AIP | Physics of Fluids

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Citation: Physics of Fluids **20**, 1031 (1977); doi: 10.1063/1.861974
View online: http://dx.doi.org/10.1063/1.861974
View Table of Contents: http://scitation.aip.org/content/aip/journal/pof1/20/6?ver=pdfcov
Published by the AIP Publishing

Plasma wave instability in the field of an intense electromagnetic wave

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(Received 12 October 1976)

The plasmon scattering by electrons in the presence of an intense laser field is discussed. It is shown that in the strong field limit the plasmon population may become unstable.

There has been recent interest in the study of the interaction of intense laser fields with plasmas.[1–4] In particular, the effect of laser-cyclotron resonance on Landau damping has been discussed.[4]

Here, we extend the theory of Ref. 4 by considering the plasma wave scattering by electrons in the presence only of an intense laser field. Our approach follows closely that of Ref. 4. The laser beam is treated as a classical plane electromagnetic wave in the dipole approximation. The electron states are described by the solution to the Schrödinger equation for an electron in the laser field. The scattering of plasma waves by electrons is treated using first-order perturbation theory.[5] The kinetic equation for the plasmons is then derived using the quantum approach of Harris[5] based upon the transition probabilities. It is found that, in the strong-field limit, only multiphoton processes are significant and as a result the plasmon population may grow with time.

We start with the solution to the time-dependent Schrödinger equation for an electron in the electromagnetic field of the laser beam, namely,[1]

$$\psi(\mathbf{x}, t) = L^{-3/2} \exp\left\{ i\mathbf{p}\cdot\mathbf{x} - \frac{i}{2m\hbar}\int^{t} dt' \left[\hbar\mathbf{p} - \frac{e}{c}\mathbf{A}(t')\right]^2 \right\}. \tag{1}$$

Here, $\mathbf{p}$ is the electron wave vector such that in the absence of the radiation field the electron energy ϵ_p is $\hbar^2 p^2/2m$ and $\mathbf{A}(t) = (c\mathbf{E}_0/\omega)\cos\omega t$ is the vector potential of the laser field of frequency ω, within the dipole approximation.

Treating the electron-plasmon interaction[6] as a perturbation and proceeding as in Refs. 1–4, the transition probability per unit time, $T_\nu(1\to 2;\, \mathbf{k})$, for a transition from a state 1 ($\mathbf{p}_1 = \mathbf{p}+\mathbf{k}$) to a state 2 ($\mathbf{p}_2 = \mathbf{p}$) due to a collision with a plasmon $\mathbf{k}$ with absorption ($\nu < 0$) or emission ($\nu > 0$) of $|\nu|$ photons may be written as

$$T_\nu(1\to 2;\mathbf{k}) = (2\pi/\hbar)\,|V_k|^2 J_\nu^2(\lambda/\hbar\omega)\,\delta(\epsilon_{p+k} - \epsilon_p - \hbar\omega_k - \nu\hbar\omega), \tag{2}$$

where $V_k^2 = 2\pi e^2\hbar\omega_k/Vk^2$ is the electron-plasmon vertex,[6,7] J_ν is the Bessel function of order ν, $\lambda = e\hbar\mathbf{k}\cdot\mathbf{E}_0/m\omega$ is the field parameter, and ω_k is the plasmon dispersion relation.

The change in N_k the number of plasmons of wavenumber $\mathbf{k}$, is then given in terms of the transition probability T_ν as[1–5]

$$dN_k/dt = \gamma_k N_k\,,$$

where

$$\gamma_k = (2\pi/\hbar) V_k^2 \sum_{\nu=-\infty}^{+\infty} \sum_{p} J^2(\lambda/\hbar\omega)\left[f(\epsilon_{p+k}) - f(\epsilon_p)\right] \times \delta(\epsilon_{p+k} - \epsilon_p - \hbar\omega_k - \nu\hbar\omega)\,. \tag{3}$$

In Eq. (3), $f(\epsilon_p)$ is the electron distribution function. If $\gamma_k > 0$, the plasmon population grows with time whereas for $\gamma_k < 0$ we have damping.

We now assume a Maxwellian distribution for the electrons and only consider the strong-field limit, namely, $\lambda \gg \hbar\omega$. The assumption of a Maxwellian distribution for the electrons is valid provided the electron heating in the radiation field may be neglected. The latter is valid if $e^2E_0^2/2m\omega^2 < \langle E\rangle$ where $\langle E\rangle = k_BT$ is the average energy of an electron in the absence of the radiation field. Therefore, our results are restricted to radiation fields, E_0, smaller than the threshold heating field $E_h = (2m^2k_BT/e^2)^{1/2}$. Under the strong-field conditions ($\lambda \gg \hbar\omega$) the argument of the Bessel function is large. For large values of argument, the Bessel function is small except when the order ν is equal to the argument. The sum of ν in Eq. (3) may then be written approximately as[1,2]

$$\sum_{\nu=-\infty}^{\infty} J_\nu^2(\lambda/\hbar\omega)\,\delta(\epsilon - \nu\hbar\omega) \simeq \tfrac{1}{2}\left[\delta(\epsilon - \lambda) + \delta(\epsilon + \lambda)\right],$$

where $\epsilon = \epsilon_{p+k} - \epsilon_p - \hbar\omega_k$. The first δ function corresponds to the emission and the second to the absorption of $\lambda/\hbar\omega$ photons. Since $\lambda \gg \hbar\omega$, only multiphoton processes are significant. The damping rate then becomes

$$\gamma_k = (\pi/\hbar) v_k^2 \sum_p \Big(f(\epsilon_p)\left\{\exp[-(\lambda + \hbar\omega_k)/k_BT] - 1\right\} \times \delta(\epsilon_{p+k} - \epsilon_p - \hbar\omega_k - \lambda) + f(\epsilon_p)\left\{\exp[(\lambda - \hbar\omega_k)/k_BT] - 1\right\} \times \delta(\epsilon_{p+k} - \epsilon_p - \hbar\omega_k + \lambda)\Big)\,. \tag{4}$$

Furthermore, assuming that $\lambda \gg k_BT$, the contribution of processes in which photons are emitted is negligible compared with the contribution of the processes in which photons are absorbed. Under these circumstances, Eq. (4) becomes

$$\gamma_k = (\pi/\hbar)v_k^2 \sum_p f(\epsilon_p)\{\exp[(\lambda - \hbar\omega_k)/k_B T] - 1\}$$
$$\times \delta(\epsilon_{p+k} - \epsilon_p - \hbar\omega_k + \lambda). \qquad (5)$$

We now take the classical limit of Eq. (5) by letting $\hbar \to 0$ such that[1,2,5]

$$h\mathbf{p} \to m\nu,$$
$$\sum_p (\cdots) f(\epsilon_p) \to V \int d^3\nu (\cdots) f(\nu).$$

Hence, expanding Eq. (5) in powers of $\hbar$ and retaining only the lowest-order terms, one gets

$$\gamma_k = \frac{2\pi^2 e^2 \omega_k}{k^2 k_B T}(\mathbf{k}\cdot\mathbf{v}_0 - \omega_k)$$
$$\times \int d^3\nu f(\boldsymbol{\nu})\delta(\mathbf{k}\cdot\mathbf{v} - \omega_k + \mathbf{k}\cdot\mathbf{v}_0), \qquad (6)$$

where we have written λ as $\hbar\mathbf{k}\cdot\mathbf{v}_0$ with $\mathbf{v}_0 = eE_0/m\omega$.

For $E_0 = 0$ $(v_0 = 0)$, Eq. (6) reduces to the well-known expression of the Landau damping.[5,6] Now, replacing $f(\mathbf{v})$ by $n_0(\pi v_T^2)^{-3/2}\exp(-v^2/v_T^2)$, where $v_T^2 = 2k_B T/m$ and performing the integrations one has

$$\gamma_k = \pi^{1/2}\frac{\omega_p^2\omega_k}{k^2 v_T^2}\frac{(\mathbf{k}\cdot\mathbf{v}_0 - \omega_k)}{kv_T}\exp[-(\mathbf{k}\cdot\mathbf{v}_0 - \omega_k)/k_B T], \qquad (7)$$

which is maximum $(\gamma_k \sim \omega_p^2\omega_k/k^2 v_T^2)$ for $x = |\mathbf{k}\cdot\mathbf{v}_0 - \omega_p|/kv_T \simeq v_0/v_T = \frac{1}{2}$, assuming k parallel to $\mathbf{E}_0$. The condition $v_0 = v_T/2$ defines a critical field

$$E_c = (m\omega^2 k_B T/2e^2)^{1/2} < E_h,$$

for which γ_k is positive and maximum. Furthermore, increasing $E_0 (E_0 \to \infty)$, the damping rate approaches zero. Physically, this may be understood as follows: Consider the problem of one electron in an electromagnetic field described by $\mathbf{A}(t)$ and moving in a potential V (the plasmon field). We have

$$H = (-1/2m)[\mathbf{p} - (e/c)\mathbf{A}(t)]^2 + V = H_0 + V.$$

In the case of a strong field, such that $|eA| \gg |V|$, the Hamiltonian may be approximated by the first term H_0 which is the free-particle Hamiltonian. In other words, under strong fields, the electron no longer sees the plasmons; the electron-plasmon interaction becomes frozen $(\gamma_k = 0)$.

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Shear stabilization of the purely growing trapped particle instability

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(Received 12 February 1976; final manuscript received 10 January 1977)

The collisionless trapped particle instability in toroidal systems is studied as a radial boundary value problem. It is shown that in the presence of a small amount of magnetic shear, electrons can flow along the field lines to stabilize the mode.

The trapped particle modes in toroidal machines were first studied by Kadomtsev and Pogutse, who showed that a purely growing instability can be excited by the bad curvature drift of particles trapped in the toroidal magnetic field.[1] They found that stability requires either an amount of magnetic shear strong enough to reverse the direction of the drift, or a certain condition on the density and temperature radial profiles. Another stabilization mechanism was proposed by Pogutse who suggested that the finite trapped ion radial displacement (banana size) can stabilize the short wavelength limit of the mode.[2,3] In obtaining this result, which is similar to the finite ion Larmor radius effect on the flute mode in a slab geometry,[4] the radial wavenumber was considered constant. We remove this limitation by letting the magnetic shear, together with the appropriate boundary conditions, determine the radial mode dependence. The parallel inertia of the circulating electrons and finite banana size effects are also included.

Consider a static magnetic field of the form

$$\mathbf{B} = B_t(1 - \epsilon\cos\theta)\hat{e}_\phi + B_p(r)\hat{e}_\theta,$$

where ϕ and θ are the toroidal and poloidal angles, respectively, with the corresponding unit vectors $\hat{e}_\phi$ and $\hat{e}_\theta$. $\epsilon \equiv r/R$ is the toroidicity (inverse aspect ratio) assumed much less than 1. The equilibrium distribution functions of the circulating and trapped particles of