# Geomagnetic Attitude Control of the Brazilian Scientific Satellite - SACI-1

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***Summary -*** This papers deals with the attitude dynamics and control of the Brazilian Scientific Application Satellite (SACI-1). It is a 60 Kg small satellite carrying experiments for scientific purposes. The satellite orbit is polar at 750 Km altitude. The attitude control combines passive spin stabilization with active Geomagnetic Attitude Control. The main task of the Attitude Control Subsystem (ACS), is to: a) maneuver the vehicle to point its solar panel towards the Sun; b) to execute the satellite spin up; c) to keep the attitude and spin rate close to the nominal specifications during the satellite lifetime.

## INTRODUCTION

The Brazilian Scientific Satellite, SACI-1 is to be launched as a piggyback satellite by Chinese Long March 4 launcher. The ACS combines passive attitude control (spin stabilization) with active magnetic attitude control. The subsystem comprises: one analogue sun sensor (ASS); a three-axis magnetometer sensor (MAG); three torque coils and their electronics; a viscous ring nutation damper, and interfaces. There is also an onboard computer to execute the automatic attitude control. The main objectives of the ACS are: to spin-up the satellite; to drive the satellite from after-separation attitude conditions to payload-operating conditions; to keep the nominal attitude and spin rate close to the specification during the satellite lifetime. This paper focus on the magnetic attitude control.

## TORQUE COILS

The use of the geomagnetic field yields one of the simplest and most practical means of satellite attitude control. The interaction between the onboard electromagnetic dipole moment and the geomagnetic field provides the control torque. The torque **N** upon a satellite produced by a magnetic moment **M** interacting with a magnetic field **B** is

$$\mathbf{N}_m = \mathbf{M} \times \mathbf{B} \qquad (1)$$

Thus the torque will have no component in the direction of **M**. For attitude acquisition and attitude control, the required torque shall be normal to the spin axis. This torque can be obtained with a magnetic moment aligned to the spin axis. On the other hand, for satellite spin up and spin speed corrections the required torque shall be aligned with the spin axis. This can be accomplished with a magnetic moment on the spin plane. For the SACI-1 two of the torque coils will be attached to the satellite lateral sides with the magnetic moment axis parallel to the x and y axes respectively (spin plane coils). The third one will be parallel to the z-axis (spin axis coil). Fig. 1 shows the torque coils attached to the satellite.
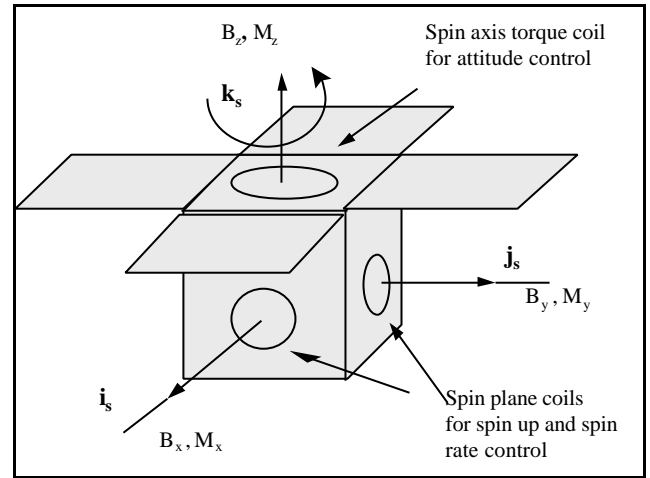


Fig. 1. Torque coils configuration on the satellite.

The whole satellite active attitude control will be performed by using torque arisen from the coils magnetic moment interaction with the Earth magnetic field. The spin plane coils will be used mainly to spin-up the satellite and to execute the spin rate control. The spin axis coil will be used for two basic maneuvers: to point the solar panels towards the Sun and to keep the nominal attitude during the satellite lifetime. The spin axis torque coil shall provide a magnetic moment of $8.0 \pm 1.0$ A.m$^2$ in the positive or negative sense. The spin plane coil shall provide a

magnetic moment of 4.0 ± 1.0 A.m$^2$ in the positive or negative sense. The torque coils specifications are shown in Table I.

The magnetic attitude control requires a sensor, the magnetometer. This sensor detects the magnetic field magnitude and direction. This information is used by the onboard computer to calculate the control.

Table I: Torque coils specifications

| Specifications | Spin axis coil | Spin plane coils |
|---|---|---|
| Magnetic Moment | 8 ± 1 A.m$^2$ | 4 ± 1 A.m$^2$ |
| Inner diameter | 298 mm | 234 mm |
| External diameter | 324 mm | 264 mm |
| Height | 13 mm | 12.5 mm |
| Voltage | 28 Volts | 28 Volts |
| Operational temperature | -20 $^0$ C to 40$^0$ C | -20 $^0$ C to 40$^0$ C |
| Number of turns | 900 | 1080 |
| Attachment flange | 8 holes | 8 holes |
| Hole diameter | 3.2 mm | 3.2 mm |
| Wire AWG | 30 (Termofix) | 32 (Termofix) |
| Nominal current | 70 mA | 100 mA |
| Power | 2 W | 2.7 W |
| Resistance | 400 Ω | 270 Ω |
| Weight | 0.8 Kg | 1.2 Kg |

For an altitude of 750 km and assuming the worst-case polar magnetic field, M=0.44 gauss, the torque generated by a 4 A.m$^2$ coil is about 1.76x10$^{-4}$ N.m. The gravity gradient torque is about 2.28x10$^{-6}$ N.m for the same altitude and an angle of 30$^0$ with respect to the local vertical. Thus the magnetic torque is almost 80 times the gravity gradient torque.

### MATHEMATICAL MODEL AND CONTROL LAWS

The mathematical model takes into account a partially filled viscous ring nutation damper. The details of a satellite and the damper mathematical modeling can be found in [1]. The rotational equations are derived by the Lagrangian formulation for quasi-coordinates [2]. By this approach it is necessary to derive the kinetic energy, which is a function of the components of the angular velocity vector and the fluid rotational speed, $\dot{\beta}$. It depends also on the angular position of the fluid ($\textbf{b}$) inside the nutation damper annular ring. The movement of the fluid causes variations on the satellite moments and products of inertia. Thus the kinetic energy expression can be written as

$$T = T(\omega_x \omega_y, \omega_z, \dot{\beta}, \beta)$$

where $\omega_x$, $\omega_y$, and $\omega_z$ are the components of the angular velocity vector.

The Lagrange's formula for quasi-coordinates is:

$$\frac{d}{dt}\left\{\frac{\partial T}{d\omega}\right\} - [\tilde{\omega}]\left\{\frac{\partial T}{d\omega}\right\} = \{N\}$$

where $\left\{\dfrac{\partial T}{d\omega}\right\} = \left\{\begin{matrix}\dfrac{\partial T}{d\omega_x} & \dfrac{\partial T}{d\omega_y} & \dfrac{\partial T}{d\omega_z}\end{matrix}\right\}^T$

$[\tilde{\omega}] = $ is a skew symmetric matrix

$\{N\} = \left\{\begin{matrix}N_x & N_y & N_y\end{matrix}\right\}^T$ is the total disturbing torque vector plus the control torque.

The Lagrange's formula for generalized coordinates is used to derive the dynamic equation of the fluid associated to the partially filled viscous ring nutation damper. The formula is

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\beta}}) - (\frac{\partial T}{\partial \beta}) = Q_\beta$$

where $\beta$ is the generalized coordinate representing the position of the fluid slug inside the nutation damper annular ring at time $t$ and $Q_b$ is the generalized force associated to the fluid viscosity, Reynolds' number and the nutation damper parameters. It can be written as $Q_\beta = c_d R\dot{\beta}$, where $c_d$ is a damping coefficient. By using the Lagrange formulation for quasi-coordinates the rotational equations can be written as

$$
\begin{aligned}
&(I_x + I_{xf})\dot{\omega}_x + [(I_z - I_y) + (I_{zf} - I_{yf})]\omega_y\omega_z + \\
&-I_{xy}\dot{\omega}_y - I_{xzf}(\dot{\omega}_z + \ddot{\beta}) + I_{az}\ddot{\beta} + \dot{I}_x\omega_x - \dot{I}_{xyf}\omega_y + \\
&-\dot{I}_{xzf}(\omega_z + \dot{\beta}) + I_{yz}(\omega_z^2 - \omega_y^2) + I_{xy}\omega_x\omega_z + \\
&-I_{xz}\omega_x\omega_y + I_{yz}\omega_z\dot{\beta} + (I_{zf} - I_{ax})\omega_y\dot{\beta} = N_x
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
&(I_y + I_{yf})\dot{\omega}_y + [(I_x - I_z) + (I_{xf} - I_{zf})]\omega_x\omega_z + \\
&-I_{xyf}\dot{\omega}_x - I_{yzf}(\dot{\omega}_z + \ddot{\beta}) + \dot{I}_{yf}\omega_y - \dot{I}_{xyf}\omega_x - \dot{I}_{yzf}(\omega_z + \dot{\beta}) + \\
&+I_{xzf}(\omega_x^2 - \omega_z^2) + I_{yzf}\omega_x\omega_y - I_{xyf}\omega_y\omega_z + \\
&+(I_{az} - I_{xzf})\omega_z\dot{\beta} - (I_{zf} - I_{ax})\omega_x\dot{\beta} = N_y
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
&(I_z + I_{zf})\dot{\omega}_z + [(I_x - I_y) + (I_{xf} - I_{yf})]\omega_x\omega_y + \\
&+(I_z - I_{ax})\ddot{\beta} - I_{xzf}\dot{\omega}_x - I_{yzf}\dot{\omega}_y + \dot{I}_{zf}\omega_z + (\dot{I}_z + \dot{I}_{ax})\dot{\beta} + \\
&-\dot{I}_{xzf}\omega_x - \dot{I}_{yzf}\omega_y + I_{xyf}(\omega_y^2 - \omega_x^2) + \\
&+[I_{xzf}\omega_y - I_{yzf}\omega_x](\omega_z + \dot{\beta}) + I_{az}\omega_y\dot{\beta} = N_z
\end{aligned}
\tag{4}
$$

By using the Lagrange's formula for generalized coordinates the dynamic equation for the fluid slug (nutation damper) becomes

$$
\begin{aligned}
&m_f R^2 \ddot{\beta} + (I_{zf} - I_{ax})\dot{\omega}_z - (I_{xzf} - I_{az})\dot{\omega}_x - I_{yzf}\dot{\omega}_y + \\
&-(\dot{I}_{zf} - \dot{I}_{ax})\omega_z - \dot{I}_{xzf}\omega_x - \dot{I}_{yzf}\omega_y + \\
&-\frac{1}{2}\frac{\partial}{\partial\beta}(I_{xf}\omega_x^2 + I_{yf}\omega_y^2 + I_{zf}\omega_z^2) + \\
&-\frac{\partial}{\partial\beta}\left[\left(I_{zf} - I_{ax}\right)\omega_z - I_{xzf}\omega_x - I_{yzf}\omega_y\right]\dot{\beta} + \\
&+\frac{\partial}{\partial\beta}\left[-I_{yzf}\omega_y\omega_z - I_{xzf}\omega_x\omega_z - I_{xyf}\omega_x\omega_y\right] = Q_\beta
\end{aligned} \tag{5}
$$

The control torque (magnetic) $\mathbf{N}$ is given by Eq.(1) where $\mathbf{M}$ can be written as

$$
\mathbf{M} = \begin{cases} U\mathbf{k}_s, \\ V\mathbf{i}_s \end{cases} \quad U, V = \text{magnetic dipole magnitude.} \tag{6}
$$

where $\mathbf{k}_s$ is the unit vector along the spin axis and $\mathbf{i}_s$ is the unit vector along one of the satellite transversal axis.

SPIN AXIS ATTITUDE CONTROL

The spin axis can be steered between two given attitudes by conveniently switching the axis magnetic coil. The control policy derived here is based on [3]. Denoting by $\mathbf{k}_f$ the unit vector corresponding to the spin axis desired attitude and by $\mathbf{H}$ the actual angular momentum vector, the error vector can be written as

$$
\mathbf{E} = \mathbf{k}_f - \frac{\mathbf{H}}{H} \tag{7}
$$

where $H$ is the magnitude of $\mathbf{H}$. The objective of the control is to reduce the error $\mathbf{E}$ to zero. Differentiating the error $\mathbf{E}$ with respect to time and taking into account that $\mathbf{N}_m$ is equal to the rate of change in time of $\mathbf{H}$, one obtain

$$
\frac{d\mathbf{E}}{dt} = \left(-\frac{1}{H}\right)\mathbf{N}_m + \left(\frac{\dot{H}}{H^2}\right)\mathbf{H} \tag{8}
$$

Considering $\mathbf{N}_m$ orthogonal to $\mathbf{H}$ for spin axis control, that is, $\mathbf{N}_m = U\mathbf{k}_f \times \mathbf{B}$, it can be checked after some algebraic manipulation that $\dot{H} = 0$. Then Eq.(9) can be written as

$$
\frac{d\left(E^2\right)}{2dt} = -\frac{\mathbf{E}\cdot\mathbf{N}_m}{H} \tag{9}
$$

The sufficient asymptotic stability condition for $\mathbf{E}$ is

$$
\frac{dE^2}{dt} \leq 0 \tag{10}
$$

That yields the control criteria

$$
U\mathbf{E}\cdot(\mathbf{k}_s \times \mathbf{B}) \geq 0 \tag{11}
$$

By assuming $U$ as a bang-bang controller and defining a switching function as

$$
s_1 = \mathbf{E}\cdot(\mathbf{k}_s \times \mathbf{B}) \tag{12}
$$

then the control criteria to govern the polarity of $U$ are expressed as

$$
U = \begin{cases} \alpha^2 & \text{when } s_1 > 0 \\ -\alpha^2 & \text{when } s_1 < 0 \end{cases} \tag{13}
$$

By selecting the polarity of the dipole moment according to the sign of $s_1$, the magnitude of $\mathbf{E}$ always decreases. Therefore, the desired orientation can be achieved from any initial state.

SPIN RATE CONTROL

The spin rate control is performed by using the spin plane magnetic coil whose axis is orthogonal to the spin axis. The formulation is similar to that for the spin axis control but the error $\mathbf{E}$ is given by:

$$
\mathbf{E} = \mathbf{H}_f - \mathbf{H}, \quad \mathbf{H}_f = I_s\omega_s\mathbf{k}_s \tag{14}
$$

The control torque in this case is given by $\mathbf{N}_m = V(\mathbf{i} \times \mathbf{B})$ so that the switching function for spin rate control is given by

$$
\begin{aligned}
s_2 &= \mathbf{E}\cdot(\mathbf{i}_s \times \mathbf{B}) \quad \text{and} \\
V &= \begin{cases} \beta^2 & \text{when } s_2 > 0 \\ -\beta^2 & \text{when } s_2 < 0 \end{cases}
\end{aligned} \tag{15}
$$

SIMULATION RESULTS

The equations of motion presented here are written in a reference frame aligned to the satellite principal axes of inertia. It is necessary to write these equations and the control torque, in terms of the satellite orbital position and the spin axis attitude. This attitude is given by the spin axis azimuth and elevation angles with respect to the orbital plane. The equations must be written also in the spin angle variable.

In practice every time the spin rate decays below 5 rpm the onboard computer will activate and control the coils polarity to execute the spin rate control increasing the speed up to this nominal value (6 rpm). The polarity of the spin plane torque coils must be changed twice per rotation at a constant phase angle. A mathematical model of the geomagnetic field, the Sun angle and the environmental disturbances are also necessary in order to simulate the attitude dynamics and control. The knowledge of the Sun position is necessary because in the normal mode operation the satellite must point the solar panels towards the Sun. In practice the Sun Sensor will detect the Sun and will inform its position to the onboard computer. The computer will check for the errors and will activate the torque coils to correct the satellite attitude. The gravity-gradient torque, the torque related to the Foucault current and the residual magnetic moment inside the satellite are also taken into account by the computer simulations. Details of the mathematical models can be found in [1,3,4].

A simulation program [4] has been developed to test the control laws. The results are shown in the Fig. [2-4]. The main input dada used to simulate the attitude and control are

Altitude: 750 km:
Satellite initial spin: $\approx 0.001$ rpm
Principal Moments of Inertia): $I_x = I_y = 2.7$ Kg.m$^2$ and
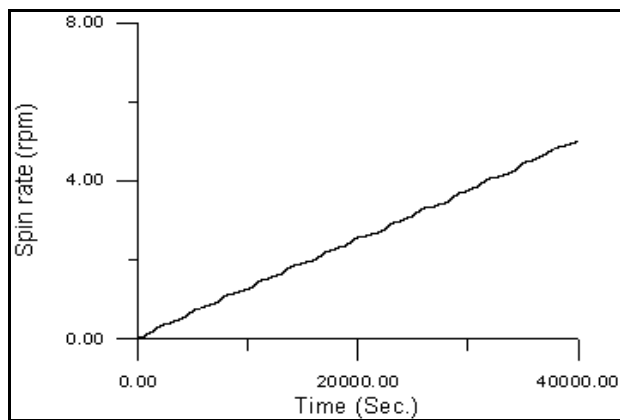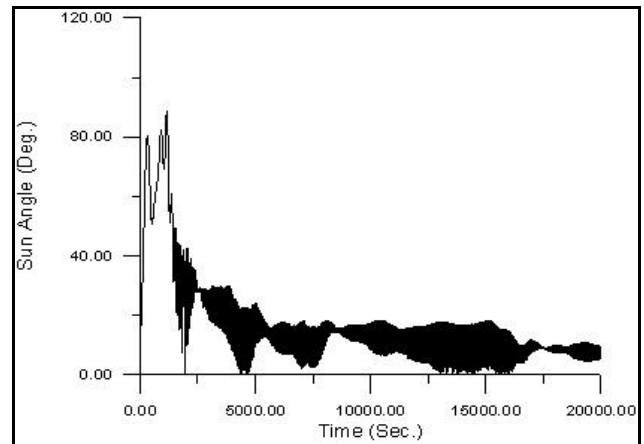$I_z = 4.6$ Kg.m$^2$



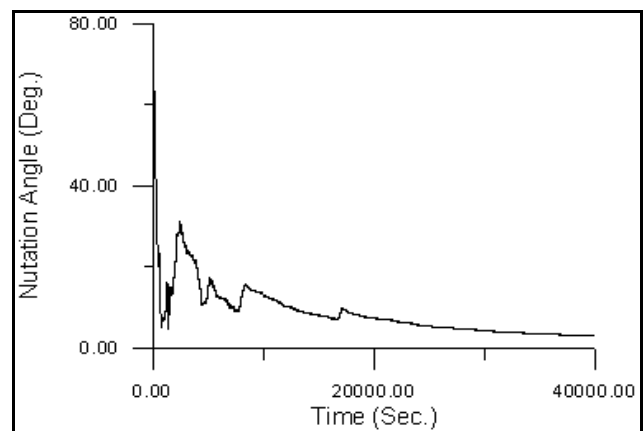Fig. 2: The satellite spin up



Figure 3 - Sun acquisition



Fig. 4: Nutation Angle decay during the Sun acquisition phase.

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