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MULTI-OBJECTIVE OPTIMIZATION APPLIED TO SATELLITE CONSTELLATIONS I: FORMULATION OF THE SMALLEST LOSS CRITERION

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ABSTRACT

In this work the problem of orbital maintenance of symmetrical constellations of satellites, with minimum fuel consumption, is studied using impulsive maneuvers with time constraint. To perform the station keeping of a constellation of *n* satellites we have the problem of simultaneously optimizing the maneuvers for *n* satellites using scarce resources. When we consider all the satellites it is not simple to determine the optimal maneuver strategy that minimizes the fuel consumption with time constraint. Therefore, the goal of this work is to formulate and to study maneuver strategies that makes possible to obtain solutions with small fuel consumption considering all the satellites in the constellation. The problem can be formulate as a multi-objective problem due to the nature of the station-keeping of a satellite constellation. Thus, the multi-objective problem applied to satellite constellations is defined and a new method of multi-objective optimization is presented. This method can consider *n* conflicting objectives simultaneously without reducing the problem to an optimization of only one objective, as occur with most of the methods found in the literature. This new method, called the smallest loss criterion, was compared with other existent methods and it was verified that it is capable to supply better results to the problem of the station-keeping of satellite constellations.

THE MULTI-OBJECTIVE PROBLEM

The analysis of multi-objective problems was developed, mainly, in Economy, in Sociology, in Psychology and in Operational Research. However, it is possible to find multi-objective problems in many other areas. Actually, we worked with multi-objective problems the whole time: when we make most of the daily decisions we are working with problems of this type. The simple act of choosing which plate to order during the lunch could become a big problem if we tried to optimize the choice using a multi-objective approach: which factors should be taken in consideration in the choice? The price, the flavor, the appearance or the nutrients that the plate can supply? Perhaps the cheapest plate is the less tasty, the most expensive is the less nutritious and the tastiest has the worse appearance. Considering these factors, it is difficult to choose a solution that optimize all the factors. But of course, we didn't waste so much time everyday in the choice of our lunch. In some way we found the solution for this problem with less effort, but it is not possible to affirm that it is the optimal solution, it is simply the solution that in that moment, for some reason, perhaps external to the problem, seemed to be the best one. But for more important applications than the choice of the best plate, it would be good if we could find the optimal solution systematically. In engineering, for example, we cannot choose the solution for a problem in an aleatory or uncertain way. It would be convenient to apply a methodology capable to find the solution that assists all the objectives in the best possible way. But that it is a very complex task, mainly when we worked with problems whose objectives are conflicting, that is, to assist a certain objective obligatorily the other objectives will be pained. According to the Theory of Differential Games (Isaacs[1]) we can affirm that we have a game of the lose-win type: for a player to be victorious, the other necessarily has to be defeated. In the algorithm developed in this work we considered an intermediary case. Perhaps for applications in engineering it is more convenient a draw among the players.

According to Cohon[2], the static optimization of problems with one objective can be defined in the following way:

$$\begin{array}{ll} \text{Maximize} & \mathrm{Z}(\boldsymbol{x}) \text{ with relation of } \boldsymbol{x} \in \boldsymbol{R}^n \\ \text{Subject to} & \mathrm{g_i}(\boldsymbol{x}) \leq 0 \quad i = 1, 2, ..., m \\ & \boldsymbol{x} \geq 0 \\ \text{Given} & \mathrm{Z}(\cdot),\ g_i(\cdot) \end{array} \tag{1}$$

or

$$\begin{array}{ll} \text{Maximize} & \mathrm{Z}(\boldsymbol{x}) \text{ with relation of } \boldsymbol{x} \in \boldsymbol{R}^n \\ \text{Subject to} & \boldsymbol{x} \in \boldsymbol{F}_d \\ \text{Given} & \mathrm{Z}(.),\ \boldsymbol{F}_d \end{array} \tag{2}$$

where $\boldsymbol{F}_d$ is the feasible area of the decision space, defined by:

$$\boldsymbol{F}_d = \left\{ \boldsymbol{x} \in \boldsymbol{R}^n \mid \mathrm{g_i}(\boldsymbol{x}) \leq 0, i = 1, 2, \ldots, m; \boldsymbol{x} \geq 0 \right\} \tag{3}$$

The multi-objective problem can be defined by:

$$\text{Maximize } \mathbf{Z}(\boldsymbol{x}) = \left[\mathrm{Z}_1(\boldsymbol{x}), \mathrm{Z}_2(\boldsymbol{x}), ..., \mathrm{Z}_p(\boldsymbol{x}) \right] \tag{4}$$

Subject to $\boldsymbol{x} \in \boldsymbol{F}_d$

Therefore, in this case, the objective function, is a vector with dimension p.

In problems of unidimensional optimization (when we have one objective), the possible solutions ($\boldsymbol{x} \in \boldsymbol{F}_d$) can be compared by means of the objective function, that is, given two solutions $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ we can compare $Z(\boldsymbol{x}^1)$ with $Z(\boldsymbol{x}^2)$ and to determine the optimal solution so that $\boldsymbol{x} \in \boldsymbol{F}_d$ doesn't exist such that $Z(\boldsymbol{x}) > Z(\boldsymbol{x}^*)$. In problems of multi-dimensional optimization (multi-objective problem), in general, it is not possible to compare all the possible solutions because the comparison on the basis of one objective can be contradicted with the comparison based on another objective. Namely, supposing that:

$$\begin{aligned} \mathbf{Z}(\boldsymbol{x}^1) &= \left[\mathrm{Z}_1(\boldsymbol{x}^1), \mathrm{Z}_2(\boldsymbol{x}^1) \right] \\ \mathbf{Z}(\boldsymbol{x}^2) &= \left[\mathrm{Z}_1(\boldsymbol{x}^2), \mathrm{Z}_2(\boldsymbol{x}^2) \right] \end{aligned} \tag{5}$$

$\boldsymbol{x}^1$ is better than $\boldsymbol{x}^2$ if and only if:

$$\mathrm{Z}_1(\boldsymbol{x}^1) > \mathrm{Z}_1(\boldsymbol{x}^2) \text{ and } \mathrm{Z}_2(\boldsymbol{x}^1) \geq \mathrm{Z}_2(\boldsymbol{x}^2) \tag{6}$$

or

$$\mathrm{Z}_1(\boldsymbol{x}^1) \geq \mathrm{Z}_1(\boldsymbol{x}^2) \text{ and } \mathrm{Z}_2(\boldsymbol{x}^1) > \mathrm{Z}_2(\boldsymbol{x}^2) \tag{7}$$

If $\mathrm{Z}_1(\boldsymbol{x}^1) > \mathrm{Z}_1(\boldsymbol{x}^2)$ and $\mathrm{Z}_2(\boldsymbol{x}^1) < \mathrm{Z}_2(\boldsymbol{x}^2)$ we cannot conclude anything regarding $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$, that is, $\boldsymbol{x}^1$ e $\boldsymbol{x}^2$ cannot be compared

EXISTENT APPROACHES IN THE LITERATURE FOR THE SOLUTION OF THE MULTI-OBJECTIVE PROBLEM

The Weighting Method

A possible solution for the multi-objective problem would be to combine all the objectives to obtain an objective that is formed by the average of the original objectives multiplied by an influence factor:

$$Z_{Médio}(\boldsymbol{x}, \boldsymbol{w}) = \sum_{i=1}^{n} w_i Z_i(\boldsymbol{x}), \text{ for } w_i > 0 \tag{8}$$

$$w_i = \frac{\partial Z_{Médio}(\boldsymbol{x})}{\partial Z_i(\boldsymbol{x})} \tag{9}$$

where w_i are the Lagrange multiplier, or the influence factors, and $Z_{Médio}(\boldsymbol{x}, \boldsymbol{w})$ a dependent function that substitutes the set of objectives. Thus, the problem can be treated as an optimization problem with only one objective and therefore the multi-objective formulation doesn't need to be applied. In the practice, most of the multi-objective problems have been treated in this way: initially a multi-objective approach is used, and later, the influence factors are applied, reducing the problem to a unidimensional optimization.

The use of the influence factors eliminates the necessity of the use of a more complex algorithm of optimization, but it introduces new parameters $\boldsymbol{w}$ that should be found and optimized. In this way, the solution depends on the correct determination of these factors. Therefore, this determination becomes an optimization process by itself. On the other hand, if in a certain multi-objective problem the parameters $\boldsymbol{w}$ are known, the use of the influence factors becomes efficient, but in this case the problem is not a multi-objective problem, because in fact, we just have the objective $Z_{Médio}(\boldsymbol{x}, \boldsymbol{w})$ to be optimized.

The Constraint Method

In this approach the solution for the multi-objective problem is found obtaining the optimal solution for a certain objective while the other objectives are constrained. For the case of the maximization of the objectives we have:

$$\text{Maximize } \mathbf{Z}(\boldsymbol{x}) = \left[\mathrm{Z}_1(\boldsymbol{x}), \mathrm{Z}_2(\boldsymbol{x}), ..., \mathrm{Z}_p(\boldsymbol{x}) \right] \tag{10}$$

Subject to $\boldsymbol{x} \in \boldsymbol{F}_d$

The constraint problem is given by:

$$\begin{array}{ll} \text{Maximize} & Z_h(\boldsymbol{x}) \\ \text{Subject to} & \boldsymbol{x} \in \boldsymbol{F}_d \\ & Z_k(\boldsymbol{x}) \geq L_k \end{array} \tag{11}$$

where $k = 1, 2, \ldots, h-1$ e L_k is a value a priori determined The objective to be maximized is chosen arbitrarily. To illustrate this approach, consider Figure 1, where we intended to maximize simultaneously Z_1 and Z_2. If the objective Z_1 was constrained we have a reduction of the feasible space of solutions for the problem. In this way, for a certain

Z_1 we easily obtain Z_2. However, this is not a multi-objective formulation, because when we chose one of the objectives to be optimized and we constrained all the others, the problem will be transformed in an optimization process with only one objective. Furthermore, this approach can only be used if all the L_k were known and, in the same way that in the Weighting Method, the determination of the L_k can be an optimization process by itself.

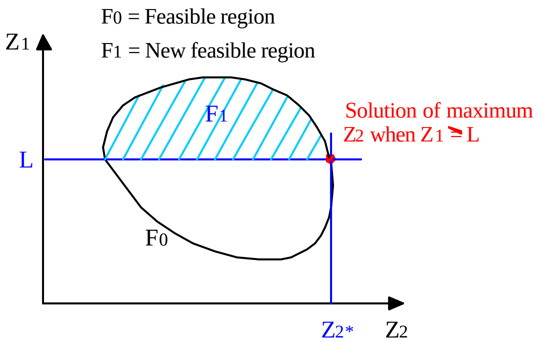

Fig. 1 – Criterion of the constraint objectives.

The Surrogate Worth Tradeoff Method

This approach is based on local approximations made by a specialist (decision maker) during the optimization process. The optimization procedure is a variation of the Constraint Method presented in the previous item. Given a problem with n objectives, $n-2$ objectives are fixed in values initially defined, and the other two are optimized. But in fact, one of those objectives is controlled by the specialist while the other is optimized. Assuming that Z_2 should be maximized and that Z_1 can assume different values, the problem can be formulated in the following way:

$$\text{Maximize} \quad Z_2(\boldsymbol{x}) \tag{12}$$

$$\text{Subject to} \quad \boldsymbol{x} \in \boldsymbol{F}_d$$

$$Z_1(\boldsymbol{x}) \geq L_1$$

$$Z_k(\boldsymbol{x}) = L_k$$

where $k = 3, 4, \ldots, n$ and L_k is a predefined value.

However, this approach presents the same deficiencies of the Constraint Method, and in this case, we have to consider the specialist's performance during the optimization process. In this way the solution for the problem is particular for each specialist. Thus, we cannot affirm that this is the optimal solution for the problem.

The Multi-Objective Simplex Method

The Multi-Objective Simplex Method is an algorithm that supplies the group of non inferior solutions for a multi-objective problem. In fact, the method doesn't supply the best solution, but it finds the worst solution inside of the group of the alternatives and it classifies all the others as non inferior solutions. However, this method can only be used for the linear cases. Therefore it cannot be used in the maneuvers optimization problem, applied in a satellite constellation.

Method Based on Geometrical Definitions of Best

The bases of this approach (Yu[3] and Zeleny[4]) is the idea that the best solution presents the smallest distance with relationship to an ideal solution (utopian solution) previously defined. The method begins with the definition of the ideal solution, composed by the individual solutions. In general, this ideal solution is out of the feasible space of solutions for convex domains. Figure 2 illustrates this method for the case of a problem where we want to maximize Z_1 and Z_2 simultaneously.

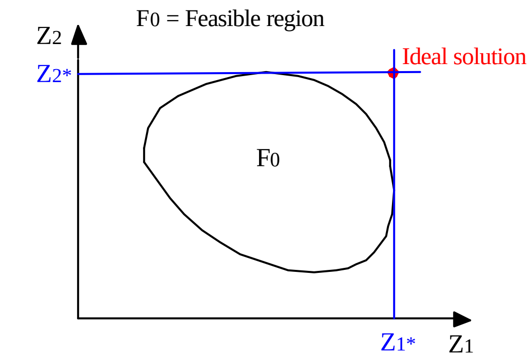

Fig. 2 – Ideal solution for a multi-objective problem.

The optimal solution would be the solution of the feasible region that presented the smallest distance related to the ideal solution, as it can be seen in Figure 3.

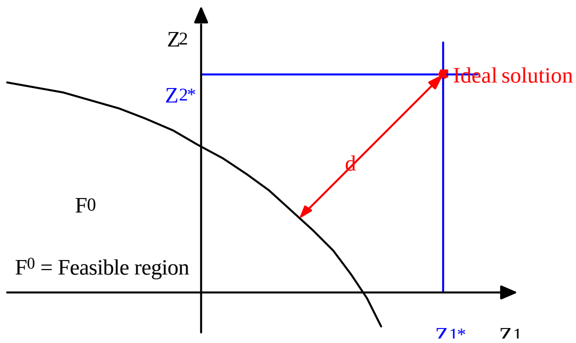

Fig. 3 – Distance from the ideal solution.

The calculation of the distance d between two points $x_1, \ldots, x_p$ and $y_1, \ldots, y_p$, according with this method, it is given by:

$$d_a = \left[\sum_{k=1}^{p} \left| x_k - y_k \right|^a \right]^{\frac{1}{a}} \tag{13}$$

where α varies from 1 to infinite, according to the metric chosen, for $\alpha = 2$ we have the definition of the Euclidean distance between two points.

However, this approach presents three basic deficiencies: 1) it doesn't use normalized measures, thus, the solution depends on the dimension of each objective; 2) the ideal solution composed by the individual solutions is, in general, out of the feasible space of solutions for convex domains, as shown in the Figure2; 3) in general, the definition of the ideal solution, which is the objective of the problem, is not a trivial task, in some cases it is practically impossible to determine, especially when the objectives are conflicting. For example, in the case of maneuvers, where we should minimize the time spent in the maneuver and the fuel consumption, it may be concluded that the ideal solution would be (0;0). But this solution, besides being utopian, is also incorrect because the objectives are conflicting and therefore it would be impossible to obtain a solution like this. And, supposing that the solution with time and fuel consumption equal to zero is the optimal solution for the problem, it would be easily noticed that the satellite would not be maneuvered. Therefore, it should be found another ideal solution. But, in the same way that in the Constraint Method and in the Surrogate Worth Tradeoff Method, will depend on a specialist (decision maker) and in this way the solution for the problem would be particular for each specialist.

The Goal Programming Method

This approach can be considered as a variation of the approach presented in the previous item. The idea that the best solution is the solution of the smallest distance from an ideal solution is also the base of this approach, but in this case, the ideal solution is predefined by a specialist (decision maker), based on his personal experience or in any other subjective method defined by the specialist. In this way, the ideal solution is not the solution that represents the maximum (or minimum) value for each one of the objectives. Therefore it can be inside the feasible space of solutions. Generally, the distance d is calculated by the Equation 13, but with $\alpha = 1$. Thus, the problem is defined in the way:

$$\text{Minimize} \quad d_1 = \sum_{k=1}^{p} \left| G_k - Z_k(\boldsymbol{x}) \right| \tag{14}$$

Subject to $\boldsymbol{x} \in \boldsymbol{F}_d$

where G_k represents an ideal solution defined by the specialist.

But besides the deficiencies 1 and 3 of the previous method, in this method the determination of the ideal solution is an optimization process by itself, and it will depend on each specialist, in this way the solution for the problem becomes particular for each of them.

Pareto Method (Non Inferiority)

Another possibility would be the use of the algorithm of Pareto optimization (Carroll and Mason[5]). This algorithm can be used in problems where the objectives compete to each other. The optimal solution is an element of a group of solutions that are considered equally good in relation to the objective vector. The algorithm should select, starting from the feasible region of the decision space, a group of solutions that support one “a priori” defined criterion. This can be made through a methodology that systematically makes a comparison among candidates. A solution $\boldsymbol{x}$ can only be considered optimal for a certain group of objectives, if a better solution $\boldsymbol{y}$ in all the objectives doesn't exist. This solution $\boldsymbol{x}$ is called *non-dominated* (or *non-inferior*). A solution $\boldsymbol{x}$ is *non-dominated* if it doesn't exist a possible solution $\boldsymbol{y}$ such that:

$$\mathbf{Z}(\boldsymbol{y}) \geq \mathbf{Z}(\boldsymbol{x}) \tag{15}$$

or

$$Z_k(\boldsymbol{y}) \geq Z_k(\boldsymbol{x}), \quad k = 1, 2, ..., p \tag{16}$$

If $\boldsymbol{y}$ exists, then $\boldsymbol{x}$ is a *dominated* solution (or *inferior*) and in this way it is considered as not being a Pareto optimal solution, in other words, it is not the best solution in any of the objectives.

According to Kuhn-Tucker[6], if $\boldsymbol{x}$ is a *non-dominated* solution then multipliers $u_i \geq 0,\ i = 1, 2, \ldots, m$ and $w_k \geq 0,\ k = 1, 2, \ldots, p$ should exist such that:

$$\boldsymbol{x} \in \boldsymbol{F}_d \tag{17}$$

$$u_i g_i(\boldsymbol{x}) = 0, \quad i = 1, 2, \ldots, m \tag{18}$$

$$\sum_{k=1}^{p} w_k \nabla Z_k(\boldsymbol{x}) - \sum_{i=1}^{m} u_i \nabla g_i(\boldsymbol{x}) = 0 \tag{19}$$

The conditions of Kuhn-Tucker, (17) to (19), are the necessary conditions for the solution $\boldsymbol{x}$ to be *non-dominated*. They are also sufficient if $Z_k(\boldsymbol{x})$ are concave for $k = 1, 2, \ldots, p$, $\boldsymbol{F}_d$ is convex, and $w_k > 0$ for all k.

Example 1: To illustrate this procedure, consider the problem with two objectives presented in Table 1 where we have some alternative solutions for the problem. We should choose the alternative that maximize Z_1 and Z_2 simultaneously.

In Table 1 we can observe that the alternative A is the optimal solution only considering the objective Z_2, the alternative B is the optimal solution considering the objective Z_1 and the alternative *C* is not a optimal solution in relation to any objective. Thus, the alternatives *A* and *B* are *non-dominated* solutions, and the alternative *C* is a *dominated* solution or *inferior*. According to Pareto, the alternatives *A* and *B* represent the highest optimal degree in the resolution of the problem.

TABLE 1 – 1º Example of the Pareto Methodology.

Alternative	Z_1	Z_2
A	10	11
B	12	10
C	9	8

Example 2: Another example is given by Table 2, where we should choose the alternative that maximize simultaneously the three objectives. In the table, we have the alternative *I* that is sharply the optimal solution for the problem, because it presents the maximum value for the three objectives Z_1, Z_2 and Z_3. Whether in a multi-objective problem an alternative solution like that exists, the multi-objective approach becomes unnecessary. But a solution of this type can only be found in problems where the objectives are not conflicting. In most of the problems this doesn't happen. Generally, when an objective is maximized it is not possible to do the same to the others.

TABLE 2 – 2º Example of the Pareto Methodology.

Alternative	Z_1	Z_2	Z_3
A	5	8	7
B	4	9	2
C	4	4	4
D	3	10	6
E	2	9	8
F	5	10	1
G	5	3	8
H	1	10	8
I	5	10	8

The alternatives *F*, *G* and *H* maximize two objectives simultaneously. The alternative *F* maximizes Z_1 and Z_2. The alternative *G* maximizes the objectives Z_1 and Z_2. And the alternative *H* maximizes the objectives Z_2 and Z_3. Solution of this type requires two non conflicting objectives. Generally, it is difficult to happen. However, even if it happens, it cannot be affirmed that a solution that maximizes two objectives is better than another that not maximizes any objective or maximizes only one, because, different from the case where the three objectives are maximized, now, one of the objectives does not present the maximum value but a value that can be very small for a certain practical application. For example, in a biological experiment where it want a cavy colony to grow to the maximum, we should supply the three basic elements for the maintenance of life in the colony: water, food and oxygen. If we consider that Z_1 represents the water supply, Z_2 represents the food supply and Z_3 represents the oxygen supply, it can analyzed the alternatives *F*, *G* and *H* in relation to the chances of survival of the colony. The alternative *F* supplies the maximum of water and feeding propitiating necessary conditions for the growth of the colony, however, the alternative *F* supplies the minimum supply of oxygen; therefore, as the growth of the colony happens it is very probable that the supply of oxygen becomes insufficient, and it would cause the death of the whole colony. The same reasoning can be applied to the alternatives *G* and *H*. Thus, the alternatives *F*, *G* and *H*, depending on the application and of the value of the objective not optimized, could become the worst solution alternatives.

In fact, the solutions for multi-objective problems, of the type of the solutions represented by the alternatives *I*, *F*, *G* and *H* are very rare. This kind of solutions occur only in some cases where the objectives are not conflicting. In this way, those alternatives will not be consider in this example.

Analyzing Table 2 again, it can be observed that the alternative *A* is the optimal solution considering only the objective Z_1; the alternatives *B* and *C* are not optimal solutions in relation to any objective; the alternative *D* is the optimal solution only considering the objective Z_2 and the alternative *E* is the optimal solution considering only the objective Z_3. Thus, the alternatives *A*, *D* and *E* are *non-dominated* solutions, and the alternatives *B* and *C* are *dominated* solutions or *inferior*. So, according to Pareto, the alternatives *A*, *D* and *E* present the highest optimal degree in the resolution of the problem, when we do not consider the alternatives *I*, *F*, *G* and *H*.

Example 3: However, we can notice that different domain levels exist. A *dominated* solution will always be inferior in relation to some *non-dominated* solution, but a *dominated* solution can be dominated by other *dominated* solution. Those domain levels provide a characterization of the feasible area of the decision space, sorting the solutions in categories with different levels of optimality. This case is shown in Table 3. The alternatives *A*, *D* and *E* are *non-dominated* solutions of the level 1. Eliminating those alternatives and continuing the process, we determined that the alternatives *B*, *C* and *H* are *non-dominated* solutions of the level 2, because they are the optimal solutions for each one of the objectives considered separately when we excluded the solutions of the level 1. Therefore, the alternatives *F* and *G* are *non-dominated* solutions of the level 3 and the alternative *I* is a solution of the level 4.

TABLE 3 – 3º Example of the Pareto Methodology.

Alternative		Z_1	Z_2	Z_3
A	Level 1	5	8	7
B	Level 2	4	9	2
C	Level 2	4	4	4
D	Level 1	3	10	6
E	Level 1	2	9	8
F	Level 3	1	7	5
G	Level 3	2	5	3
H	Level 2	3	3	7
I	Level 4	1	6	1

Using this procedure, it can be verified that the Pareto optimization process can be basically described as being a search for *non-dominated* solutions. This search consists in sorting the candidates of solutions in several optimality levels and, in this way, to obtain a group of solutions for the multi-objective optimization problem. A variation of the Pareto method would be the search of the inferior solution instead of the search of the solutions of level 1. This procedure, known by the Noninferior Set Estimation Method (NISE), was developed by Cohon et al.[7] and it was mainly applied for problems with two objectives.

Example 4: The search procedure of the inferior solution is shown in Table 4, where we wanted to maximize the three objectives Z_1, Z_2 and Z_3. In this case, all the alternatives of solution must be compared. To facilitate the comparison we will adopt the following approach: if two alternatives present the same value for a certain objective each one receives 1 point; if an alternative is better than the other, the largest receives 2 points and the smallest none. Thus, comparing the alternatives *A* and *B* in relation to the objective Z_1, *A* is larger than *B*, therefore it receives 2 points; in relation to the objective Z_2, *B* is larger than *A*, and now *B* receive 2 points; and in relation to the objective Z_3, *A* is larger than *B*, so, *A* receives more 2 points. Continuing this procedure we obtain: *A* = 16 points; *B* = 10 points; *C* = 7 points; *D* = 14 points; and *E* = 13 points. Therefore the *inferior* is the alternative *C*. However, this method is clearly worse than the Pareto Method, because this method supplies a group of *non-inferior* solutions, and this group is larger than the group of *non-dominated* solutions of level 1 determined by the Pareto Method, that is, according to Cohon et al.[7], given a group of alternatives for solution of a multi-objective problem, the NISE Method just selects the worst alternative.

TABLE 4 – Search of the *Inferior* Solution.

Alternative		Z_1	Z_2	Z_3
A	*Non-Inferior*	5	8	7
B	*Non-Inferior*	4	9	2
C	*Inferior*	4	4	4
D	*Non-Inferior*	3	10	6
E	*Non-Inferior*	2	9	8

Example 5: To illustrate the Pareto Method applied in the orbital maneuvers, we presented in Table 5 several maneuvers that can be executed by a satellite. Each one of the maneuvers, needs a velocity increment Dv, a time T and it generates a position error dq. We wanted to find a maneuver that minimizes Dv, T and dq, but these objectives are conflicting. Therefore, a solution that minimizes the three objectives simultaneously doesn't exist. Thus, we adopted the Pareto methodology and selected the *non-dominated* solutions for the problem.

Examining Table 5, we can select the maneuvers 4, 12, and 1 as *non-dominated* solutions. Thus, each one of them minimizes dq, Dv and T respectively. However, we can continue the classification of the solutions to obtain other groups with different optimality levels: level 1, maneuvers 4, 12 and 1; level 2, maneuvers 5, 11 and 2; level 3, maneuvers 6, 10 and 3; level 4, maneuvers 7 and 9; level 5, maneuvers 8. With this classification we can choose the maneuver that best satisfies the constraints. We could choose any one of the maneuvers of the optimality level 1 to obtain the highest Pareto optimality level in the satisfaction of the problem.

TABLE 5 – Optimal Maneuvers of the 5º Example.

	dq	Dv	T
1	0,565637	1,07333	1660
2	0,445267	1,02667	1700
3	0,314329	0,99600	1800
4	0,118357	0,88733	2110
5	0,217551	0,85493	2250
6	0,241902	0,84267	2305
7	0,275673	0,80400	2550
8	0,278017	0,79867	2600
9	0,291305	0,78667	2705
10	0,289064	0,77867	2800
11	0,314594	0,76400	2910
12	0,337307	0,76212	2990

This methodology has been used, without practically any improvement, since 1909 when it was presented by Pareto[8]. Some authors use the Pareto Methodology to find the solutions of level 1 but after this, they adopt weights or influence factors to choose one of the solutions of the level 1. Therefore, the application of the Pareto Methodology becomes unnecessary, because in fact, the method of the influence factors, that was presented in the previous item, that is being used. Thus, the Pareto Methodology supplies a group of solutions, candidates to the optimal solution, but it is not capable to find the best solution from this group.

TABLE 6 – Normalized Optimal Maneuvers of the 5° Example

$\delta\theta_{máx} = 0,7$ rad $\quad \Delta v_{máx} = 1,2$ km/s $\quad T_{máx} = 3000$ s

	$\delta\theta/\delta\theta_{máx}$	$\Delta v/\Delta v_{máx}$	$T/T_{máx}$
1	0,808053	0,894442	0,533333
2	0,636096	0,855558	0,566667
3	0,449041	0,830000	0,600000
4	0,169081	0,739442	0,703333
5	0,310787	0,712442	0,750000
6	0,345574	0,702225	0,768333
7	0,393819	0,670000	0,850000
8	0,397167	0,665558	0,866667
9	0,416150	0,655558	0,901667
10	0,412949	0,648892	0,933333
11	0,449420	0,636667	0,970000
12	0,481867	0,635100	0,996667

THE SMALLEST LOSS CRITERION

In practical applications, it would be convenient to apply a methodology capable to find the solution that assists all the objectives. The Pareto Optimization Method supplies a group of solutions where, according to Pareto, each solution present the same optimality degree. In the 5^{th} Example the solutions of the optimality level 1 are the maneuvers 4, 12 and 1. But which maneuver should be chosen? All of them are optimal solutions for a certain objective; If we choose any of those solutions, we would also be choosing a certain objective as priority, and in this way, we would also be choosing a certain objective as priority. In this way we would be going back to the problem of choosing influence factors to the objectives, which is an optimization process by itself. Thus, the Pareto methodology becomes unnecessary because we could selected the influence factors before the Pareto optimization process and decide which is the best maneuver to be used. Therefore, it seems that the optimal solution is not possible to be found, unless we change the definition of optimal solution for the multi-objective problem. The definition for multi-objective optima solution could be: the solution of smaller loss for all the objectives. We can call this solution, sub-optimal solution of the multi-objective problem, since the optimal "classic" solution is not possible to obtain.

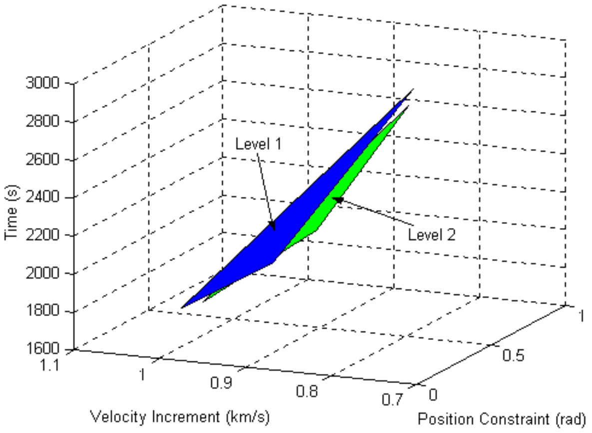

Fig. 4 – Levels 1 and 2 of the 5° Example.

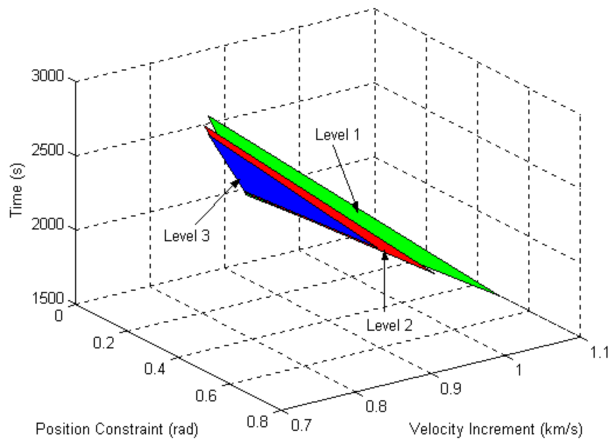

Fig. 5 – Levels 1, 2 and 3 of the 5° Example.

However, if we draw in a three-dimensional graph the points of the maneuvers 4, 12 and 1, and 5, 11 and 2, for the levels 1 and 2, respectively, it will be obtained the graph shown in Figure 4. The triangles blue and green represent the levels of Pareto optimality. Figure 5 shows the triangles that symbolize the levels 1, 2 and 3. The vertices of the triangles represent the optimal solutions for each one of the objectives for the different levels. These solutions are extreme solutions, because each one of them consider only one objective as priority. To obtain a solution that does not assume any objective as priority, it would be necessary to choose an intermediary solution. Therefore, we should adopt an approach for this choice, because otherwise, considering the objectives equally, the choice would be aleatory. This approach can be based on the following argument:

<u>Argument 1</u>: We can observe that Nature, presents solutions with an accentuated symmetry: the bodies of the living beings are generally presented in a bilateral symmetry; the format of the crystals; the

symmetry of the subatomic particles; the seasons; the mass distribution in the planets and in the planetary systems, are examples of symmetry that is resulting of the interactions among the several elements of Nature. These interactions, generate many phenomenon that can be treated as multi-objective problems, which are solved in some way, generating solutions that are the optimal solutions found by Nature after billions of years of evolution and interactions. Therefore, the multi-objective problem can be considered as a resolvable problem. And, we can infer by the observation of Nature, that perhaps, this kind of problem has only one solution. The best example of this is our own body. We lived in a environment where our survival depends on many factors that generate a complex multi-objective problem. However, we are perfectly adapted to this environment, that is, we are the optimal solution of the multi-objective problem of the human evolution.

Thus, it is expected that in the Engineering multi-objective problems should be found solutions with the same symmetry. For this reason, in multi-objective problems, an extreme solution cannot be considered as the optimal solution. Only an intermediary solution is capable to take in consideration the symmetry between the candidates to the optimal solution. A possibility to find this solution is given by the following criterion:

Criterion 1: An attempt to find this intermediary solution would be to find the barycenter of the triangle shown in the Figure 4 and 5, normalized by the maximal of the objectives. This normalization (Table 6) is necessary because otherwise the solution depends on the dimension of each objective. The barycenter is the solution which generates the smallest loss in relation to all the objectives. Thus, the optimal solution for the multi-objective problem would be the central point of the figure that has as vertexes the optimal solutions for each objective. Therefore, for problems with three objectives the solution would be in the center of a normalized triangle, for *n* objectives the solution would be in the center of a normalized *n*-dimensional figure. This criterion we call the Smallest Loss Criterion. In Figure 6 we have an example of this criterion applied to a problem with three conflicting objectives. In this example, *S*1, *S*2 and *S*3 are the optimal solutions for each one of the objectives, considered separately. B is the barycenter of the triangle formed by the segments $\overline{S1S2}$, $\overline{S2S3}$ and $\overline{S3S1}$. By the barycenter definition, the distance from *B* to the vertexes of the triangle represented by the solutions *S*1, *S*2 and *S*3 is the same. So, if the barycenter *B* is adopted as a solution for the multi-objective problem, the segment $\overline{S1B}$ represents the loss in relation to the objective 1, and in the same way, the segments $\overline{S2B}$ and $\overline{S3B}$ represent the loss in relation to the objectives 2 and 3 respectively. Thus, from the Figure 6 we can conclude that if the three objectives are equally considered, the best solution is that one which coincides with the barycenter of the triangle.

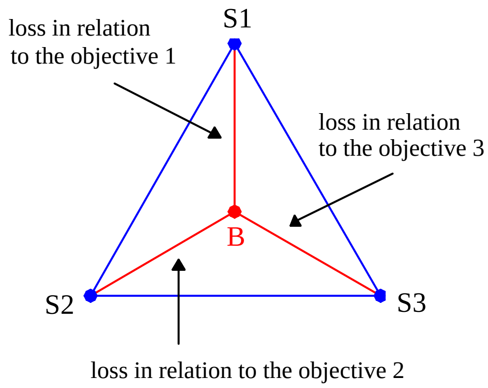

Fig. 6 – The Smallest Loss Creiterion.

Returning to the 5th Example and using the Smallest Loss Criterion, the best solution considering the triangle of level 1 and making the calculations with normalized measures (Table 6), is given by: $\delta\theta$ = 0,340434 rad; Δv = 0,907593 km/s; t = 2233,333 s (Criterion 1). However, we don't have this maneuver in the Table 5. Thus, the best maneuver seems to be that whose parameters are the closest of the coordinates of the barycenter (Criterion 1A). Calculating the distances between the barycenter and the points that represent the maneuvers, using the Table 6, we concluded that the best maneuver to be applied is the maneuver 6. Other possibility, would be to use the program that calculates the maneuvers again, with the time of 2233,333 s as entrance, to calculate a maneuver that approaches the coordinates of the barycenter (Criterion 1B).

Example 6: To exemplify this methodology applied in a satellite constellation, we will use as example a constellation composed by three satellites with circular and equatorial nominal orbits. Therefore, the nominal elements of the satellites are given by:

e	=	0,000000	a	=	7010,000000 km
l	=	7010,000000 km	i	=	0,000000 rad
ω	=	0,000000 rad	Ω	=	0,000000 rad

To assist the specifications of the mission, the satellites should be positioned in such a way that the difference between the true longitudes ($\Delta\theta_1$, $\Delta\theta_2$ and $\Delta\theta_3$) should be 2,09439435 rad (120 degrees). It may be assumed that in the initial instant the satellite 1 is entering in the visibility cone of the ground tracking station, and the measures of X, Y, Z, $\dot{X}$, $\dot{Y}$ and $\dot{Z}$ in the inertial reference system are given by:

X	=	7000,00000000	$\dot{X}$	=	0,00000000
Y	=	0,00000000	$\dot{Y}$	=	7,54605517
Z	=	0,00000000	$\dot{Z}$	=	0,00000000

Thus, we determined the actual elements of the satellite 1, where M is the mean anomaly, u is the eccentric anomaly, f it is the true anomaly and θ is the true longitude.

Satellite 1:

a_1	=	7000,00000000	Ω_1	=	0,00000000
e_1	=	0,00000000	ω_1	=	0,00000000
i_1	=	0,00000000	M_1	=	0,00000000
u_1	=	0,00000000	f_1	=	0,00000000
θ_1	=	0,00000000			

The difference between the semi-major axis a_1 of the actual orbit and the semi-major axis of the nominal orbit, is Δa_1 = -10 km. The other elements are in agreement with the nominal orbit. However we will consider that Δa_1 is larger than the allowed variation and therefore, the orbit should be corrected. With the orbital propagation we can obtain the actual elements of the others satellites.

Satellite 2:

a_2	=	7002,00000000	Ω_2	=	0,00000000
e_2	=	0,00000000	ω_2	=	0,00000000
i_2	=	0,00000000	M_2	=	1,91986218
u_2	=	1,91986218	f_2	=	1,91986218
θ_2	=	1,91986218			

Satellite 3:

a_3	=	7005,00000000	Ω_3	=	0,00000000
e_3	=	0,00000000	ω_3	=	0,00000000
i_3	=	0,00000000	M_3	=	4,36332313
u_3	=	4,36332313	f_3	=	4,36332313
θ_3	=	4,36332313			

With the actual true longitudes θ_1, θ_2 and θ_3 we can calculate the position constraints $\delta\theta_1$, $\delta\theta_2$, $\delta\theta_3$ and $\delta\theta$, which represent the position error of the satellites, as shown in the Figure 7:

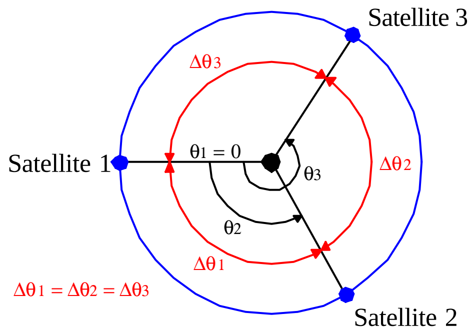

Fig. 7 – Nominal position of the satellites.

$$\theta_1 \le \theta_2 : \quad \delta\theta_1 = (\theta_2 - \theta_1) - \frac{2\pi}{3} \tag{20}$$

$$\theta_2 < \theta_1 : \quad \delta\theta_1 = (2\pi - \theta_1 + \theta_2) - \frac{2\pi}{3} \tag{21}$$

$$\theta_2 \le \theta_3 : \quad \delta\theta_2 = (\theta_3 - \theta_2) - \frac{2\pi}{3} \tag{22}$$

$$\theta_3 < \theta_2 : \quad \delta\theta_2 = (2\pi - \theta_2 + \theta_3) - \frac{2\pi}{3} \tag{23}$$

$$\theta_3 \le \theta_1 : \quad \delta\theta_3 = (\theta_1 - \theta_3) - \frac{2\pi}{3} \tag{24}$$

$$\theta_1 < \theta_3 : \quad \delta\theta_3 = (2\pi - \theta_3 + \theta_1) - \frac{2\pi}{3} \tag{25}$$

$$\delta\theta = \frac{|\delta\theta_1| + |\delta\theta_2| + |\delta\theta_3|}{3} \tag{26}$$

If the difference between the nominal and actual elements, and the position constraint $\Delta\theta$ do not satisfy the tolerance previously specified, at least a correction maneuver becomes necessary. Otherwise we consider the actual elements and propagate the orbit of the satellite 1 to predict which will be the elements of this satellite in the next passage by the ground tracking station. After this, the control program waits the next satellite to enter in the visibility area of the ground tracking station, and in this way, the process described above repeats.

Considering that it is necessary to execute the maneuver, several possible maneuvers are calculated, each one of them with different values of the semi-major axis of the final orbit and different values of the time spent for the maneuver. The semi-major axis and the time, vary from predefined values belonging to an operation range for the satellite. Thus we obtain the orbital elements of the transfer orbit, where α_1 is the true anomaly of the location of the first impulse, α_2 is the true anomaly of the location of the second impulse, Δv_1 and Δv_2 are the velocity increment generated by the first and second impulses, Δv is the total velocity increment and t is the time spent.

Maneuver 1: $a_{final} = a_{nominal} = 7010$ km

a = 7006,23867081 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01326311 $\quad \boldsymbol{a}_2$ = 0,10768541

w = 4,76633701 $\quad t$ = 100,00000000

Dv_1 = 0,09993920 $\quad Dv_2$ = 0,09994003

Dv = 0,19987923

Maneuver 2: $a_{final} = 1{,}0001 a_{nominal} = 7010{,}701$ km

a = 7006,63796680 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01351801 $\quad \boldsymbol{a}_2$ = 0,11306121

w = 4,76903787 $\quad t$ = 105,00000000

Dv_1 = 0,10184595 $\quad Dv_2$ = 0,10184694

Dv = 0,20369289

Maneuver 3: $a_{final} = 1{,}0002 a_{nominal} = 7011{,}402$ km

a = 7007,44799235 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01575290 $\quad \boldsymbol{a}_2$ = 0,10336250

w = 4,76418552 $\quad t$ = 96,00000000

Dv_1 = 0,11870424 $\quad Dv_2$ = 0,11870527

Dv = 0,23740951

Maneuver 4: $a_{final} = 1{,}003 a_{nominal} = 7012{,}103$ km

a = 7007,55340389 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01459562 $\quad \boldsymbol{a}_2$ = 0,11842733

w = 4,77174271 $\quad t$ = 110,00000000

Dv_1 = 0,10994625 $\quad Dv_2$ = 0,10994758

Dv = 0,21989383

Maneuver 5: $a_{final} = 1{,}0004 a_{nominal} = 7012{,}804$ km

a = 7008,02403816 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01516596 $\quad \boldsymbol{a}_2$ = 0,12057153

w = 4,77282558 $\quad t$ = 112,00000000

Dv_1 = 0,11423413 $\quad Dv_2$ = 0,11423564

Dv = 0,22846977

Maneuver 6: $a_{final} = 1{,}0005 a_{nominal} = 7013{,}505$ km

a = 7008,88406240 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01739282 $\quad \boldsymbol{a}_2$ = 0,11087445

w = 4,76797259 $\quad t$ = 103,00000000

Dv_1 = 0,13103180 $\quad Dv_2$ = 0,13103334

Dv = 0,26206514

Maneuver 7: $a_{final} = 1{,}006 a_{nominal} = 7014{,}206$ km

a = 7010,12115852 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,02070642 $\quad \boldsymbol{a}_2$ = 0,09794970

w = 4,76149997 $\quad t$ = 91,00000000

Dv_1 = 0,15603114 $\quad Dv_2$ = 0,15603265

Dv = 0,31206379

Maneuver 8: $a_{final} = 1{,}0007 a_{nominal} = 7014{,}907$ km

a = 7009,54050147 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01719814 $\quad \boldsymbol{a}_2$ = 0,12377333

w = 0,01719814 $\quad t$ = 115,00000000

Dv_1 = 0,12952163 $\quad Dv_2$ = 0,12952373

Dv = 0,25904536

Maneuver 9: $a_{final} = 1{,}0008 a_{nominal} = 7015{,}608$ km

a = 7010,17329534 $\quad \boldsymbol{a}_1$ = 0,00000000

e = 0,01832564 $\quad \boldsymbol{a}_2$ = 0,12161165

w = 4,77338020 $\quad t$ = 113,00000000

Dv_1 = 0,13801639 $\quad Dv_2$ = 0,13801865

Dv = 0,27603504

TABLE 7 – Normalized Optimal Maneuvers of the 6° Example.

$dq_{m\acute{A}x}$ = 0,7 rad $\quad Dv_{m\acute{A}x}$ = 1,1 km/s $\quad T_{m\acute{A}x}$ = 200 s

	$dq/dq_{\max}$	$Dv/Dv_{\max}$	$T/T_{\max}$
1	0,33237709	0,18170839	0,5000000
2	0,33237378	0,18517535	0,5250000
3	0,33237971	0,21582683	0,4800000
4	0,33237048	0,19990348	0,5500000
5	0,33236917	0,20769979	0,5600000
6	0,33237510	0,23824104	0,5150000
7	0,33238301	0,28369435	0,4550000
8	0,33236720	0,23549578	0,5750000
9	0,33236851	0,25094094	0,5650000

Examining Table 7 we can select the maneuvers 8, 1 and 7 as *non-dominated* solutions for the entire group of candidates solution (level 1). The coordinates of the barycenter, using normalized values, are (0,33237577; 0,23363284; 0,510), which are equivalent to dq = 0,23266304 rad; Dv = 0,25699613 km/s; *Time* = 102 s. Calculating the distances between the barycenter and the points determined by the maneuvers (Criterion 1A), it is found:

d(B, m1) = 0,05287862 $\quad$ d(B, m6) = 0,00678772

d(B, m2) = 0,05072601 $\quad$ d(B, m7) = 0,07437173

d(B, m3) = 0,03488630 d(B, m8) = 0,06502669
d(B, m4) = 0,05232275 d(B, m9) = 0,05765909
d(B, m5) = 0,05632516

Using the program to calculate the optimal maneuver again, with time constraint fixed in 102 s, which is the time of the maneuver equivalent to the barycenter (Criterion 1B), we obtain:

a	=	7008,92581157	α_1	=	0,00000000
e	=	0,01756316	α_2	=	0,10979799
ω	=	4,76743295	t	=	102,00000000
Δv_1	=	0,13231807	Δv_2	=	0,13231961
Δv	=	0,26463768			

which generate a position constraint $\delta\theta/\delta\theta_{\max}$ = 0,22571391, a velocity increment $\Delta v/\Delta v_{\max}$ = 0,24057971 and the time spent in the maneuver $T/T_{\max}$ = 0,510. Thus, the distance between the barycenter and the point determined by the maneuver is given by: d (b, m) = 0,10688784.

Comparing all the distances calculated, it can be noticed that the maneuver that better approaches the coordinates of the barycenter is the maneuver 6. In this case, even calculating the transfer maneuver again, using as input the time specified by the barycenter (Criterion 1B), it was not possible to obtain a better maneuver than those that had already been calculated (Criterion 1A). In this way, we chose the maneuver 6 as the best maneuver.

CONCLUSION

In this article, which is part of the work developed by Rocco[9], the problem of orbital station keeping of satellite constellations was studied as a problem of multi-objective optimization. We considered as example, a constellation composed by n = 3 satellites. However, the multi-objective optimization method and the software developed for the control of the constellation allow easily to consider more than 3 satellites. But the concept for the problem for n = 3 is identical to the problem for n > 3, but in this case, we would have a larger computer effort. So, we opted to consider n = 3.

The methodologies found in the literature, generally began the problem with an multi-objective approach but ended reducing the problem to the mono-objective case, by means of simplifications or influence factors. Or, when the approach was really multi-objective, the found result was a group of solutions candidates to the optimal solution, and in this case, for the choice of the optimal solution we should use other approaches external to the problem. The best methodology found in the literature, that bases on Pareto[8], present this deficiency, as it was shown. Until today, several authors use the Pareto Criterion with small variations. Therefore, it seems that doesn't exist in the revised literature any method really capable to accomplish the multi-objective optimization, considering all the objectives of the problem equally. Thus, it was tried in some way, to create a methodology that at least to consider equally all the objectives. This methodology is based on what we called Smallest Loss Criterion. Using this criterion, it was possible to consider equally conflicting objectives and to obtain a single solution for the problem, which for Engineering applications is very interesting. It was considered in the examples presented in this work three objectives, but the Smallest Loss Criterion allow to consider so many objectives as necessary.

The software developed was tested and it was proven that it is capable to generate reliable results. It was developed in modules to allow to be enlarged considering a larger number of satellites and/or other geometric configurations of the constellation according to the need.

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