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Magnon instability in ferromagnetic semiconductors under strong fields*

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The problem of magnon instability in an electron-magnon system is studied in the presence of a static uniform magnetic field. It is found that the magnons may become unstable when certain threshold values for the drift velocity and the magnetic field are exceeded. However, even after the thresholds are exceeded, it is predicted that there exist alternate bands of the magnetic field, in which the magnons are unstable in one band, stable in the next band, unstable again in the following band, etc. These alternate bands have their origin in the discreteness of the Landau levels of the electrons.

I. INTRODUCTION

In recent papers[1-4] the possibility of spin-wave amplification has been discussed in ferromagnetic semiconductors when an external dc electric field is applied. Originally this problem was investigated by Akhiezer *et al*[5] and Vural.[6] According to their theories there is a net gain for the spin wave when a dc electric field is applied such that the drift velocity of the carriers exceeds the phase velocity of the spin wave in complete analogy with the sound amplification in semiconductors.[7] On the other hand, the recent development of high-mobility ferromagnetic semiconductors has enhanced the possibility of observing such effect. Despite this fact no direct observation of amplification seems to have been made up to date. Recent measurements of magnetoresistance[8] and microwave transmission[9] only qualitatively indicate the existence of spin-wave amplification. Although the earlier papers[1-4] were directed more towards explaining the measurements of Balberg and Pinch,[8] the suggestion has also been made in these previous papers[1-4] of another alternative observation of spin-wave amplification in parallel pumping experiments[10,11] in which the sample is subject to an additional dc electric field. The amplification would, in this case, be observed through a distortion of the "butterfly curve," indicating a decrease in the effective linewidth.

In Refs. 1–4, however, the amplification coefficient found is essentially independent of the external magnetic field. On the other hand, it is natural to expect that external fields changing the spectrum and the occupation number of the electron states will influence the spectrum and damping of the spin waves. In the case of weak applied magnetic field the electron motion is not affected at all and the amplification coefficient should be that of the previous papers.[1-4] However, in the opposite case of strong magnetic field, the electron motion is considerably affected and one should expect the magnon growth rate to depend on the field. Indeed, we shall find that, at temperatures $k_BT \ll \epsilon_F$, $\hbar\omega_c$, where ϵ_F and ω_c denote the Fermi energy and the electron cyclotron frequency, respectively, the magnons may become alternately unstable and stable as the magnetic field strength is varied, i.e., there exist bands of the magnetic field in which the magnons are unstable in one band, stable in the next band, unstable again in the following band, etc. These alternate bands have their origin in the discreteness of the Landau levels of the electron.

The existence of well-defined Landau levels depends of course on whether $\omega_c\tau > 1$ can be realized. Here τ is the electron scattering rate. This in turn entails in restricting our choice to high-mobility materials. In general the $3d$ transition-metal compounds show a low Hall mobility (1–10 $cm^2/V\,sec$) with the exception of the chromium chalcogenide spinels like, for instance, $CdCr_2Se_4$ doped with Ag ($\sim 10^4$ $cm^2/V\,sec$).[8,12-15] The relatively high mobility of these compounds has been associated with their covalent character.[8,15] We shall make contact with this point again in Sec. III where it is suggested that in fact $CdCr_2Se_4$ is the most suitable ferromagnetic semiconductor to perform the experiments.

Although this problem of magnon instability is itself of sufficient interest for an independent investigation, particularly in the low-temperature regions, our main motivation is to find out not only the changes induced by the strong field but also if the condition of a net growth of the magnon population would be enhanced by increasing the field.

Our model for a magnetic semiconductor is that of an interacting conductor-electron–localized-moment system.[16,17] The carriers and the localized moments are interacting by their exchange interaction which is taken to have the familiar s-d contact form.

In Sec. II, we develop the Hamiltonian for the electron-magnon system and set up a kinetic equation for the magnon distribution function which is then linearized to establish the criteria for the onset of the magnon instability. In Sec. III, the growth rate of the magnon population is derived, and a summary and conclusions are given in Sec. IV.

II. FORMULATION

The total Hamiltonian of the system will comprise the conduction-electron part, the exchange-coupled-local-moment part, and the interaction term. We assume that the localized moments experience a ferromagnetic exchange interaction only with their Z nearest neighbors, and consider only the exchange part of the conduction-electron–local-moment interaction, which will be represented by a spin-dependent contact potential. Also, since we are interested in studying the system below the Curie temperature, we shall introduce the magnon variables straightway. Finally, the effect of the external magnetic field and the dc electric field are taken into account by replacing the usual parabolic energy by the Landau levels,[18] and using a drifted distribution function[19] as the carrier distribution, respectively. Thus, in the second quantization formalism, the total Hamiltonian is given by[16–18]

$$\mathcal{H}=\sum_{\alpha\sigma}\epsilon_{\alpha\sigma}c^{\dagger}_{\alpha\sigma}c_{\alpha\sigma}+\sum_{\vec{k}}\hbar\omega_k b^{\dagger}_k b_k + M_{sd}\sum_{\alpha\alpha'\vec{k}}(\langle\alpha|e^{i\vec{k}\cdot\vec{r}}|\alpha'\rangle c^{\dagger}_{\alpha\uparrow}c_{\alpha'\downarrow}b_k+\text{c.c.}) \ . \quad (1)$$

Here

$$\epsilon_{\alpha\sigma}=(n+\tfrac{1}{2})\hbar\omega_c+p_z^2/2m-(\hbar\omega_c+JS)\sigma \quad (2)$$

and

$$\hbar\omega_k=g\mu_B H+2ZIS\left(1-\frac{1}{Z}\sum_{\vec{\delta}}e^{i\vec{k}\cdot\vec{\delta}}\right) \quad (3)$$

denote the electron and magnon energies, respectively, and

$$M_{sd}=-J(S/2N)^{1/2},\quad \omega_c=eH/mc \ .$$

J is the exchange parameter between the localized spin and the conduction electron, N is the number of magnetic atoms of ionic g value g, μ_B is the Bohr magneton, H is the external magnetic field, assumed along the z direction, I is the exchange constant between the Z nearest-neighbor localized spins, and $\vec{\delta}$ is a vector to a nearest neighbor. $c_{\alpha\sigma}$ and $c^{\dagger}_{\alpha\sigma}$ are the usual annihilation and creation operators for an electron in the state $\alpha=n,p_x,p_z$ with spin σ in the Landau representation.[18] Here, $\sigma=+1$ for up conduction-electron moments and $\sigma=-1$ for down moments. The b_k and $b^{\dagger}_k$ are the magnon annihilation and creation operators. In arriving at Eqs. (1)–(3) we have kept only terms bilinear in the magnon operators.

By using the Golden rule one can get the transition probability per unit time for the emission and absorption of a magnon of wave vector $\vec{k}$ while the electron goes from an initial state $|n,p_x,p_z\rangle$ to a final state $|n',p_x-\hbar k_x,p_z-\hbar k_z\rangle$. Once we have the emission and absorption rates the kinetic equation for the magnon distribution is readily written as

$$\frac{\partial N_k}{\partial t}=\frac{2\pi}{\hbar}\sum_{\alpha\alpha'}M^2_{sd}|\langle\alpha'|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle|^2[(N_k+1)\times\tilde{f}_{\alpha'\downarrow}(1-\tilde{f}_{\alpha\uparrow})-N_k\tilde{f}_{\alpha\uparrow}(1-\tilde{f}_{\alpha'\downarrow})]\times\delta(\epsilon_{\alpha'\downarrow}-\epsilon_{\alpha\uparrow}-\hbar\omega_k) \ , \quad (4)$$

where N_k represents the number of magnons with wave vector k, $\alpha=n,p_x,p_z$, and $\alpha'=n',p'_x,p'_z$. The meaning of various terms in Eq. (4) is clear. The change of the magnon number is a result of the emission and absorption of magnons by electrons in all possible states. The first term on the right-hand side of Eq. (4) arises from the emission process. The factor (N_k+1) accounts for the presence of N_k magnons in the system when the additional magnon is being emitted. The factor $\tilde{f}_{\alpha'\downarrow}(1-\tilde{f}_{\alpha\uparrow})$ represents the probability that the electron $\alpha'\downarrow$ state is occupied and the final electron $\alpha\uparrow$ state is empty. Similarly, the second term on the right-hand side of Eq. (4) arises from the absorption process, and the factor $N_k\tilde{f}_{\alpha\uparrow}\times(1-\tilde{f}_{\alpha'\downarrow})$ again takes care of the boson and fermion statistics. Furthermore, in Eq. (4)

$$\tilde{f}_{\alpha\sigma}=[\exp\beta(\epsilon_n-(\hbar\omega_c+JS)\sigma+(1/2m)(p_z-mv_d)^2-\epsilon_F)+1]^{-1} \ ,$$

with $\epsilon_n=(n+\frac{1}{2})\hbar\omega_c$, $\beta=1/k_BT$ is the unperturbed but shifted electron distribution in a magnetic field, both the magnetic field and the drift velocity $\vec{v}_d$ being assumed to be parallel to the z axis. As we have previously mentioned the effect of the dc electric field is supposed to be taken into account by using the drifted distribution. Strictly speaking, the electron-distribution functions in Eq. (4) should be the exact, time-dependent functions $F_{\alpha\uparrow}(t)$ and $F_{\alpha'\downarrow}(t)$ instead of the unperturbed $\tilde{f}_{\alpha\uparrow}$ and $\tilde{f}_{\alpha'\downarrow}$. One should then write another equations of motion governing $F_{\alpha\uparrow}(t)$ and $F_{\alpha'\downarrow}(t)$ and then solve them together with Eq. (4). However, to lowest order in the electron-magnon coupling, one can approximate $F_{\alpha\uparrow}$ by $\tilde{f}_{\alpha\uparrow}$.

After some manipulations, Eq. (4) can be rewritten in a more convenient form,

$$\frac{dN_k}{dt}=\gamma(k)N_k+\gamma'(k) \ , \quad (5)$$

where the magnon generation rate $\gamma(k)$ is given by

$$\gamma(k)=\frac{2\pi}{\hbar}M_{sd}^2\sum_{\alpha\alpha'}|\langle\alpha'|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle|^2(\tilde{f}_{\alpha'\downarrow}-\tilde{f}_{\alpha\uparrow})\times\delta(\epsilon_{\alpha'\downarrow}-\epsilon_{\alpha\uparrow}-\hbar\omega_k) \tag{6}$$

and $\gamma'(k)$ is the spontaneous-emission term.

Having reached this point let us now establish the criterion for the onset of the instability. However, before we actually do this one notices that in our discussion so far, we have completely neglected other mechanisms such as multimagnon, magnon-phonon, etc., which may interact with magnons and lead to a finite magnon lifetime even in the absence of electron-magnon interaction. We may take them into account by introducing a phenomenological magnon decay rate $\nu(k)$ due to other processes than magnon emission and absorption by electrons. We shall see presently that the threshold magnetic field for the onset of the magnon instability becomes finite (rather than zero) for a finite $\nu(k)$

Equation (6) can still be written in a simpler form by changing p_z and p_z' into p_z+mv_d and $p_z'+mv_d$, respectively. Performing this change of variables in Eq. (6) one obtains

$$\gamma(k)=\frac{2\pi}{\hbar}M_{sd}^2\sum_{\alpha\alpha'}|\langle\alpha'|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle|^2(f_{\alpha'\downarrow}-f_{\alpha\uparrow})\times\delta(\epsilon_{\alpha'\downarrow}-\epsilon_{\alpha\uparrow}-\hbar(\omega_k-k_zv_d)) , \tag{7}$$

where

$$f_{\alpha\sigma}=[\exp\beta(\epsilon_{\alpha\sigma}-\epsilon_F)+1]^{-1}$$

is the equilibrium Fermi-Dirac function. In other words, the effect of the drift velocity is equivalent to a Doppler shift in the magnon frequency. By recalling that $f(\epsilon')\geq f(\epsilon)$ when $\epsilon'\leq\epsilon$ it is easy to see that

$$\gamma \gtrless 0 \quad \text{for } \omega_k-k_zv_d \lessgtr 0 . \tag{8}$$

Therefore, for a particular magnon wave vector $\vec{k}$ the criterion for the onset of the magnon instability is just the Čerenkov condition

$$\omega_k-k_zv_d<0 , \tag{9}$$

since in this case the magnon population grows exponentially at a rate given by Eq. (7). Equation (8) is independent of the existence of an external magnetic field and is, in fact, the same as the one found elsewhere,[1–5] i.e., there is only a threshold value for the drift velocity but no threshold value for the magnetic field. If, on the other hand, $\nu(k)$ is finite, the instability criterion then becomes

$$\gamma>\nu \tag{10}$$

which generally depends on the magnetic field via Eq. (7). It is clear from Eqs. (7) and (10) that, in addition to a drift-velocity threshold whose component in the k direction still has to exceed the magnon phase velocity, there is now also a magnetic field threshold whose value depends on ν. Finally, it should be mentioned that the exponential growth behavior as predicted by the linearized Eq. (5) is valid only at the initial stage after the onset of the instability. Thereafter, the nonlinear terms which have been neglected so far will limit the amplitude of the magnon fluctuations.

III. GROWTH RATE

To evaluate $\gamma(k)$ we need to know the quantity $|\langle\alpha'|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle|^2$. Using the Landau wave function[18] it can easily be seen that

$$|\langle\alpha'|e^{i\vec{k}\cdot\vec{r}}|\alpha\rangle|^2=\delta_{p_x',p_x+\hbar k_x}\delta_{p_z',p_z+\hbar k_z}\chi_{n'n}(q) , \tag{11}$$

where

$$\chi_{n'n}(q)=\Theta(n-n')(n'!/n!)q^{n-n'}e^{-q}[L_{n'}^{n-n'}(q)]^2+\Theta(n'-n)(n!/n'!)q^{n'-n}e^{-q}[L_n^{n'-n}(q)]^2+\delta_{n,n'}e^{-q}[L_n(q)]^2 . \tag{12}$$

Here $L_n^\alpha(x)$ are the associated Laguerre polynomials, Θ is the step function

$$\Theta(n-n')=\begin{cases}1, & n>n'\\ 0, & n\leq n'\end{cases} ,$$

and $q=\hbar k_\perp^2/2m\omega_c$, $k_\perp$ being the component of the magnon wave vector $\vec{k}$ perpendicular to the magnetic field.

Substituting Eqs. (11) and (2) into Eq. (7), we obtain

$$\gamma(k)=\frac{m\omega_cL^2M_{sd}^2}{\hbar^2}\sum_{nn'p_z}\chi_{n'n}(q)\times(f_{n',p_z+\hbar k_z,\downarrow}-f_{n,p_z,\uparrow})\delta\left(\frac{(p_z+\hbar k_z)^2}{2m}-\frac{p_z^2}{2m}+(n'-n)\hbar\omega_c+2(\hbar\omega_c+JS)-\hbar(\omega_k-k_zv_d)\right) , \tag{13}$$

where the sum over p_x has yielded[18] a factor $L^2m\omega_c/2\pi\hbar$, L being the linear dimension of the crystal. The δ function in Eq. (13) determines the value for p_z. Thus, we have

$$\gamma(k)=\frac{VM_{sd}^2m^2\omega_c}{2\pi\hbar^4|k_z|}\sum_{n'n}\chi_{n'n}(q)\times[f(n',p_0+\hbar k_z,\sigma=-1)-f(n,p_0,\sigma=+1)] , \tag{14}$$

where

$$p_0=\frac{m\omega_k}{k_z}-p_d-\frac{\hbar k_z}{2}-(n'-n)\frac{m\omega_c}{k_z}-\frac{2m}{\hbar k_z}(\hbar\omega_c+JS) ,\qquad p_d=mv_d . \tag{15}$$

This is the most general expression for the growth rate γ. The analysis of the above equation is very complicated for arbitrary $\vec{k}$ because, in general, both the $n \to n$ and the $n \to n'$ $(n \neq n')$ transitions are possible. However, by making use of the properties of the Laguerre polynomials,[20] Eq. (12) can be drastically simplified in the case when $q \ll 1$ or $\hbar k_\perp^2 \ll m\omega_c$. Under these conditions $(q \ll 1)$

$$L_n^r(q) \approx \frac{(n+r)!}{n!\,r!} - \frac{(n+r)!}{(n-1)!\,(r+1)!}\,q \;.$$

Using this result for L_n^r and comparing the terms in Eq. (12), it is seen that the $n \to n'$ transitions become unimportant when $q \ll 1$. In particular, for spin-wave propagating parallel to field this is an exact result. Physically this means that when the momentum transfer $\hbar k_\perp$ is much smaller than, the momentum change corresponding to a jump from one orbit to the next, transitions involving different orbits are not important. Condition $q \ll 1$ is also equivalent to $k_\perp R_c \ll 1$ where $R_c = v_\perp/\omega_c \sim (\hbar/2m\omega_c)^{1/2}$ is essentially the radius of the circular orbit. Accordingly, if we assume

$$k_\perp v_F \ll \omega_c \;, \tag{16}$$

where v_F is the Fermi velocity, $\chi_{nn'}(q)$ can be approximated by $\delta_{n'n}$. The next important terms in $\chi_{n'n}$ involve transitions between the neighboring levels $(n'-n=\pm 1)$. However, for the special case of magnon propagating parallel to the field, $\chi_{n'n}(q)$ is exactly $\delta_{nn'}$. The above condition is well satisfied for magnetic semiconductors under possible experimental conditions as we shall see later on. For the sake of simplicity in our discussion, we shall from now on assume that this condition is satisfied with the result that Eq. (14) then becomes

$$\gamma(k) = \frac{VM_{sd}^2 m^2 \omega_c}{2\pi\hbar^4 |k_z|} \sum_n \left[f(n, p_0 + \hbar k_z, -1) - f(n, p_0, +1) \right], \tag{17}$$

where p_0 is now

$$\frac{m\omega_k}{k_z} - p_d - \frac{\hbar k_z}{2} - \frac{2m}{\hbar k_z}(\hbar\omega_c + JS) \;.$$

If we define two energies

$$\epsilon_A = \epsilon_F - p_0^2/2m + (\hbar\omega_c + JS) \;,$$
$$\epsilon_B = \epsilon_F - (p_0 + \hbar k_z)^2/2m - (\hbar\omega_c + JS) \;, \tag{18}$$

then at $T = 0$ °K, the nth Landau level contributes a positive term to the sum over n on the right-hand side of Eq. (17) if

$$\epsilon_A < \epsilon_n < \epsilon_B \;, \tag{19}$$

it contributes a negative term if

$$\epsilon_B < \epsilon_n < \epsilon_A \;, \tag{20}$$

and it contributes nothing otherwise. Thus, at zero temperature, Eq. (17) becomes

$$\gamma(k) = \frac{VM_{sd}^2 m^2 \omega_c}{2\pi\hbar^4 |k_z|} \sum_{\substack{n \\ (\epsilon_A < \epsilon_n < \epsilon_B)}} 1 \geq 0 \quad \text{if } \epsilon_B > \epsilon_A \tag{21}$$

and

$$\gamma(k) = -\frac{VM_{sd}^2 m^2 \omega_c}{2\pi\hbar^4 |k_z|} \sum_{\substack{n \\ (\epsilon_B < \epsilon_n < \epsilon_A)}} 1 \leq 0 \quad \text{if } \epsilon_B < \epsilon_A \;. \tag{22}$$

Hence, γ is seen to be proportional to the numbers of levels which lie between the two limits ϵ_A and ϵ_B. We note from Eq. (21) that the criterion for amplification, $\epsilon_B > \epsilon_A$ reduces to

$$\Delta\epsilon = \epsilon_B - \epsilon_A = \hbar(k_z v_d - \omega_k) > 0 \;, \tag{23}$$

which is the same as before.

To see how $\gamma(k)$ depends on the magnitude of the magnetic field, let us focus our attention on the magnon instability, Eq. (21), in the following cases.

(i) $\epsilon_A > \frac{1}{2}\hbar\omega_c$ and $\epsilon_B > \frac{1}{2}\hbar\omega_c$ such that

$$0 < \Delta\epsilon < \hbar\omega_c \;. \tag{24}$$

In this case, the above conditions assure us that there is at most one level n such that $\epsilon_A < \epsilon_n < \epsilon_B$, i.e.,

$$\sum_{\substack{n \\ [\epsilon_A < \epsilon_n < \epsilon_B]}} 1 = \begin{cases} 1 & \text{for } n \text{ such that } \epsilon_A < \epsilon_n < \epsilon_B \\ 0 & \text{otherwise .} \end{cases} \tag{25}$$

In Fig. 1, some Landau levels and the limits ϵ_A, ϵ_B are shown schematically. It is seen in Fig. 1 that when the Landau level, with $\epsilon_n = (n+\frac{1}{2})\hbar\omega_c$, lies within the $\Delta\epsilon$ interval but just above ϵ_A, the $(n-1)$th level must lie below ϵ_A and the $(n+1)$th level must lie above ϵ_B. If we keep $\vec{k}$ and $\vec{v}_d$ fixed

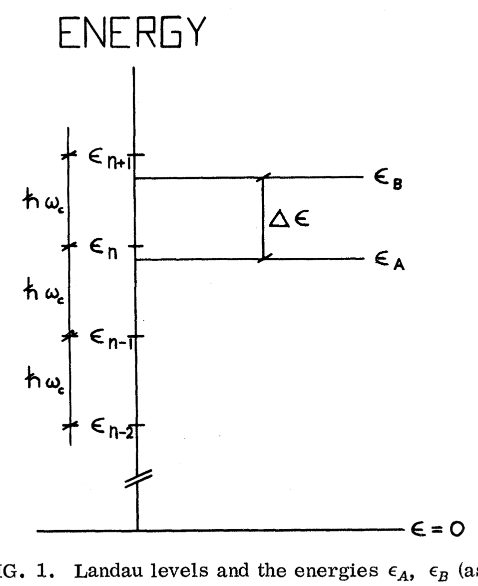

FIG. 1. Landau levels and the energies ϵ_A, ϵ_B (assumed positive here).

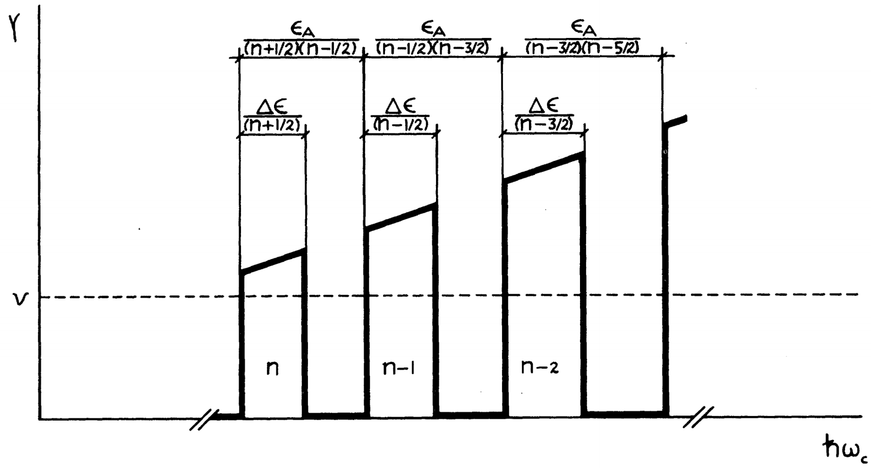

FIG. 2. Growth rate γ vs $\hbar\omega_c$ showing a few alternate stable and unstable bands.

and vary ω_c we find that as ω_c increases, the nth level gradually rises and eventually leaves the $\Delta\epsilon$ interval when ω_c has increased by an amount $\Delta\omega_c = \Delta\epsilon/(n+\frac{1}{2})\hbar$. At this value of ω_c, γ drops abruptly from a positive finite value to zero. This rate remains zero until ω_c has increased by an amount $\Delta\omega_c = \epsilon_A/(n+\frac{1}{2})(n-\frac{1}{2})\hbar$ when the $(n-1)$th level is just about to enter the $\Delta\epsilon$ interval. Thus, as long as Eq. (24) is satisfied, the growth rate γ for the $\vec{k}$ magnons takes on finite positive values proportional to ω_c and zeros on alternate bands of the magnetic field as shown in Fig. 2. Also, as ω_c increases, these bands become wider and wider. These bands are of course due to the discreteness of the Landau levels. In order to observe this phenomenon the system has to be kept at low temperatures $k_BT \ll \hbar\omega_c$ so that the levels are not smeared by temperature broadening. One also notices that in this region of ω_c, the growth rate becomes independent of the drift velocity, although the widths of the alternate bands of the magnetic field still depend on the drift velocity.

(ii) ω_c is small such that $\epsilon_B, \epsilon_A > \hbar\omega_c$ and $\Delta\epsilon \gg \hbar\omega_c$. In this case, there will be one or more levels within $\Delta\epsilon$. This in turn entails that we will no longer have regions in which as shown above. In this case

$$\sum_{\substack{n\\(\epsilon_A<\epsilon_n<\epsilon_B)}} 1 = \mathfrak{N} \simeq \frac{\Delta\epsilon}{\hbar\omega_c} ,$$

$\mathfrak{N}$ being the number of levels in $\Delta\epsilon$. Thus, Eq. (21) becomes

$$\gamma(k) = (VM_{sd}^2 m^2/2\pi\hbar^4 |k_z|)(k_z v_d - \omega_k) . \tag{26}$$

That is, the growth rate becomes independent of ω_c and is identical to the magnon growth rate in a weak magnetic field discussed elsewhere.[1-4]

(iii) ω_c is large enough such that $\epsilon_A, \epsilon_B < \frac{1}{2}\hbar\omega_c$. In this case $\gamma = 0$ since there will be no levels within $\Delta\epsilon$. This region of ω_c follows the alternate bands region, depicted in Fig. 2. The widths of the alternate bands become larger and larger as ω_c increases until finally the width of the last band in which $\gamma = 0$ becomes infinite.

In the cases discussed above, we have implicitly assumed both ϵ_A and ϵ_B to be positive energies. The cases in which one (or both) of these energies is (are) negative can be analyzed similarly.

In our discussion of the growth rate $\gamma(k)$ so far we have ignored the losses of the spin waves $\nu(k)$ due to other processes. These can be taken into account by defining an effective growth rate as $\gamma(k) - \nu(k)$. Therefore, even if $\gamma(k)$ is positive when $v_d > v_s$ there may or may not be an instability, depending on whether

$$\gamma(k) \gtrless \nu(k) , \tag{27}$$

respectively. Equation (27) determines the magnetic field threshold for an actual magnon instability. It is clear that for case (ii) there is no magnetic field threshold. Substituting Eq. (21) for $\gamma(k)$ into Eq. (27) one gets

$$H > H_c(k) \tag{28}$$

as the instability criterion for the magnetic field. The threshold magnetic field $H_c(k)$ is given by

$$H_c(k) = \frac{4\pi\hbar^4 c(N/V)}{emSJ^2} |k_z| \nu(k) . \tag{29}$$

Equation (29) indicates that the instability is more easily achieved in high-mobility materials ($\omega_c\tau > 1$) for relatively small values of $\vec{k}$. Of the existing ferromagnetic semiconductors the best candidate would be $CdCr_2Se_4$ doped with Ag for it has the highest known mobility.[8,15] To get an estimate of the magnetic field threshold we take the following values for the physical parameters for $CdCr_2Se_4$ doped with Ag[8,15,21-24]: $J = 10^{-14}$ erg, $S = \frac{3}{2}$, $N/V = 10^{23}$ cm^{-3}, $\mu = 10^4$ cm^2 V^{-1} sec^{-1}, ϵ_F

$= 1.2\times10^{-13}$ erg, $n_0 = 10^{20}$ electrons/cm^3. It turns out that for $k = 10^6$ cm^{-1} H_c is of the order of 52 kG and for $k = 10^7$ cm^{-1} H_c is 520 kG, assuming that $\nu(k)$ is typically of the order of 10^8 sec^{-1} at T near zero. Hence, in principle, one might observe the predicted bands as dc fields up to 200 kG can presently be realized in the laboratory whereas fields up to 10^6 G can be obtained using pulsed techniques. Note that for a field of 100 kG one should observe well-defined Landau levels in $CdCr_2Se_4$ as $\omega_c\tau = 10$.

IV. CONCLUSIONS

It was found in this paper that magnons in an electron magnon system such as doped ferromagnetic semiconductors may become unstable when the drift velocity of the electrons and the static magnetic field exceed certain threshold values which are given in Eqs. (23) and (29), respectively. It was predicted here that there exist some alternate bands of the magnetic field so that magnons of wave vector $\vec{k}$ are stable in one band, unstable in the next band, but stable again in the following band, etc., while the drift velocity is kept fixed. The physical origin of these bands is the discreteness of the electron levels in a quantizing magnetic field. In order that our prediction be valid, these discrete Landau levels must not be smeared by other mechanisms, such as temperature or collision broadening. This implies that one should work at very low temperatures, and with high-mobility magnetic semiconductors, like $CdCr_2Se_4$.

To observe the magnon instability discussed in this paper it may be more suitable to perform either magnetoresistance or microwave transmission experiments. However, due to the eventual experimental difficulties of working in the frequency region of 10^{11}–10^{12} sec^{-1} one could alternately perform spin-wave pumping experiments in antiferromagnetic semiconductors. The reason for this is that in these materials one of the spin-wave branches decreases as one increases the external magnetic field. In particular, for fields of the order of 10^4–10^5 G the spin-wave frequency lies in the microwave region (X band), and hence one may observe the predicted bands of amplification in parallel pumping experiments as suggested in Refs. 1–4. Unfortunately, there seems to exist no antiferromagnetic semiconductor of high enough mobility which could enable us to actually observe the Landau levels.

Regarding the magnetoresistance experiments,[8] one could use the same procedure as that of the recently reported[25] measurements of electrical resistivity and Hall effect on n type EuSe single crystals in magnetic fields up to 150 kG. This paper followed the high-magnetic-field studies of electrical transport in Eu chalcogenides by the same authors. The Hall mobilities in their EuSe samples lead to the estimate of $\hbar/\tau \sim 0.02$ eV ($\mu \sim 10^2$ cm^{2}V^{-1}sec^{-1}). This, in turn, gives $\omega_c\tau \simeq 0.2$ for the highest field used in their experiments. In order to observe the Landau levels in these samples one therefore ought to use quite high fields, of the order of 10^6 G, which can only be obtained by pulsed techniques. On the other hand, in the case of $CdCr_2Se_4$ as the mobility is of about two orders of magnitude larger, one could, in principle, observe Landau levels using the presently available high dc fields.

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