Kalman Filter Robustness in Estimating an Elastic Parameter of a Flexible Space System

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1. ABSTRACT

Nowadays, control systems of satellites with rigid and flexible components are increasingly being extended to advanced applications, such as space uninhabited vehicles with very demanding pointing accuracy. The control design methods presently available, including parameters and states estimation, robust and adaptive control, need more investigation to know their capability and limitations. In that context, the guaranty of the controller performance depends not only on its good design but also on the knowledge of all states to be fed-back in order to improve the overall control system efficiency. In this paper, a Kalman filter methodology is used to recover all the unmeasured states (elastic displacement and its rates) considering that only the states associated with rigid motion are measured (angle and angular velocity). To investigate the robustness of the Kalman filter, one considers in the measurements model phase a satellite model with three flexible modes, whereas in the time and measurement update phases the satellite model will consider just one flexible mode. Through the simulations, one observes that the fidelity of the estimation process enhances with the inclusion of more modes into the satellite model. However, one observes as well that even with a reduced model in the update phase the robustness of the Kalman filter is preserved once it is properly tuned.

2. KEY WORDS

Kalman Filter, Robustness, Flexible Space System

3. INTRODUCTION

The use of small satellites has been a fast, simple and of a low cost way of reaching the space [1]. However, in the order to conquer the space it's necessary to launch spacecrafts that involves rigid/flexible structures. In that type of spacecrafts, the influence of flexibility plays an important role in the dynamics behavior as well as in the performance of the Attitude Control System (ACS). Other important aspect in the study of the dynamics and control of flexible space structure are: the degree of interaction between the rigid and flexible motion [2], maintenance of a ACS performance in face the uncertainties of the mathematical model [3], damping residual vibrations in order to keep pointing precision and states estimations [4]. This paper introduces a state estimation procedure using the Kalman filter methodology to recover the flexible coordinates from measurements of the rigid part (angle and velocity angular). Section 2 presents a mathematical model of a simple spacecraft based on a two flexible Euler-Bernoulli beam connected to a rigid hub. The equations of motion are derived considering the torque as input, and angle and angular velocity as outputs. Section 3 presents the Kalman filter state estimation problem. Section 4 presents the simulation

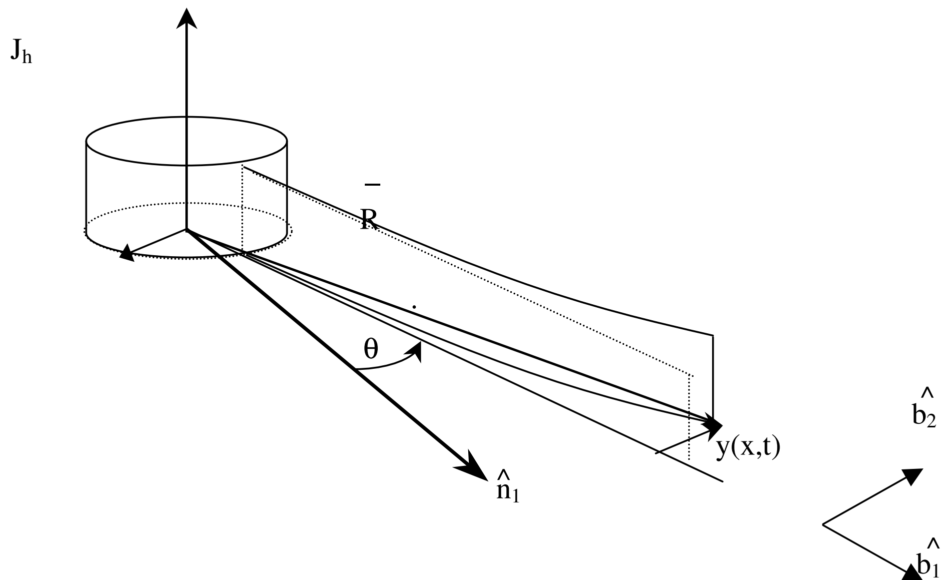

Figure 1: Satellite Mathematical Model

of the problem. Section 5 concludes the paper.

4. SATELITE MATHEMATICAL MODEL

The satellite mathematical model used is composed of a rigid platform with two flexible appendices (see Figure 1). The appendices are identical and opposite, being considered as beam connected to the platform, subject to rotational and vibrational motion. The equations of motion are derived using the Lagrange methodology, starting from the expression of the kinetics and potential energy of the system.

The inertial reference system is represented by the axes $\hat{n}_1$, $\hat{n}_2$, $\hat{n}_3$, which coincides with the center of mass of the rigid body characterized by the axes $\hat{b}_1$, $\hat{b}_2$, $\hat{b}_3$. The vector r is the radius of the rigid body. The vector **x** represents the position along the axis $\hat{b}_1$ in no deformed form. The vector position in the appendage relative to the inertial reference system is given by $\bar{R}$. The vector of elastic deformation, perpendicular to the axis $\hat{b}_1$,is represented by y(x,t), and $\dot{\theta}$ is the satellite angular velocity. Therefore, the vector velocity of any point in the deformed appendage form, relative to the inertial reference system is given by:

$$\dot{\bar{R}} = -\dot{\theta} y \hat{b}_1 + \left[\dot{\theta}(r+x) + \dot{y}\right]\hat{b}_2 \tag{1}$$

5. EQUATIONS OF MOTION

The total kinetics energy of the system is given by

$$T_T = \frac{1}{2} J_h \dot{\theta}^2 + 2\int_0^L \rho\left\{\dot{y}^2 + 2\dot{y}\dot{\theta}(r+x) + \dot{\theta}^2 (r+x)^2\right\} dx \tag{2}$$

where J_h is the rotary inertial of the hub, ρ is the mass density of the appendages, L is the length of the appendage and y(x,t) represents the elastic displacement. The potential energy is given by:

$$V_T = \int_0^L (EI)\left(\frac{d^2 y}{dx^2}\right)^2 dx \tag{3}$$

where E is the modulus of elasticity and I the moment of inertia of the beam. The discretization of the system is done using assumed mode method [4]. Therefore, the elastic displacement y(x,t) is given by

$$y(x,t)=\sum_{j=1}^{n}\phi_j(x)q_j(t) \tag{4}$$

where $q_j(t)$ are the generalized coordinates and $\phi_j(x)$ are the admissible functions. The equations of motion for the rigid $\theta(t)$ and the elastic q(t) motion, are found using the Lagrange formulation:

$$\frac{d}{dt}\left(\frac{dT_T}{d\dot{x}_i}\right)-\frac{dT_T}{dx_i}+\frac{dV_T}{dx_i}=F_i \tag{5}$$

where F_i is the generalized force, and x_i is the ith element of the vector **(x)**. After derivation the equations of motion in matrix form is given by

$$M\ddot{x}+Kx=Du \tag{6}$$

where M represents the mass matrix, K is the stiffness matrix and D in known as control influence matrix. Transforming Equation (6) in space state modal form, one has:

$$\tilde{M}\ddot{\eta}+\tilde{C}\dot{\eta}+\tilde{K}\eta=\tilde{D}u \tag{7}$$

Here $\tilde{M}$, $\tilde{C}$, $\tilde{K}$ and $\tilde{D}$ represents mass, damping, stiffness and control influence matrices in modal form, respectively.

6. KALMAN FILTER METHODOLOGY

The complete dynamical model is represented by:

$$\dot{\eta}_1=A\eta_1+G\omega \tag{8}$$

where $\eta_1=[\eta,\dot{\eta}]^T$ is the modal coordinates, ω is white gaussian noise, G is matrix unitary and A is the system matrix that relates the state linearly by

$$A=\begin{bmatrix}0 & I\\ -\tilde{K} & -\tilde{C}\end{bmatrix}$$

The measured model is given by:

$$Y=C\eta_1+\nu \tag{9}$$

the output is the angle θ and angular velocity $\dot{\theta}$, with standard deviation of 0.05° and 0.005°/s, respectively. The matrix is $C=B^T$. The term ν represents a white noise vector with the following statical characteristic $\nu_\theta=N(0,0.05°)$, $\nu_{\dot{\theta}}=N(0,0.005°/s)$

In the time update, the states are estimates using

$$\dot{\bar{x}}=A\bar{x} \tag{10}$$

with initial conditions $\bar{x}_{k-1}=\hat{x}_{k-1}$, and the covariance is computed by

$$\dot{\bar{P}}=A\bar{P}+\bar{P}A^T+GQG^T-\bar{P}C^TR^{-1}C\bar{P} \tag{11}$$

with initial conditions $\bar{P}_{k-1}=\hat{P}_{k-1}$. Equation (11) is known as Riccati equation. In the measurement update the states and covariance matrix are calculated by

$$K_k=\bar{P}_kC^T\left(C\bar{P}_{k-1}C^T+R\right)^{-1} \tag{12}$$

$$\bar{P}_k=(I-K_kC)\bar{P}_{k-1} \tag{13}$$

$$\hat{x}_k=\bar{x}_{k-1}+K_k(y_k-C\hat{x}_{k-1}) \tag{14}$$

where K represents the Kalman gain, and $\bar{P}$ and $\hat{x}$ are the covariance and the state updated. The errors between the actual state and the estimated state is

$$\Delta\varepsilon_i=x_i-\hat{x}_i \tag{15}$$

7. SIMULATIONS

In order to investigate the robustness of the filter, it has been done the simulation with a satellite model with one, two and three elastic modes. The structural parameters are: radius r = 0.3048m, density ρ=47.89Kg/m, damping $\varsigma=0.2$, L=1.2192m, E = 7.735x10^9 Kg/m^2, I=1.293x10^{-10} Kg*m^2 J_h = 10.84 Kg*m^2, G=[0_{4x4}, I_{4x4}]^T, R= [0.05^2 ,0.005^2]^T, and $Q=\mathrm{diag}([10^{-4},10^{-6},10^{-6},10^{-6}])$.

The initials conditions, $\theta_0 = 0.1$, $\dot{\theta}_0 = 0.01$, $P0 = \mathrm{diag}([10^{-2}]_{nxn})$. Figure 2 shows the difference between the ideal state and the estimate state the "error" for the satellite model with one, two and three modes. It can be seen that angular velocity estimated, remains in all modes, under the limits of standard deviation. But for the angle it is necessary 50 seconds for the filter to adapt and have a good performance.

Figure 3 shows a significant difference between the model with one and two modes in the flexible coordinate q_1 and q_2. However, that difference is negligible for the model with two and three modes, which means that the satellite can be modeled at most with two modes without lost of accuracy. This is correct because, when more modes are included, the dynamics of the system tend to stationary values.

8. CONCLUSIONS

In this work, one applies the Kalman filter Methodoly to estimated the elastic displacement, considering that the angle and the angular velocity of a flexible satellite are sensed. Having in mind the complexity of putting a sensor on the elastic parts of the satellite, the application of the Kalman filter technique has been showed a good approach to estimate indirectly the flexible parameters of a rigid-flexible satellite. That approach becomes more promising when it is necessary to feedback the elastic measurements into the control system in order to assure better pointing conditions and/or better system performance.

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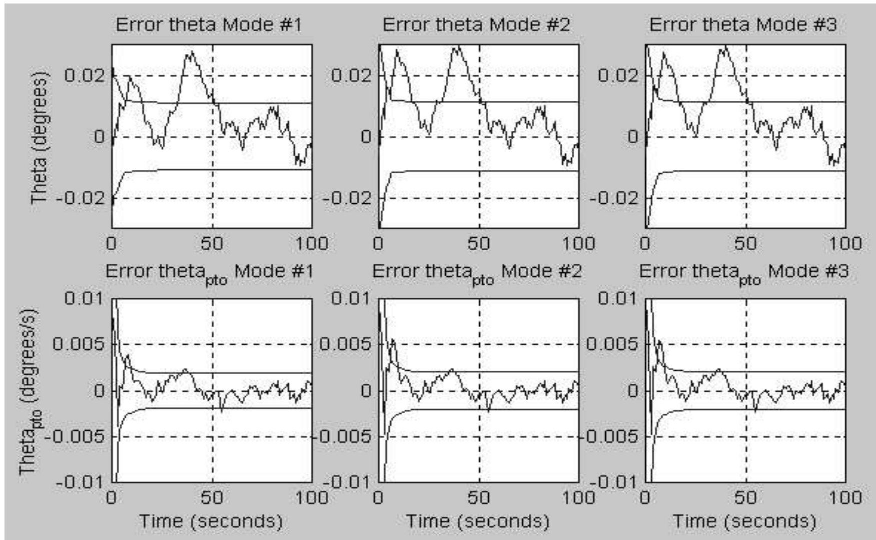

Figure 2 Errors for angle and angular velocity.

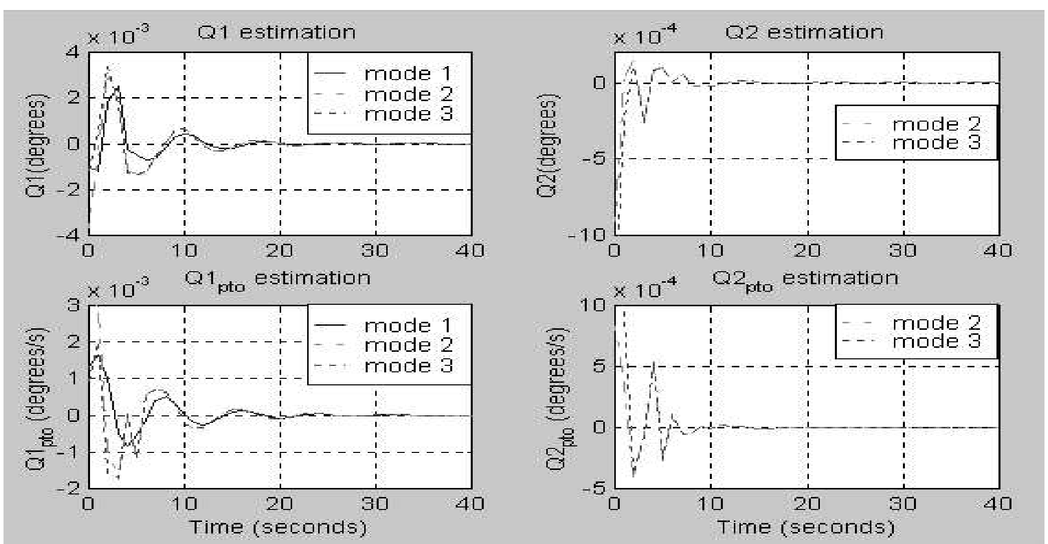

Figure 3 Estimation of the elastic displacement q_1 and q_2.