boldsymbol{p}] = \sum_{k=0}^{N} \alpha_k \, \left\|\boldsymbol{p}^{(k)}\right\|_2^2 \tag{3}$$

where $\boldsymbol{p}^{(k)}$ is the k-th derivative (difference), and the parameters $\alpha_k \leq 0$. Here, if $\alpha_k = \delta_{kj}$ (Kronecker's delta), i.e., $\Omega[\boldsymbol{p}] = \left\|\boldsymbol{p}^{(k)}\right\|_2^2$ then the method is called the Tikhonov regularization of order-j (Tikhonov-j).

1.2. Entropic Regularization

The maximum entropy principle was first proposed by Jaynes[5] on the basis of Shannon's information theory. Similar to the Tikhonov's approach, this general inference method searches a global regularity, searching the smoothest solution which is consistent with the available data.

Recently, a higher order entropic regularization has been proposed [6–10]. An expression for entropic regularization can be written as

$$\Omega(\mathbf{p}) = \sum_{m=0}^{N} \alpha_m \, S^m(\mathbf{p}) \; ; \qquad S^m(\mathbf{p}) \equiv -\sum_{q=1}^{N_p} s_q \, \log(s_q) \; , \tag{4}$$

where $s_q \equiv r_q^m / \sum_l r_l^m$, and r_q^m represents the m-th difference of the parameter vector. The function S^m attains its global maximum when all the r_q are the same. It can be shown that the Morozov's discrepancy principle can also be applied for the maximum entropy (MaxEnt) regularization of higher order [10].

2. APPLICATIONS

2.1. Hydrologic Optics

The transmission of light can be described from the linear non-charged particle transport theory, if the light is understood as a beam of particles (photons). Therefore, the linearized Boltzmann equation, known also as transport equation or radiative transfer equation, for a given wavelength, is written as

$$\mu \frac{dL_\lambda(\zeta,\xi)}{d\zeta} + L_\lambda(\zeta,\xi) = \omega_0(\zeta) \int_\Xi L_\lambda(\zeta,\xi') \, \beta(\xi' \to \xi) \, d\xi' + S_\lambda(\zeta,\xi) \tag{5}$$

where L denotes the radiance; β the scattering phase function; $\omega_0 = b/c$ the single scattering albedo, $c = a + b$ the beam attenuation coefficient, a and b the absorption and scattering coefficients, respectively; ζ is the optical depth; $\xi'(\theta', \phi')$ and $\xi(\theta, \phi)$ are the incident and scattered directions for an infinitesimal beam; θ is the polar angle; ϕ is the azimuthal angle, S is the source term, and $\mu = \cos(\theta)$. An outline of the physical process is depicted in figure 1, where S stands for the direction of the scattered photon, Φ is the scattering angle, and t is the transmitted beam. Three techniques have been applied to solve the forward problem: invariant imbedding [11]; $\mathrm{LTS_N}$ Method [12, 13]; and analytical $\mathrm{S_N}$ method [14].

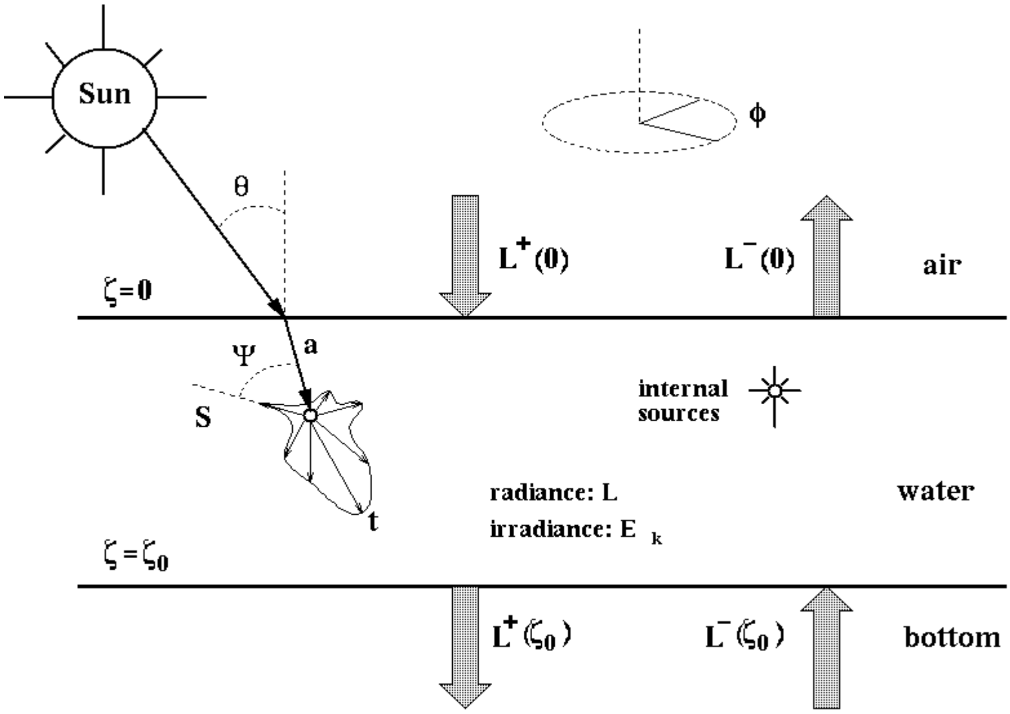

Figure 1. Pictorical representation of the forward radiative problem.

Reconstructions were obtained using *in situ* radiometric data (radiance and irradiances) and remote sensing data (radiance). Estimations obtained with the present inverse analysis are summarized in Table 1, showing the property estimated, forward technique, regularization operator, and optimizer used. The properties of interest in hydrological optics are: internal source term; IOP (inherent optical properties): absorption and scattering coefficients, phase function; and boundary conditions.

An interesting aspect in Case-1 is that no regularization was necessary [15], and the source term is approximated as the sum of Gaussian distributions. In Case-2 the alternate step-by-step strategy was introduced, meaning that a, b coefficients are estimated first, then the source term is estimated, and finally the convergence is checked [16]. The phase function was identified in Cases 3 and 4 [17, 18]. Boundary conditions are identified in Case-5. Finally, the case-6 represents our first result in inverse hydrologic optics using remote sensing data [19].

Table 1. Estimation of properties in hydrologic optics.

Cases	Property	Forward method	Optimizer	Regularization
(1)	$S(\zeta)$	Inv. imbedding	Q-Newton (NAG) / GA	–
(2)	$S(\zeta)$, a, b	Inv. imbedding	Q-Newton (NAG)	MaxEnt-0
(3)	$\beta(\xi)$	standand $\mathrm{S_N}$	Q-Newton (NAG)	MaxEnt-0
(4)	$\beta(\xi)$, ω_0	$\mathrm{AS_N}$	L-M (IMSL)	L-M method
(5)	B. C.	$\mathrm{LTS_N}$	Q-Newton (NAG)	Tikhonov-0
(6)	$S(\zeta)$	$\mathrm{AS_N}$–multispectral	L-M (IMSL)	L-M method

2.2. Atmospheric Temperature Profile from Satellite Data

Considering the infrared radiances captured by the satellite, the integral form of the radiative transfer, for a given wavelength λ, can be simplified for following expression [23]:

$$I_\lambda(\tau_\lambda) = I_\lambda(\tau_\lambda^s) - \int_{\tau_\lambda^s}^{\tau_\lambda} B_\lambda[T(z)]\, \mathrm{d}\tau_\lambda \tag{6}$$

where $I_\lambda(\tau_\lambda)$ is the radiance at height z, τ_λ and τ_λ^s are the transmittance at level z and at surface, and B_λ is the Planck function [23]:

$$B_\lambda(T) = \frac{2\,h\,c^2\,\lambda^{-5}}{e^{hc/k_B \lambda T} - 1} \tag{7}$$

being h the Planck constant, c the light speed, k_B the Boltzmann constant, and T the temperature at level z.

This inverse problem was solved using 7 satellite channels for retrieving the atmospheric temperature profile with 40 layers. The second order maximum entropy principle was used as regularization [24, 7]. Results are showing in figure 2. Our methodology has shown more robust than ITPP5 code, since it produces the same result for different initial guess (figures 4a and 4b).

3. FINAL REMARKS

A methodology for inverse problems appearing on space applications concerning to photon transport has been developed and applied, mainly dealing with regularized solutions. Local and global searching strategies have been efficiently employed for solving the inverse model.

Future works can include: (a) the multi-spectral reconstruction in hydrologic optics needs more studies; (b) new regularization operators, such as non-extensive entropy [25, 26], should be investigated for hydrologic optics problems; (c) new approaches deserve to be explored, such as neural networks [27], Kalman filter, and variational methods [28]; (d) new applications: satellite inverse thermal analysis.

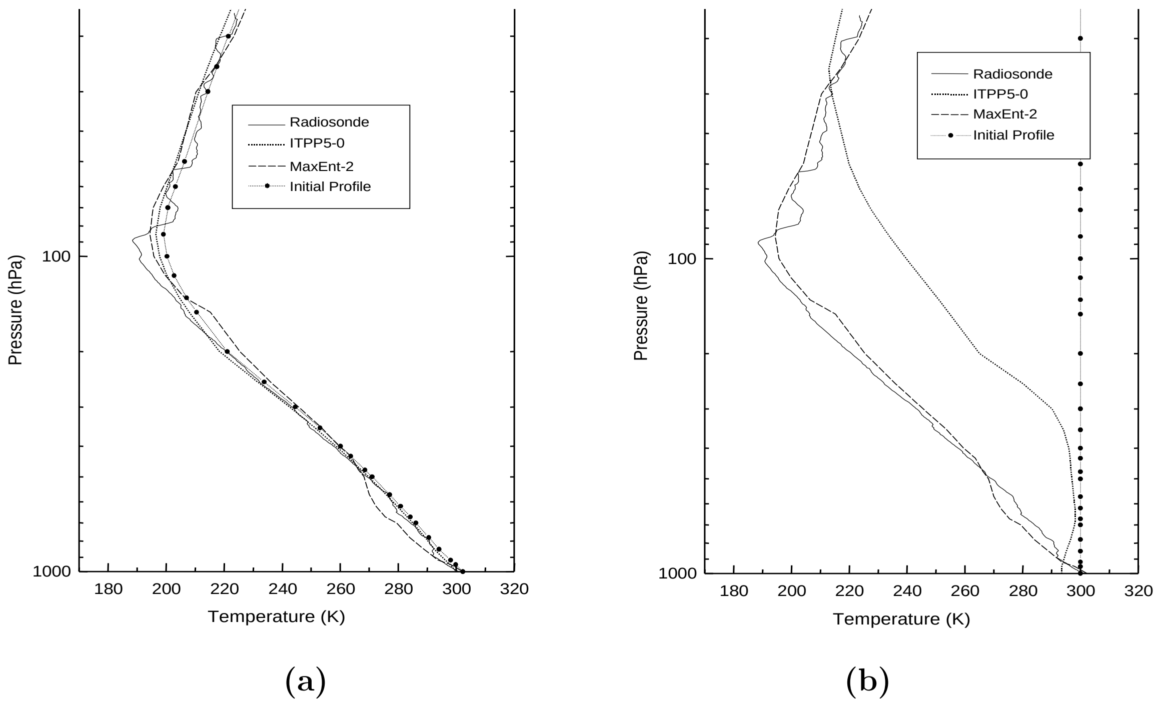

Figure 2. Atmospheric temperature profile for different first guess: (a) climate data base; (b) homogeneous atmospheric layer.

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