Resistive transition in π-junction superconductors

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The resistivity behavior of inhomogeneous superconductors with random π junctions, as in high-T_c materials with d-wave symmetry, is studied by numerical simulation of a three-dimensional XY spin-glass model. Above a concentration threshold of antiferromagnetic couplings, a resistive transition is found in the chiral-glass phase at finite temperatures and the critical exponents are determined from dynamic scaling analysis. The power-law exponent for the nonlinear contribution found in recent resistivity measurements is determined by the dynamic critical exponent of this transition.

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Inhomogeneous superconductors containing a random distribution of π junctions, as in high-T_c superconducting materials with d-wave symmetry, can display unusual frustration effects even in zero external magnetic field.[1] A π junction leads to a phase shift of π between superconducting regions, and to half flux quantum vortices on closed loops with an odd number of these junctions.[2] Interesting ordering effects are expected due to vortex interactions. Some of them have already been directly imaged on specially prepared high-T_c Josephson contacts[3] and should also be relevant for π junctions in low-T_c superconductors.[4] Much attention has been devoted to the magnetic properties of inhomogeneous superconductors arising from the orbital currents of these vortices[1,5,6] and in particular to its relevance for the explanation of paramagnetic Meissner effect.[7–10] Nevertheless, there are also important consequences for the resistivity behavior of theses systems which have not been investigated satisfactorily.

In the absence of π junctions, the phases of neighboring superconducting regions tend to be locked with zero phase shift, and a phase-coherence transition is expected for decreasing temperature into a superconducting state with vanishing linear resistivity. The critical behavior of this resistive transition is reasonable well understood. On the other hand for sufficiently large concentration of π junctions, which may occur for example in granular samples, frustration, and disorder effects leads to a vortex glassy phase and the resistive behavior is much less understood. The simplest model of the system is to consider only contributions from the Josephson coupling energy of nearest-neighbor grains,[1] $H_{ij} = -J_o\cos(\theta_i-\theta_j-t_{ij})$, where θ_i is the phase of the local superconducting order parameter $J_o>0$ and $t_{ij}=0$ or π correspond to the phase shifts of conventional and π junctions. This is equivalent to the interaction of two-component pseudospins $\vec{S}=[\cos(\theta),\sin(\theta)]$, coupled by ferro or antiferromagnetic interactions, respectively, which leads to an XY-spin-(chiral-) glass model for the granular system.[7] The chiral variable can be defined as the direction of the local circulating currents (vortices) in closed loops of junctions. Based on earlier and recent Monte Carlo (MC) simulations[7–9] for this model in three dimensions, it has been suggested that the equilibrium low-temperature state for the inhomogeneous superconductor is a chiral-glass but with no phase coherence and, therefore, the resistivity should be nonzero. This implies a chiral glass transition at a nonzero critical temperature but no resistive transition, except perhaps at zero temperature. Thus, strictly speaking, there is no true superconducting phase at low temperatures in this scenario. However, while different works agree on the existence of the proposed chiral-glass transition, the situation regarding the resistive behavior is unsettled. Results for the ground state[11,12] of the XY-spin glass model indicate that the lower critical dimension for phase ordering is between 2 and 3 and therefore a phase-coherence transition is only possible at zero temperature in two dimensions[13] but should occur at finite temperatures in three dimensions. The critical temperature for three dimensions, however, cannot be estimated from these calculations. Moreover, dynamical simulations suggest a resistive transition at finite temperature[14,15] and very recent MC calculations for a model with Gaussian couplings, expected to be in the same universality class, strongly support the occurrence of this transition.[16] The dynamical simulations were based on different representations of the same model and different dynamics. While the static exponents agree, as expected from the universality of critical behavior, the dynamic exponent $z\sim4.6$ obtained from the resistively shunted junction (RSJ) model of the dynamics in the phase representation[15] is significantly different from that obtained from MC dynamics, $z\sim3.1$, in the vortex representation,[14] suggesting a strong dependence of z on the details of the dynamics.

On the experimental side, there have been some attempts to identify the chiral-glass phase from nonlinear resistivity measurements in ceramic $YBa_2Cu_4O_8$ bulk samples[17] at zero magnetic field, near the onset of the paramagnetic Meissner effect. The nonlinear contribution ρ_2 to the resistivity was found to have a peak at the transition with power-law behavior $\rho_2\propto J^{-\alpha}$. This behavior has already been reproduced in dynamical simulations.[18] The results of the experiment have been interpreted as a chiral-glass transition attributed to the presence of π junctions, with a nonzero linear resistivity below the critical temperature, but the value of α and its possible relation with the critical exponents of the underlying transition was not found.

In this work, we study the resistivity behavior of inhomogeneous π-junctions superconductors using an XY-spin glass model with varying concentration x of antiferromagnetic bonds. An improved numerical method is used, combining MC and Langevin simulation with periodic boundary conditions. The results of a scaling analysis of extensive simulations for $x=0.5$ clearly show the existence of a resistive transition at finite temperature. A threshold $x_g \sim 0.3$ for the chiral-glass phase is estimated from the behavior of the zero-temperature critical current. The power-law exponent α for the nonlinear contribution found in resistivity measurements[17] can be related to dynamic critical exponent z of this transition. The observed value of α is within the range expected from numerical estimates of z.

We consider inhomogeneous superconductors with π junctions modeled by a three-dimensional XY-spin glass described by the Hamiltonian

$$H=-J_o \sum_{\langle ij \rangle} \cos(\theta_i-\theta_j-t_{ij}), \tag{1}$$

where the quenched phase shift t_{ij} is equal to π or 0 with probabilities x and $1-x$, respectively. The symmetric $\pm J_o$ XY spin glass[7,11,13–15] corresponds to $x=0.5$ while the unfrustrated XY model correspond to $x=0$. We use the time-dependent Guinzburg-Landau model for the dynamics given by the Langevin equations

$$\frac{1}{R_o}\frac{d\theta_i}{dt}=-J_o \sum_j \sin(\theta_i-\theta_j-t_{ij})+\eta_i, \tag{2}$$

where η_i represents uncorrelated thermal noise with $\langle \eta_i(t)\eta_i(t')\rangle = 2k_BT\delta(t-t')/R_o$ to ensure thermal equilibrium. This can also be regarded as an onsite dissipation model for the dynamics of the granular superconductor where R_o is the resistance of each point grain to the ground and J_o is the Josephson coupling. The RSJ model studied previously[15] allows only dissipation through the junction shunt resistance. We use units where $\hbar/2e=1$, $R_o=1$, $J_o=1$. To obtain the current-voltage characteristics more accurately in the glassy phase, we introduce an improved method. First, MC simulations are performed using Eq. (1) to obtain the equilibrium state (zero current bias) which is then used as initial state to integrate numerically the Langevin equations (2) for the driven system. Periodic (fluctuating twist) boundary conditions are used both for the MC simulations[19,20] and driven Langevin dynamics[21] simulations. Previous simulations used current injection with free boundary conditions[15] but periodic boundary conditions are more adequate since they avoid possible edge contributions. For systems of linear size L, the voltage V (electric field $E=V/L$) was computed as a function of the driving current I (current density $J=I/L^2$) for different temperatures and systems sizes ranging from $L=4$ to $L=12$. Calculations were performed in a cubic system, using 10^7 time steps and ten different realizations of the A_{ij} distribution, in the lowest current range. The most extensive simulations were done for $x=0.5$ while for $x<0.5$ the main purpose was to obtain the qualitative phase diagram and $T=0$ critical currents.

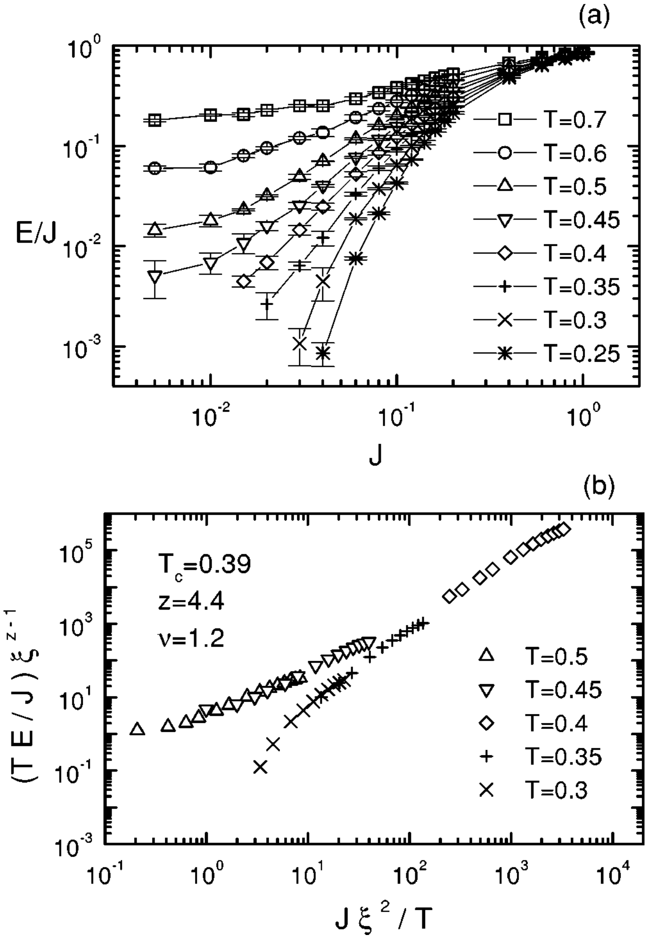

FIG. 1. (a) Nonlinear resistivity E/J for $x=0.5$ (equal distributions of 0 and π junctions), and different temperatures T, for system size $L=12$, (b) scaling plot of the data near the transition and for small currents, with $\xi \propto |T/T_c-1|^{-\nu}$.

The nonlinear resistivity $\rho=E/J$ for $x=0.5$ is shown in Fig. 1(a) for different temperatures T for the largest system size $L=12$. The behavior is consistent with a resistive transition at an apparent critical temperature in the range $T_c \sim 0.3$–0.45. At higher T, the linear resistivity $\rho_L = \lim_{J\to 0} E/J$ is finite while at lower T, it extrapolates to zero. The phase transition can be confirmed by a scaling analysis of the nonlinear resistivity which assumes the existence of a continuous equilibrium transition at $T>0$.[22] Near the transition, measurable quantities scale with the diverging correlation length $\xi \propto |T-T_c|^{-\nu}$ and relaxation time $\tau \propto \xi^z$, where ν and z are the correlation length and dynamical critical exponents, respectively. The nonlinear resistivity should then satisfy the scaling form[22]

$$TE\xi^{z-1}/J = g_{\pm}(J\xi^2/T) \tag{3}$$

in $d=3$ dimensions where $g(x)$ is a scaling function. The $+$ and $-$ signs correspond to $T>T_c$ and $T<T_c$, respectively. A scaling plot according to this equation can then be used to verify the scaling arguments and the assumption of an underlying equilibrium transition at $J=0$. The optimal data collapse provides an estimate of T_c and critical exponents. Such

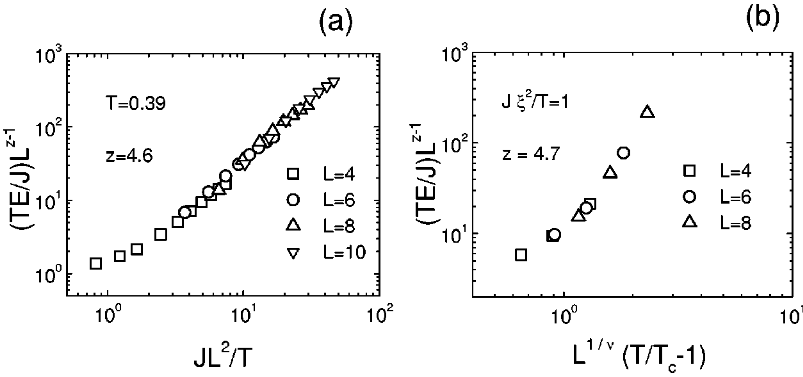

FIG. 2. (a) Finite-size scaling plot of the nonlinear resistivity at $T_c=0.39$, (b) finite-size scaling plot near T_c using current densities such that $J\xi^2/T=1$, a constant value.

scaling plot, which neglects finite-size effects, is shown in Fig. 1(b), obtained by adjusting the unknown parameters, giving the estimates $T_c=0.39(2)$, $z=4.4(3)$, and $\nu=1.2(2)$. We now show that these estimates, using the largest system size, are reliable by verifying that they give the expected finite-size behavior using smaller system sizes. Finite-size effects are particularly important sufficiently close to T_c when the correlation length ξ approaches the system size L. In particular, at T_c, the correlation length will be cut off by the system size in any finite system and the nonlinear resistivity should then satisfy a scaling form as in Eq. (3) with $\xi=L$. In fact, as shown in Fig. 2(a), the nonlinear resistivity calculated at the estimated $T_c=0.39$ for different system sizes satisfy this scaling form with $z=4.6$ which agrees within the errors. Away from T_c, the scaling function in Eq. (3) will also depend on the dimensionless ratio[22,14] L/ξ as $g(J\xi^2/T,L/\xi)$. To simplify the analysis, we consider resistivity data at current densities such that $J\xi^2/T=$constant. Then, the scaling form depends only on a single variable and the resistivity should satisfy the finite-size scaling form

$$TEL^{z-1}/J=\tilde{g}[L^{1/\nu}(T/T_c-1)]. \tag{4}$$

As shown in Fig. 2(b), the nonlinear resistivity calculated for different temperatures and system sizes such that $J\xi^2/T=1$, with the estimated $T_c=0.39$ and $\nu=1.2$, indeed satisfy this scaling form with $z=4.65$ which again agrees within the errors. Using a different constant $J\xi^2/T=2$ gives similar results.

The values of T_c, z, and ν obtained by the above scaling analysis using the onsite dynamics of Eq. (2) agree well with the previous estimate using the RSJ model[15] for the dynamics $[T_c=0.41(3), z=4.6(4)$, and $\nu=1.2(4)]$, clearly showing the existence of a phase-coherence transition at $T>0$ and also showing that the dynamic exponent z is essentially the same. Our estimate of T_c from the resistivity scaling is in good agreement with recent estimate of the critical temperature for the chiral-glass transition from MC simulations,[9] in the range $T_{ch}=0.38$–0.41. The agreement is quite intriguing since it supports the suggestion[15] that chirality and phase variables may order simultaneously. Recent MC simulations of the XY-spin-glass model with Gaussian couplings, expected to be in the same universality class, strongly support such single transition scenario.[16] Nevertheless, this transition is in sharp contrast with MC simulations of the phase-overlap distribution function.[7–9] On the other hand, a phase-coherence transition at $T>0$ is consistent with calculations of the spin stiffness exponent in the ground state showing that the lower-critical dimension for spin order in the XY-spin-glass model[11] is below 3, which implies that a phase-coherence transition at $T>0$ is possible. More recently, improved calculations in the vortex representation, also clearly shows a well-defined positive stiffness exponent.[12] In addition, calculations of the linear resistivity ρ_L (zero current bias) from MC dynamics simulations in the vortex representation,[14] shows an equilibrium resistive transition. The estimate of the static exponent ν agrees with the present estimate from the nonlinear resistivity but the dynamic exponent[14] $z=3.1$ is significantly lower. Interestingly enough, our calculations of z show the same result for the onsite and RSJ dynamics. Additional calculations using MC dynamics in the phase representation give the same result.[20] In spite of that, it is possible that the different z is a result of the particular dynamics in the vortex representation. In fact, vortex variables are collective excitations in the phase representation and thus lead to long-range correlations for the phases, suggesting that these representations may belong to different dynamic universality classes.

The dependence of T_c on the concentration of π junctions x is shown in the phase diagram of Fig. 3(a). The values of T_c for $x<0.5$ were obtained as rough estimates from the nonlinear resistive behavior, and is found to be nonzero in the whole range. We have also estimated the critical temperature from the peak of the phase susceptibility $\chi=(\langle m^2\rangle-\langle m\rangle^2)/L^3$, where $m=|\Sigma_i\vec{S}_i|$ and $\vec{S}=[\cos(\theta),\sin(\theta)]$, averaged over the disorder, which measures the onset of long-range phase coherence outside the glassy phase. As shown in Fig. 3(a), this transition temperature decreases for increasing x and extrapolates to zero at a threshold value $x_g\sim0.3$. We then expect that the range $x>x_g$ should correspond to the vortex (chiral-) glass phase. We note that for $x\ll x_g$ the susceptibility peak agrees with T_c showing that indeed the resistive transition corresponds to the phase-coherence transition. Additional evidence for the vortex glass phase is also provided by the change of the critical current with applied external field. Figure 3(b) compares the behavior of the $T=0$ critical current J_c with and without a magnetic field B applied transversely to the current direction. The external field acts as a uniform frustration $f=Ba^2/\phi_o$ in the XY-spin-glass model of Eq. (1), where a is the lattice spacing of the Josephson network and ϕ_o the flux quantum, and introduces a vortex lattice with dimensionless spacing av/a

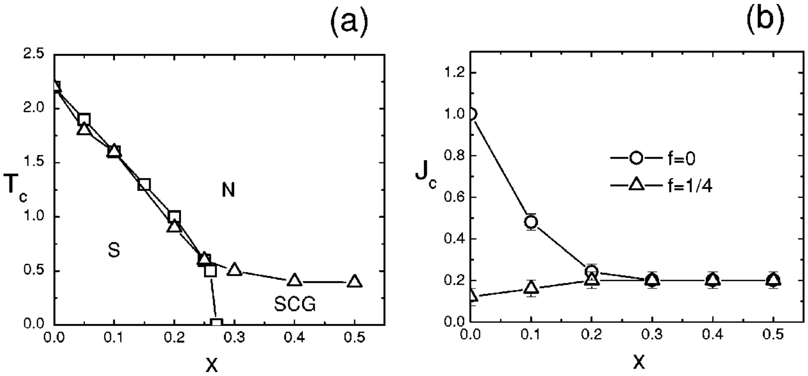

FIG. 3. (a) Phase diagram showing S (superconducting), SCG (superconducting chiral glass) and N (normal) phases as a function of temperature and concentration x of π junctions. (b) Critical currents densities J_c as a function of x without $(f=0)$ and with $(f=1/4)$ a uniform external magnetic field. In (a) triangle symbols correspond to critical temperatures estimated from the resistivity behavior and squares from the phase-susceptibility peak.

$\propto$1/f1/2. There is a large change of J_c for $x<x_g$, where some translational order at length scales large than a_v is still possible, but there is essentially no change for $x>x_g$, indicating that in this range there is only short-range order. The change of behavior gives a very rough estimate[23] of x_g.

Finally, we compare the critical properties of the resistive transition with experiments. Nonlinear resistivity measurements in ceramic $YBa_2Cu_4O_8$ bulk samples[17] near the onset of the paramagnetic Meissner effect have been interpreted as a chiral-glass transition attributed to the presence of π junctions, with a nonzero linear resistivity below T_c. In the experiments, the measured resistivity ρ was separated into a linear and nonlinear contribution through a low order expansion in the current density, $\rho=\rho_o+\rho_2 J^2+\ \ldots$. The lowest order nonlinear contribution ρ_2 was found to have a peak at the transition with power-law behavior $\rho_2 \propto J^{-\alpha}$ and exponent $\alpha \sim 1.1(6)$, while the linear contribution ρ_o appears to remain finite below this temperature. However, since the apparent linear contribution is very small and finite current bias was used, the limited accuracy of the data can not completely rule out a strict zero resistivity phase below this temperature. It is of interest to verify to which extent the resistive transition as found here is consistent with the observed peak in the nonlinear resistivity at the apparent transition temperature. If a resistive transition is assumed to occur at this temperature then the power-law behavior of ρ_2 follows directly from the current-voltage scaling near the transition temperature. Defining the nonlinear contribution as $\rho_2=\partial^2/\partial J^2(E/J)$, the scaling behavior of Eq. (3), obeyed by our numerical data, implies that $\rho_2 \propto J^{-\alpha}$, when $\xi \rightarrow \infty$ near the critical temperature, with the exponent relation $\alpha=(5-z)/2$. Using the dynamical exponent of the resistivity scaling,[24] $z=4.4(4)$ from the present work and $z=3.1$ from the vortex-representation,[14] gives the estimates $\alpha=0.3(3)$, and $\alpha=0.95$. These are comparable to the observable value in the experiments within the errors.

We should note that the model considered here neglects screening of vortices due to inductance effects. For strong screening, the finite-temperature transition is destroyed.[8,14] However, it is possible that for very weak screening a resistive transition is still possible.

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[24] If the observed resistivity scaling just reflects the critical singularity of the chiral order parameter, as proposed in Ref. 9, then, using the corresponding dynamic exponent obtained from MC dynamics (Ref. 9), $z_{ch} \sim 7.4(10)$, gives $\alpha<0$ which is inconsistent with the experimental observation (Ref. 17) and numerical simulation (Ref. 18).