The sinusoid as the longitudinal profile in backward-wave oscillators of large cross sectional area

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Abstract

This work reports on design tools for BWOs operating in symmetric TM modes since these modes are able to perturb the axial velocity and electron density density on rectilinear beams confined by an external magnetic field in slow-wave systems. Here we investigate whether a cylindrical guide with sinusoidally rippled wall can provide strong coupling between the guide surface waves and mildly relativistic (500-keV) electron beams in the 8-9 GHz frequency range for BWOs of large diameter (D ~ 3λ). For this purpose, the characteristic equation of a sinusoidally corrugated structure is derived on the basis of the Rayleigh-Fourier method, whereby the field solution is represented by a single expansion of TM eigenmodes. From the dispersion diagrams thus obtained we infer the appropriate periodic length and ripple amplitude of the guiding structure that optimize the beam-wave interaction.

DISPERSION RELATION OF THE CYLINDRICAL GUIDE WITH SINUSOIDALLY RIPPLED WALL

WAVEGUIDE PROFILE:

$$R_w(z) = R_0\left(1+\varepsilon\ \cos\left(\frac{2\pi}{d}z\right)\right)$$

FLOQUET´S THEOREM :

$$E_z(r,z,t) = \sum_{n=-\infty}^{\infty} A_n J_0(k_{\perp n} r)\exp(ik_{zn}z - i\omega t)$$

$$E_r(r,z,t) = -i\sum_{n=-\infty}^{\infty} A_n \frac{k_{zn}}{k_{\perp n}} J_1(k_{\perp n} r)\exp(ik_{zn}z - i\omega t)$$

$$k_{zn} = k_{z0} + \frac{2\pi}{d}n, \quad 0 \le k_{z0} \le \frac{2\pi}{d}$$

$$k_{\perp n}^2 = \frac{\omega^2}{c^2} - k_{zn}^2$$

Boundary Condition:

$$\left[E_z(r,z) + E_r(r,z)\frac{dR_w(z)}{dz} \right]_{r=R_w} = 0$$

Homogeneous Equations:

$$\sum_{n=-\infty}^{n=\infty} A_n C_{nm}(\omega, k_{z0}, \varepsilon, R_0, d) = 0$$

$$C_{nm} = \left[1 + \frac{2\pi}{d}(n-m) \right] \frac{k_{zn}}{k_{\perp n}^2} \int_0^d dz\, J_0(k_{\perp n} R_w(z))\, Cos\left((n-m)\frac{2\pi}{d} z\right)$$

Dispersion Equation:

$$\det \left\| C_{nm} \right\| = 0$$

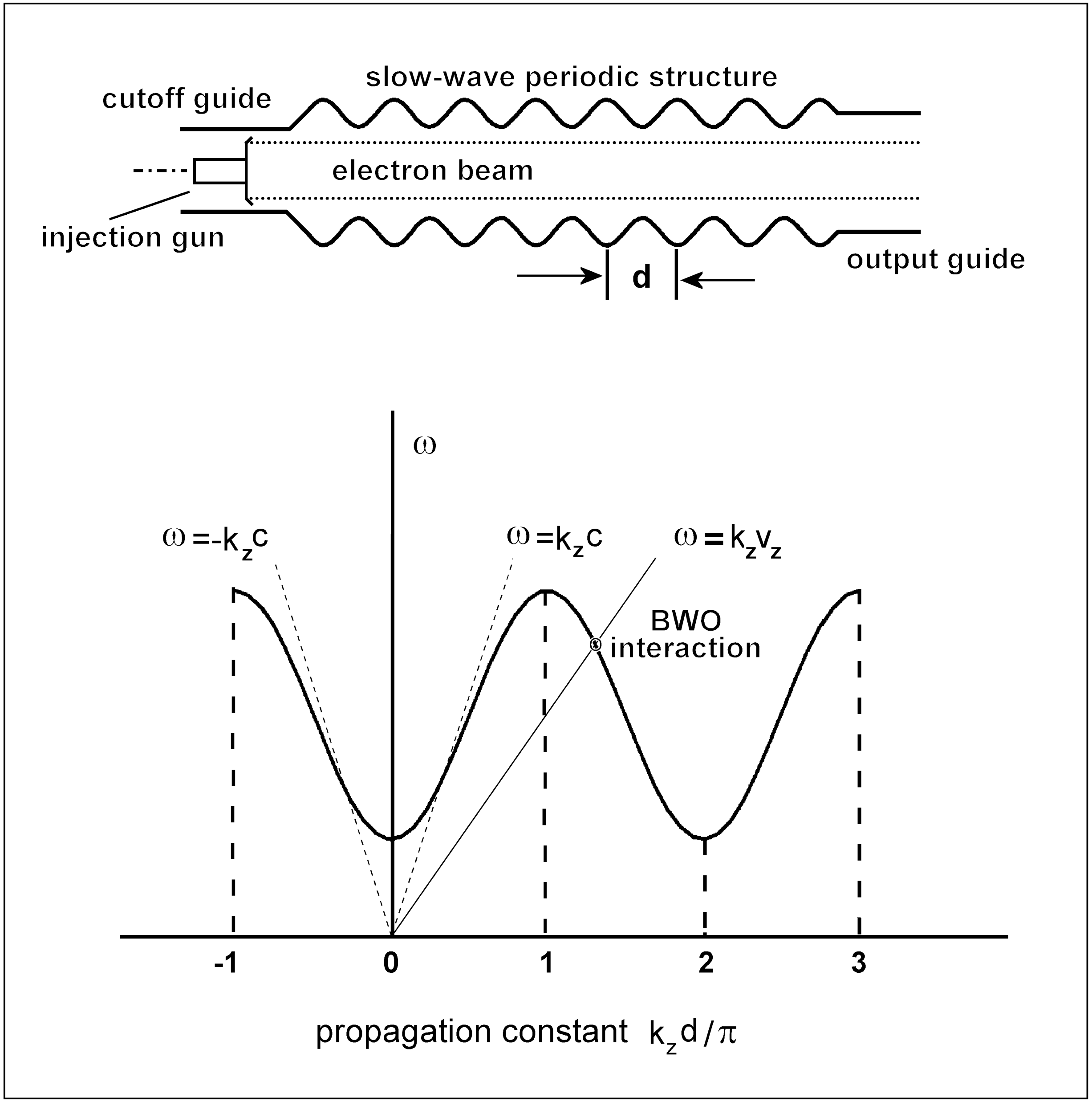

Fig. 1 BWO schematics and dispersion diagram showing the first passband

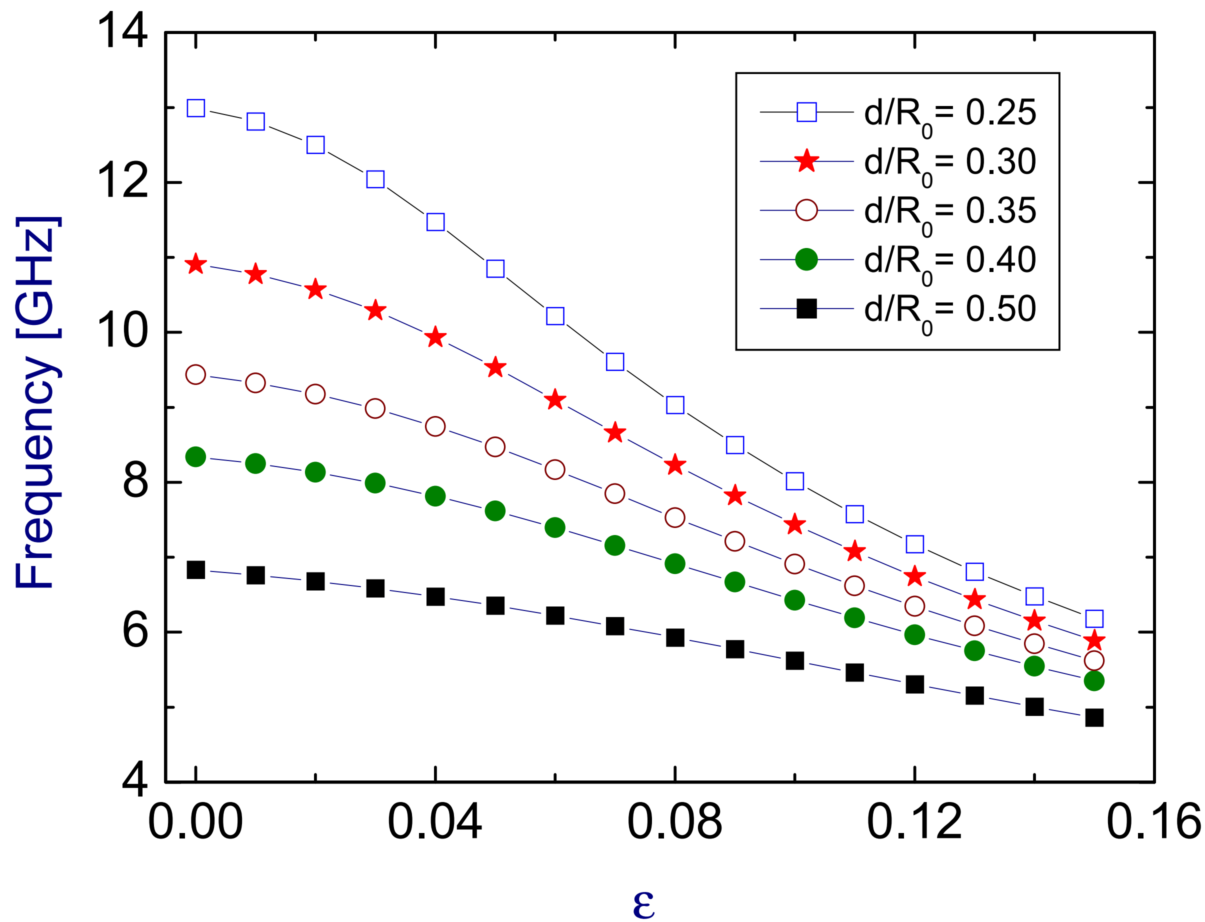

Fig. 2 TM_{01}-mode upper cutoff frequencies (π-point) as function of the ripple amplitude ε with the normalized period d/R_0 as parameter

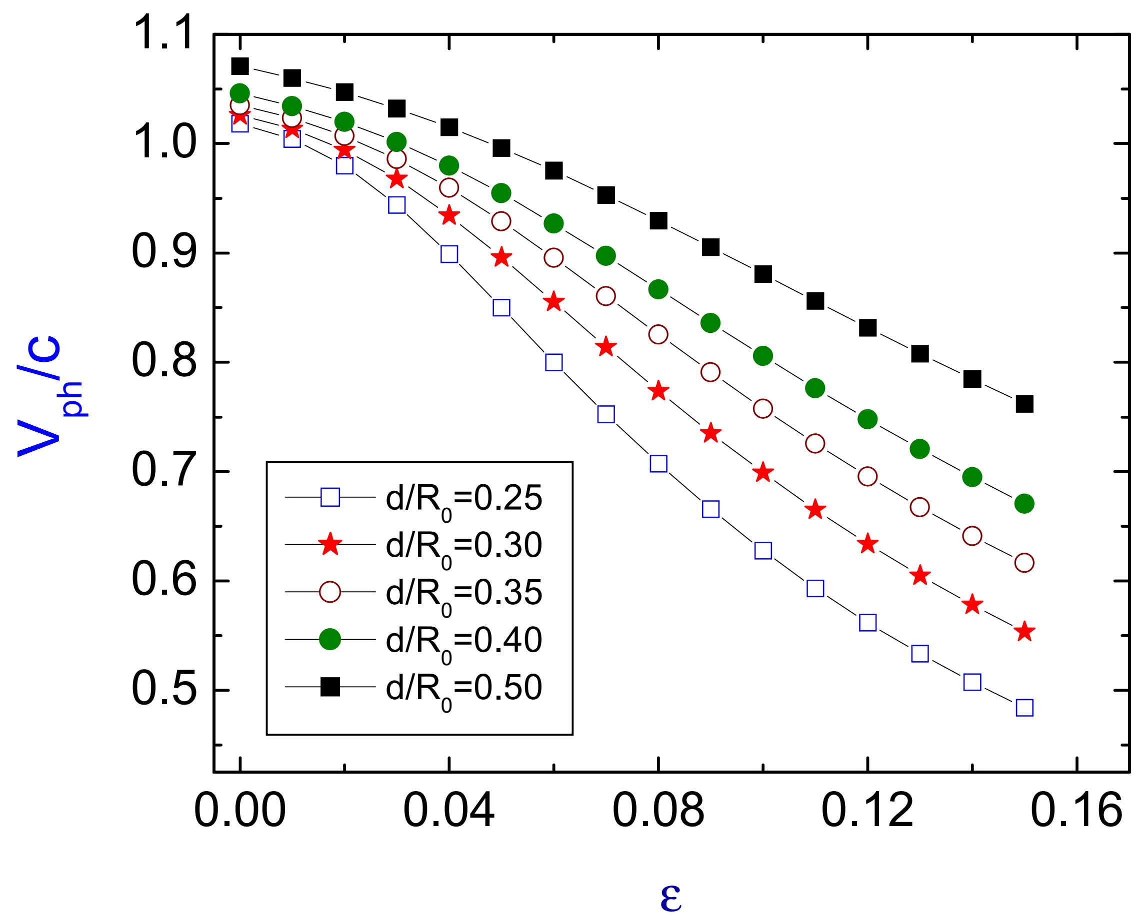

Fig. 3 TM_{01}-mode phase velocity (at the π-point) as a function of the ripple amplitude ε with the normalized period d/R_0 as parameter

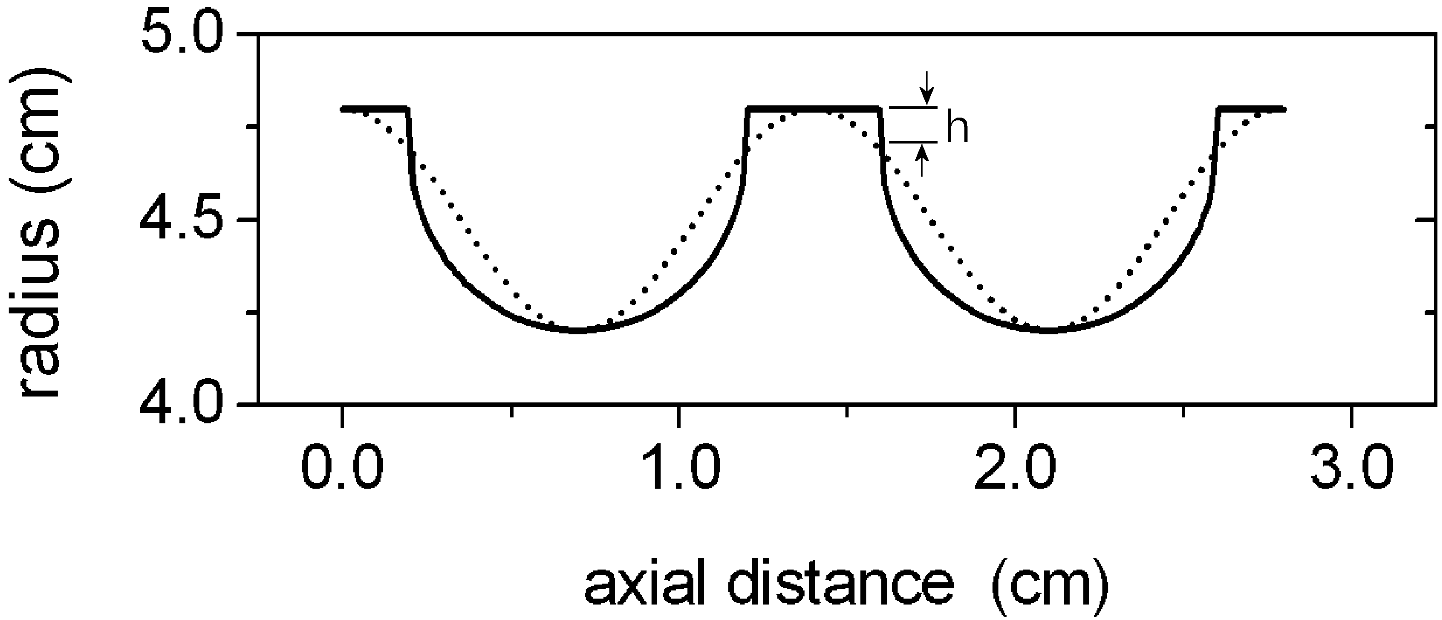

Fig. 4 Sinusoidal (R_0 =4.5 cm, ε = 1/15, R_{min} = 4.2 cm, R_{max} = 4.8 cm) and piecewise (semicircle radius 0.5 cm and rectangle height h = 0.1 cm) profiles with periodic length d=1.4 cm

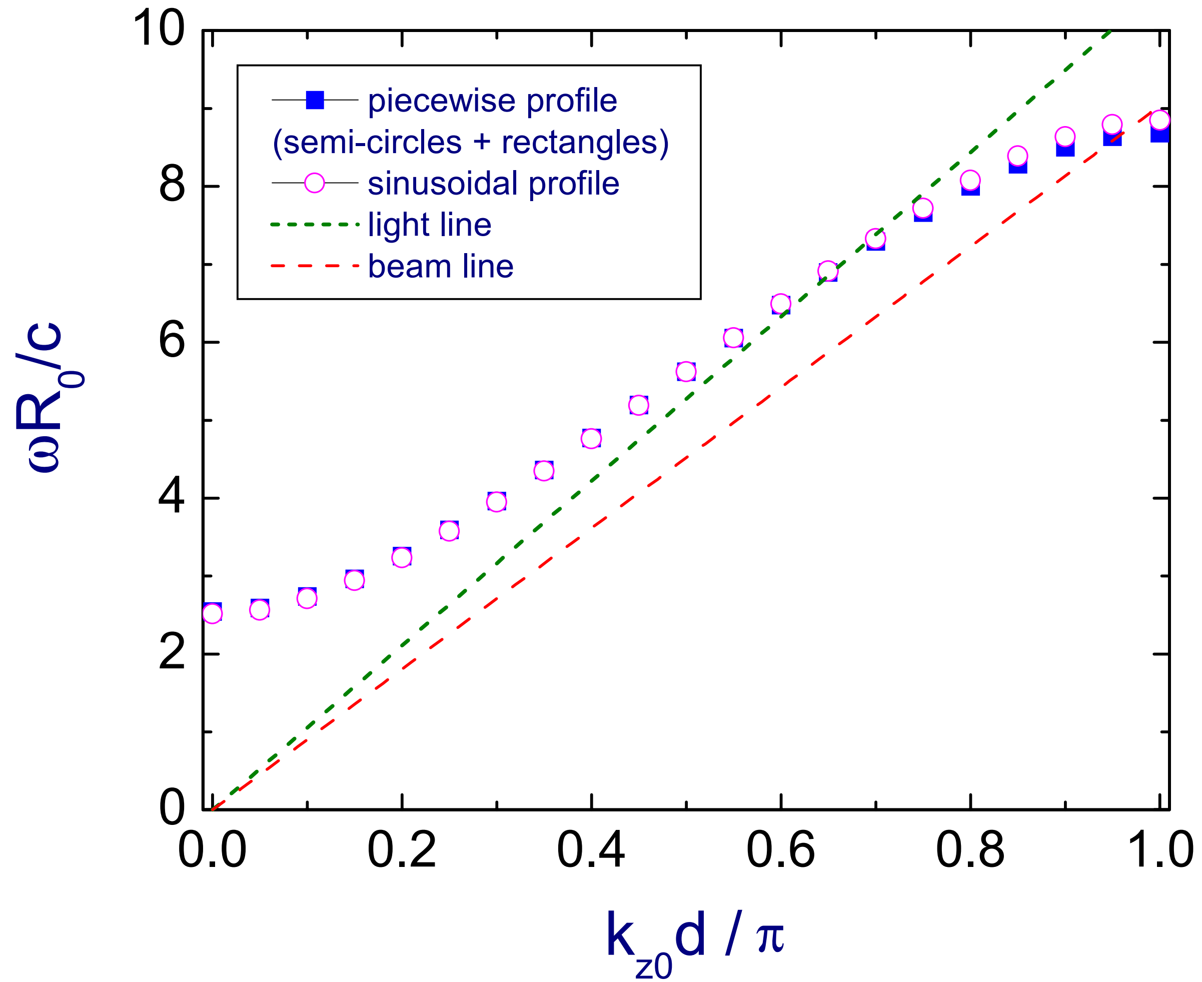

Fig. 5 TM$_{01}$-mode dispersion diagram corresponding to the sinusoidal profile in Fig. 4. Dashed and dotted lines are the light line and the 500-keV beam Doppler line (v_z =0.86 c)

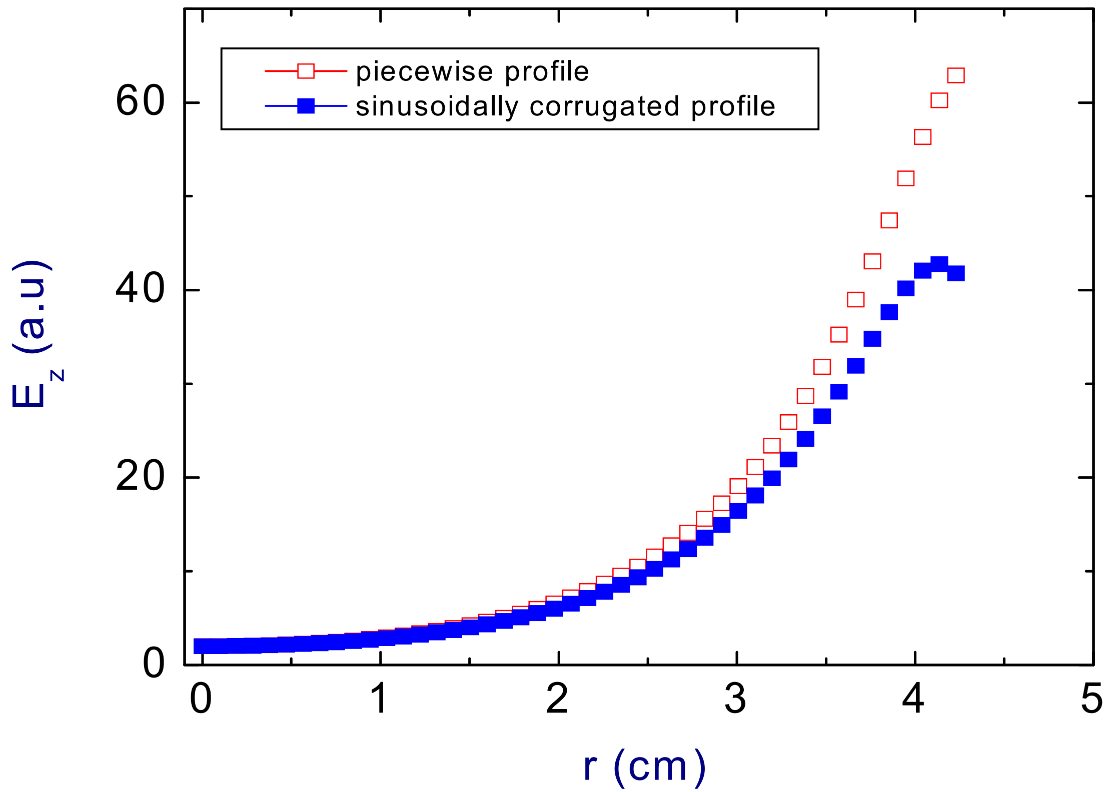

Fig. 6 Radial dependence of the TM_{01}-mode electric field on section z=0.0 of sinusoidal and piecewise profiles shown in Fig. 4

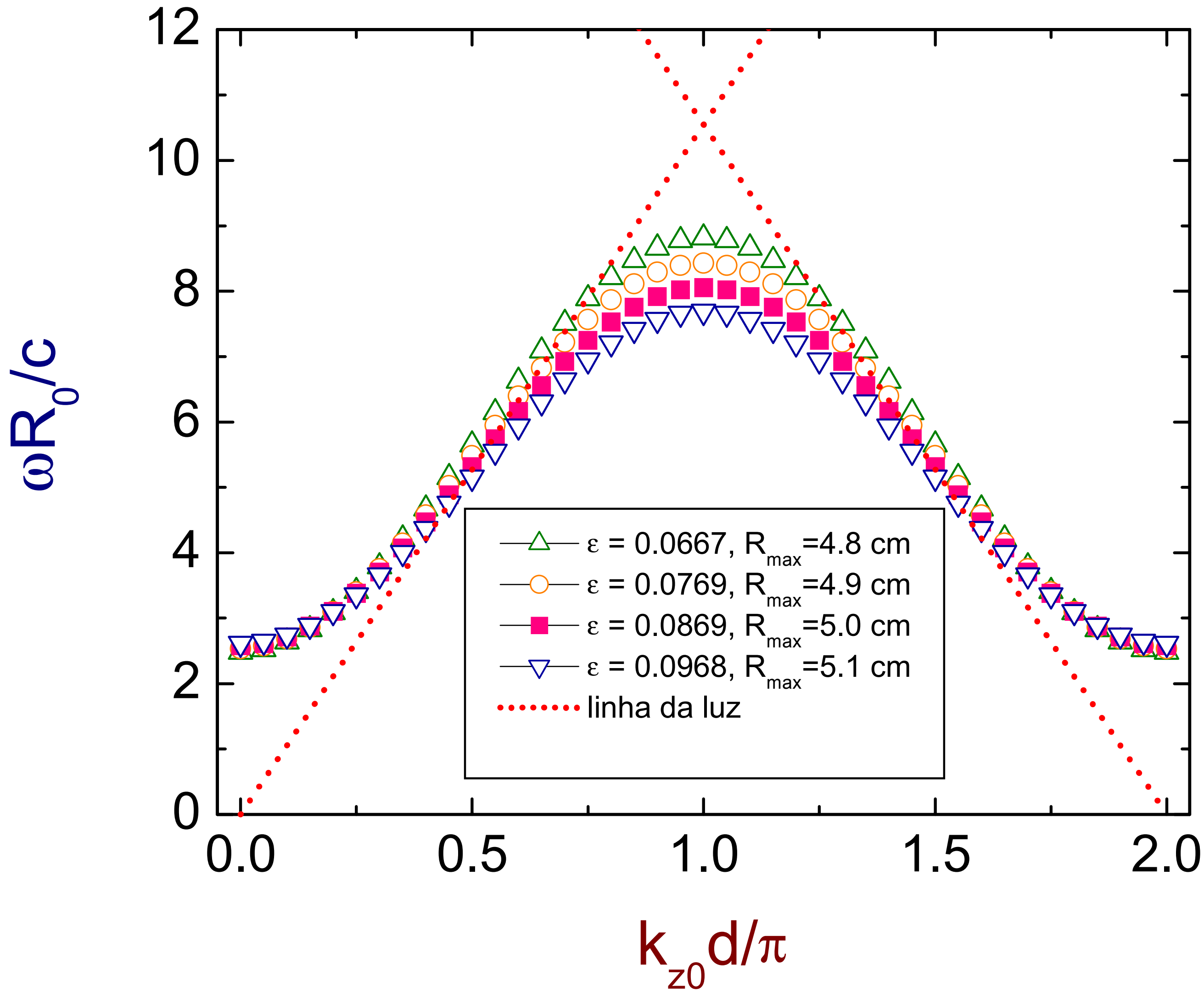

Fig. 7 TM_{01}-mode dispersion diagram for an oversized guide with longitudinal profile sinusoidally corrugated. The four curves are associated with distinct values of average radius R_0 and ripple amplitude ε, but keeping in all cases R_{min}=4.2 cm and d = 1.4 cm. Dashed line is the light line.

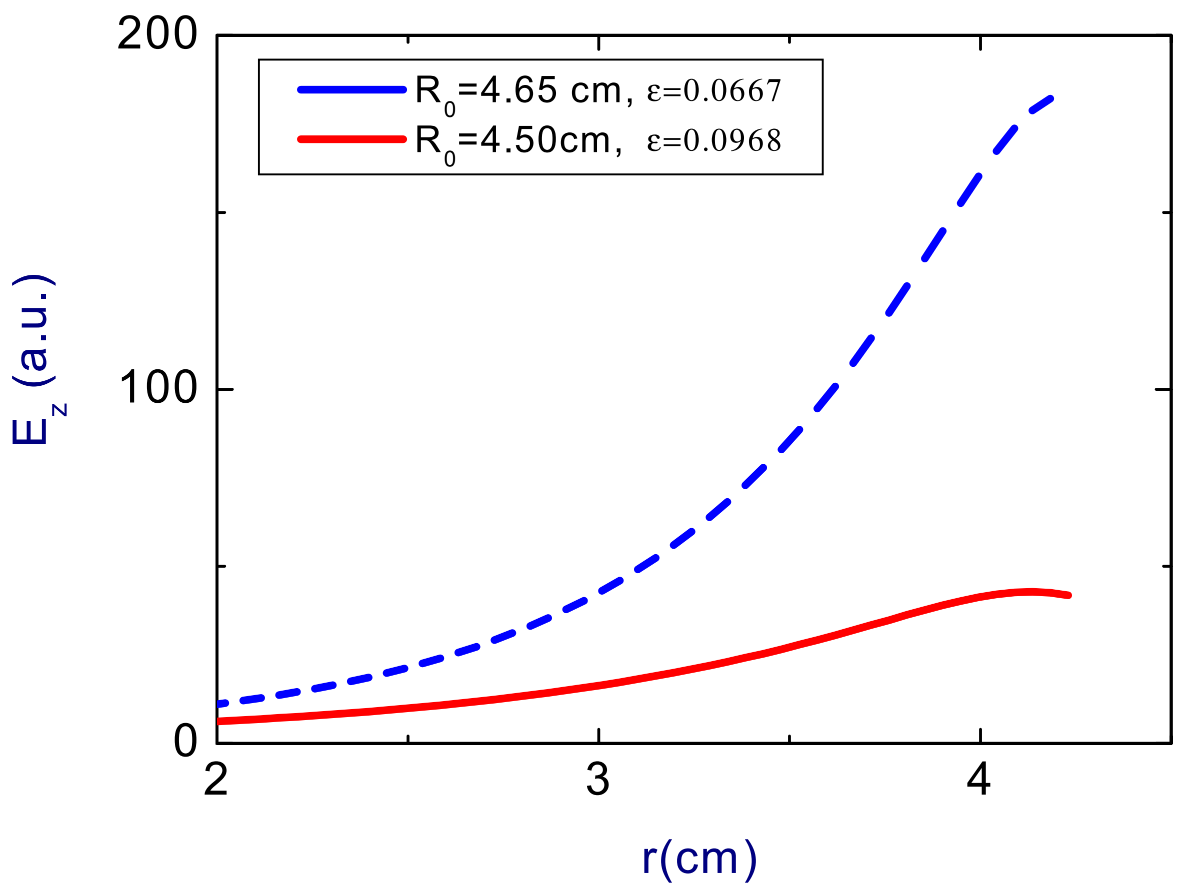

Fig. 8 Radial dependence of the TM_{01}-mode electric field (on section z=0.0) in sinusoidally rippled waveguides

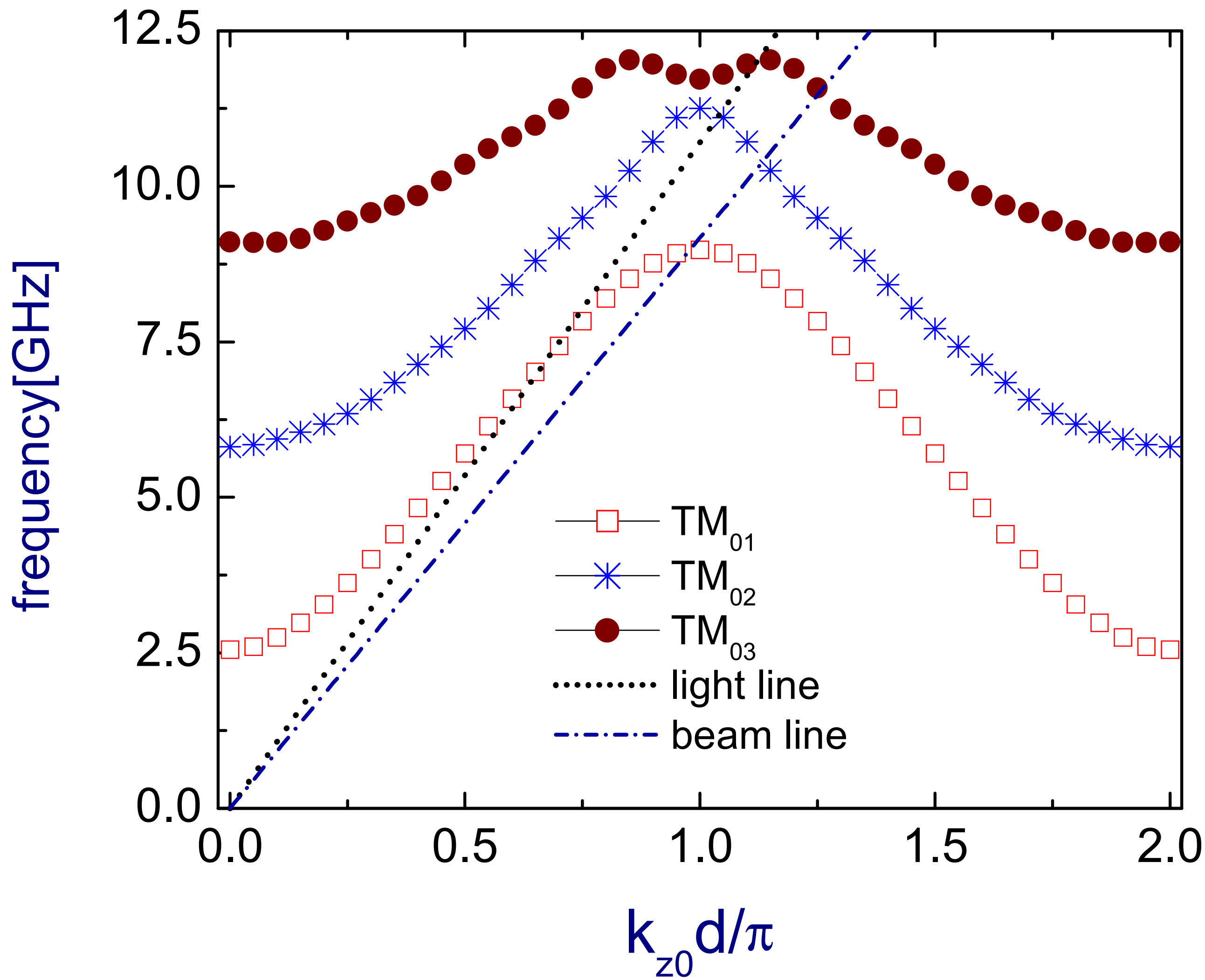

Fig. 9 Calculated dispersion diagrams for three lower-order TM_{01}-modes in the sinusoidally corrugated waveguide (R_0 =4.5 cm, d=1.4 cm, ε =1/15) shown in Fig. 4. Light and beam lines are indicated by dotted and dashed lines.

h(cm)	WaveSym Code	RF- method
0.1	8.82	8.82
0.2	8.30	8.28
0.3	7.82	7.89
0.4	7.33	7.40

Table 1 Critical frequencies , in GHz, calculated from the WaveSym code and from the RF method

Conclusions

A systematic procedure has been given to synthesize consistently with the required operating frequency and injection beam energy, an overmoded sinusoidally corrugated waveguide to be used as a slow-wave structure on backward-wave oscillators. The corrugation parameters are selected so that the waveguide when driven by a 500- keV electron beam operates close to the upper cutoff frequency on the first space harmonic of the lower order TM mode. Consistent with single-mode operation, the periodic structure here designed supports a surface wave that is synchronous with a 500-keV electron beam, and simultaneously exhibits a large coupling impedance, which is a measure of the RF field strength at the beam position. The surface wave is crucial to avoiding mode competition, while large coupling impedance is important for high-efficiency interaction piecewise continuous profile made from a combination of semicircles and rectangles.

Conclusions

- A sinusoidal profile that meets the design specifications is identified by the corrugation parameters R_0 = 4.50 cm, d=1.4 cm and ε =0.067 which yield the critical frequency of 8.9 GHz corresponding to the phase velocity of 0.82c for the synchronous space harmonic.
- The axial electric field strength (and, therefore, the coupling impedance) and the breadth of the passband can be adjusted by proper variation of the corrugation parameters ε and R_0 .The deeper the corrugation, the stronger the field and the flatter the passband become.
- The coupling impedance is substancially higher in the piecewise profile than in the sinusoidal counterpart.
- Concerning mode competition, the beam line crosses the dispersion curves of the higher-order modes (TM_{02} and TM_{03}) at frequencies above 10 GHz, significantly higher than the 8-9 GHz frequency of the nominal TM_{01} mode.
- Rayleigh-Fourier method has shown excellent performance even when used on the analysis of a piecewise continuous profile made from a combination of semicircles and rectangles