

A Preliminary Model for Growing of the Convective Boundary Layer

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Abstract

An analytical expression for the energy spectrum for growing of the convective boundary layer is derived. The Heisenberg's assumption to represent the interaction among small and large eddies is used. In addition, the time dependency of the buoyancy term is modeled as a step function.

1. INTRODUCTION

The evolution of the PBL is controlled by turbulent mixing induced by temperature difference between the atmosphere and the earth (thermal production), and by the winds in the lower levels (mechanical production). In a convective boundary layer (CBL), the turbulence is generated by heat flux from the earth to the atmosphere, and by wind field shear. The stable boundary layer (SBL) presents an inverse heat flux between the earth and the atmosphere of that in the CBL. The SBL appears in the nocturnal period, it also occurs in some cloudy days or under others special conditions. In the neutral layer the turbulence is predominantly mechanical. The Taylor's statistical theory on turbulence can be applied for expressing a parameterization for all of these stability conditions (Degrazia and Moraes, 1992; Degrazia et al., 1997; Degrazia et al., 2000). Other special PBL is the residual boundary layer – RL (Stull, 1988). Again, the RL can be parameterized using the Taylor's statistical diffusion theory (Degrazia et al., 2003; Goulart et al., 2003a).

The goal here is to derive an analytical expression for the energy spectrum for growing of the convective boundary layer. This result is obtained by solving the spectral turbulent kinetic energy equation. The Heisenberg's assumption to represent the interaction among small and large eddies is used. In addition, the time dependency of the buoyancy term is modeled as a step function.

2. EQUATION FOR THE ENERGY SPECTRUM

Homogeneous isotropic turbulence satisfies the following energy transfer relation (Hinze, 1975):

$$\frac{\partial \mathbf{E}(k, t)}{\partial t} = \mathbf{W}(k, t) + \mathbf{M}(k, t) + \mathbf{H}(k, t) - 2\mathbf{n} k^2 \mathbf{E}(k, t) \quad (1)$$

where k is the wavenumber; $\mathbf{E}(k, t)$ is the three-dimensional (3-D) energy density spectrum function (EDS); $\mathbf{W}(k, t)$ represents the contribution due to the inertial transfer of energy among different wavenumbers; $\mathbf{M}(k, t)$ is the mechanical production term, $\mathbf{H}(k, t)$ is the thermal production term; finally, the last term in equation (1) is the energy loss due to ordinary viscous dissipation. At moment, the term $\mathbf{M}(k, t)$ will be considered negligible. The Heisenberg's assumption is used for representing the non-linear term:

$$\mathbf{W}(k, t) = 2\mathbf{n}_T k^2 \mathbf{E}(k, t) \quad (2)$$

where \mathbf{n}_T is the kinematic turbulence viscosity. Its numerical value can be computed following Degrazia et al. (2003).

The key point in our analysis is to model the thermal production term $\mathbf{H}(k, t)$ as a Heaviside function:

$$\mathbf{H}(k, t) = \begin{cases} \mathbf{H}(k) & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (3)$$

Substituting equations (3) and (2) in equation (1), the spectral evolution equation becomes

$$\frac{\partial E(k, t)}{\partial t} = -2k^2 (\mathbf{n}_T + \mathbf{n}) E(k, t) + \mathbf{H}(k). \quad (4)$$

The solution for the above equation is given by

$$E(k, t) = E_0(k) e^{-k^2 (\mathbf{n}_T + \mathbf{n}) t} + \frac{\mathbf{H}(k)}{2k^2 (\mathbf{n}_T + \mathbf{n})} \left[1 - e^{-k^2 (\mathbf{n}_T + \mathbf{n}) t} \right]. \quad (5)$$

being $E_0(k)$ the spectrum at $t=0$. For $t \rightarrow \infty$ the asymptotic expression for the spectrum is

$$E(k) = \frac{\mathbf{H}(k)}{2k^2 (\mathbf{n}_T + \mathbf{n})}. \quad (6)$$

Two questions remain: who are $E_0(k)$ and $\mathbf{H}(k)$? $E_0(k)$ is the spectrum for the SBL (Degrazia and Moraes, 1992). The spectrum expressed by equation (6) is that given by fully developed CBL (Degrazia et al., 1997); this permits to obtain an expression for $\mathbf{H}(k)$. Finally, the 3-D spectrum can be obtained from 1-D spectrum using a formulation proposed by Kristensen et al. (1989).

3. FINAL REMARKS

Having an expression for growing of the CBL – equation (5), it is possible to derive a parameterization for the eddy diffusivity for this special type of the PBL. From the 3-D spectrum, the 1-D wind components can be obtained following Goulart et al. (2003b). Another parameterization for the non-linear term could also be considered (Pao, 1965):

$$W(k, t) = - \frac{\partial \left[\mathbf{a}^{-1} \mathbf{e}^{1/3} k^{5/3} E(k, t) \right]}{\partial k} \quad (7)$$

being \mathbf{a} the Kolmogorov constant, and \mathbf{e} is the rate of the molecular dissipation of TKE.

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