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STATE OBSERVER TO MULTIVARIABLE TIME VARYING SYSTEMS USING THE GUARANTEED COST CONCEPT

José Ernesto de Araujo Filho

Instituto Nacional de Pesquisas Espaciais
Av. dos Astronautas, 1758 - Cx. postal 515
São José dos Campos - São Paulo
ernesto@lit.inpe.br

Karl Heinz Kienitz

Instituto Tecnológico de Aeronautica
Praça Eduardo Gomes, 50
São José dos Campos - São Paulo
kienitz@ita.cta.br

Abstract: In this paper a state observer method that may be used with a feedback gain matrix designed to quadratic stabilization is proposed. This observer uses the guaranteed cost concept, the second method of Lyapunov and satisfies stability condition to “one-way” connected systems. This method is used to a class of discrete time varying systems with bounded time variance. State vector will be estimated if there exist solution to modified Riccati equation.

Keywords: State observer, Discrete system, Riccati equation, Multivariable system, Lyapunov equation, Time varying system

1. INTRODUCTION

The controller design problem using state feedback has been treated for a long time by state observer designs (state estimator) when the state is impossible, or inappropriate, to be measured (O'Reilly, 1983). This work want contribute with a new line of solution showing a new method to estimate the state vector of a dynamic system using controllers that make use of the guaranteed cost concept (Chang e Peng, 1972; Kienitz, 1990a) to evaluate the system performance, the second method of Lyapunov (Kalman e Bertram, 1960) to consider stability conditions and the stability condition to “one-way” connected systems (Kienitz, 1990b).

2. PROBLEM STATEMENT

Consider the global system presented at figure 1 composed by the system to be controlled, the state observer, and the feedback gain:

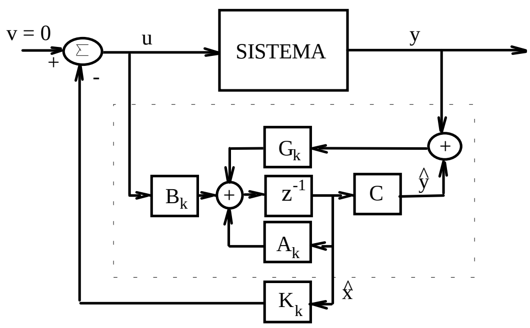

figure 1

State space equations to the system are presented bellow:

$$\mathbf{x}(k+1) = \left[A + \Delta A(k)\right]\mathbf{x}(k) + \left[B + \Delta B(k)\right]\mathbf{u}(k) \quad (1)$$

$$\mathbf{y}(k) = C\mathbf{x}(k) \quad (2)$$

$$\hat{\mathbf{x}}(k+1) = \left[A + \Delta A(k)\right]\hat{\mathbf{x}}(k) + \left[B + \Delta B(k)\right]\mathbf{u}(k) + G(k)\left\{\mathbf{y}(k) - C\hat{\mathbf{x}}(k)\right\} \quad (3)$$

where:

$\mathbf{x}(k) \in \mathbf{R}^{n}$ is the state vector,

$\mathbf{u}(k) \in \mathbf{R}^{m}$ is the control vector,

$\mathbf{y}(k) \in \mathbf{R}^{q}$ is the output vector,

$\hat{\mathbf{x}}(k) \in \mathbf{R}^{n}$ is the estimated state vector,

$A \in \mathbf{R}^{nxn}$, $B \in \mathbf{R}^{nxm}$,$C \in \mathbf{R}^{qxn}$

$\Delta A(k) = D_1 F(k) E_1, D_1 \in R^{nxp}, F(k) \in \mathbf{R}^{pxp}, E_1 \in R^{pxn}$

$\Delta B(k) = D_2 F(k) E_2, D_2 \in R^{nxp}, F(k) \in \mathbf{R}^{pxp}, E_2 \in R^{pxm}$

$C \in \mathbf{R}^{qxn}, \|F(k)\xi\|_2 \leq \|\xi\|_2 \quad \forall\ \xi \in \mathbf{R}^{p}$, e

$G(k)$ is the gain matrix to correct the estimated vector.

State vector will be assintotically estimated if the error, $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$, satisfies the condition $\lim_{k \to \infty} \mathbf{e}(k) = 0$.

Such way, the global state space equation is achieved by using equation (1) and (3), and substituting the state feedback, $\mathbf{u}(k) = -K(k)\hat{\mathbf{x}}(k)$.

had quadratic Lyapunov functions.

Although this concept had been presented by Kienitz (1990b), in 1969 Johnson (1969) showed that the estimated states of continuos time varying systems could be used substituting the real states without modify the stability property of closed loop, if a "exponential observer" were realized.

The subsystem stabilization (6) is proved in De Araujo Filho (1995a). Thus, the solution of the problem is to determine the matrix G(k) appropriate to assintotically stable subsystem (7) that represents the dynamic error.

3. PROBLEM SOLUTION

In this paper is considered that the time variance can be done matched or unmatched assumption and this one is bounded.

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \hat{\mathbf{x}}(k+1) \end{bmatrix} = \begin{bmatrix} A + \Delta A(k) & [B + \Delta B(k)]K(k) \\ G(k)C & [B + \Delta B(k)]K(k) - G(k)C + [A + \Delta A(k)] \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \hat{\mathbf{x}}(k) \end{bmatrix} \tag{4}$$

Applying a linear transformation, "T", to get $[\mathbf{x}(k+1) \quad \mathbf{e}(k+1)]^T = T[\mathbf{x}(k+1) \quad \hat{\mathbf{x}}(k+1)]^T$, the global system may be rewritten as:

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{e}(k+1) \end{bmatrix} = \begin{bmatrix} [A + \Delta A(k)] - [B + \Delta B(k)]K(k) & [B + \Delta B(k)]K(k) \\ 0 & [A + \Delta A(k)] - G(k)C \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{e}(k) \end{bmatrix} \tag{5}$$

The separation principle presents that dynamic of global system is the dynamic of system with feedback associated and the dynamic of estimator. Thus, the controller design and the state estimator design can be done independently (O'Reilly, 1983). If the system is time varying, this principle make also possible that the state observer can be used together with feedback gain matrix designed to quadratic stabilization. It is possible if a sufficient stability condition to subsystems "one-way" connected (Kienitz, 1995) is satisfied.

Subsystems with a block triangular dynamic system matrix are known as "one-way" subsystem.

Since the equation (5) presents upper triangular matrix, the system will be stable if the subsystems:

$$\mathbf{x}(k+1) = \{[A + \Delta A(k)] - [B + \Delta B(k)]K(k)\}\mathbf{x}(k) \tag{6}$$

$$\mathbf{e}(k+1) = \{[A + \Delta A(k)] - G(k)C\}\mathbf{e}(k) \tag{7}$$

Since a system needs to be completely observable in order to estimate the state vector, the duality principle may be used to determine the matrix G(k) (Ogata, 1987). Thus, if $\{[A + \Delta A(k)], C\}$ is observable, then $\{[A + \Delta A(k)]^T, C^T\}$ is controllable. In this case, the control design and state observer design solution are equivalents. This situation establish that the matrix G(k) may be founded by compute $G(k) = K_e(k)^T$. It is possible since the matrix of dynamic equation (7), $\{[A + \Delta A(k)] - G(k)C\}$, may be substituted by dynamic matrix of a dual system, that is, $\{[A + \Delta A(k)]^T - C^T K_e(k)\}$.

Therefore, the procedure developed to the stabilizing control in De Araujo Filho (1995a) may be applied to state observer design through the duality principle.

Like in (De Araujo Filho and Kienitz, 1995b), it is also necessary to introduce some notation to facilitate the understanding of the development of this new approach.

$$\left.\begin{aligned} H_e &= I - C^T\left(CP_eC^T\right)^{-1}CP_e \\ \gamma_e &= \lambda_{max}\left(E_1H_e^TP_eH_eE_1^T\right) \end{aligned}\right\} \qquad (8)$$

Considering the state equations (1) and (2) of the dynamic system, the solution of the state observer design can be sought by following theorem:

Theorem

Consider a linear discrete time varying system (1) and (2). Since the system is observable, a state estimator will assintoticaly estimate the states of the system through the quadratic stabilization of equation (7), if there exist a matrix "P_e" positive definite symmetric solution to the following modified Riccati equation:

$$(1+\varepsilon)AP_eHA^T + \left(\gamma_e + \frac{\gamma_e}{\varepsilon}\right)D_1D_1^T - P_e + Q = 0 \qquad (9)$$

that satisfy $P_e = \left(P_1^{-1} + E_1^TE_1\right)^{-1}$, for some positive definite matrix P_1 , some $\varepsilon > 0$, and considering $G(k) = K_e(k)^T$.

4. CONCLUSION

This paper is concerned with the problem of to estimate the state vector of a dynamic system. A new method based on guaranteed cost concept to solve this problem is proposed. The state vector may be estimate if there exist a solution to modified Riccati equation.

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APPENDIX

Proof of theorem:

Using the duality principle the state equation to dynamic error (7) can be rewritten as:

$$\mathbf{e}_d(k+1) = \left\{\left[A + \Delta A(k)\right]^T - C^TG(k)^T\right\}\mathbf{e}_d(k) \qquad (10)$$

Since $G(k)^T = K_e(k) = \left(CP_eC^T\right)^{-1}CP_e\left[A + \Delta A(k)\right]^T$ equation above is :

$$\mathbf{e}_d(k+1) = \left[I - C^T\left(CP_eC^T\right)^{-1}CP_e\right]\left[A + \Delta A(k)\right]^T\mathbf{e}_d(k) \qquad (11)$$

The following results is obtained using H from eq. (8).

$$\mathbf{e}_d(k+1) = H_e\left[A + \Delta A(k)\right]^T\mathbf{e}_d(k) \qquad (12)$$

Lyapunov stability will be satisfied if the difference function of Lyapunov to dynamic error is negative definite:

$$\begin{aligned}\Delta V_e(\mathbf{e}_d) = {} & \mathbf{e}_d(k+1)^TP_e\mathbf{e}_d(k+1) - \\ & - \mathbf{e}_d(k)^TP_e\mathbf{e}_d(k) < 0\end{aligned} \qquad (13)$$

Substituting equation (12) in inequation (13) results:

$$\begin{aligned}\Delta V_e(\mathbf{e}_d) = {} & \left[H_e(A + \Delta A)^T\mathbf{e}_d\right]^TP_e\left[H_e(A + \Delta A)^T\mathbf{e}_d\right] - \\ & - \mathbf{e}_d^TP_e\mathbf{e}_d < 0\end{aligned} \qquad (14)$$

or:

$$\Delta V_e(\mathbf{e}_d) = \mathbf{e}_d^T\left[(A + \Delta A)H_e^TP_eH_e(A + \Delta A)^T - P_e\right]\mathbf{e}_d \qquad (15)$$

Considering $\mathbf{e}_d^TM^TPN\mathbf{e}_d = \mathbf{e}_d^TN^TPM\mathbf{e}_d$, this yields:

$$\Delta V_e(\mathbf{e}_d) = \mathbf{e}_d^T \begin{bmatrix} AH_e{}^T P_e H_e A^T + \\ \quad + 2AH_e{}^T P_e H_e \Delta A^T + \\ \quad\quad + \Delta AH_e{}^T P_e H_e \Delta A^T - P_e \end{bmatrix} \mathbf{e}_d < 0 \qquad (16)$$

Using the lemma 3.1 (De Araujo Filho, 1995a) equation above may be rewritten as:

$$\Delta V_e(\mathbf{e}_d) = \mathbf{e}_d^T \begin{bmatrix} (1+\varepsilon) AH_e^T P_e H_e A^T + \\ + \left(1 + \frac{1}{\varepsilon}\right) \Delta AH_e^T P_e H_e \Delta A^T - P_e \end{bmatrix} \mathbf{e}_d < 0$$

(17)

Using $\Delta A(k) = D_1 F(k) E_1$, this one is equivalent to:

$$\Delta V_e(\mathbf{e}_d) = \mathbf{e}_d^T \begin{bmatrix} (1+\varepsilon) AH_e^T P_e H_e A^T - P + \\ + \left(1 + \frac{1}{\varepsilon}\right) D_1 F \lambda_{max}\left(E_1 H_e^T P_e H_e E_1^T\right) F^T D_{1\,e}^T \end{bmatrix} \mathbf{e}_d < 0$$

(18)

Using claim 3 presented in Kienitz (1995), this yields:

$$\Delta V_e(\mathbf{e}_d) = \mathbf{e}_d^T \begin{bmatrix} (1+\varepsilon) AP_e H_e A^T + \\ + \left(\gamma_e + \frac{\gamma_e}{\varepsilon}\right) D_1 D_1^T - P_e \end{bmatrix} \mathbf{e}_d < 0 \qquad (19)$$

Forcing the difference $\Delta V(\mathbf{e}_d)$ to be negative, equation (19) results:

$$\mathbf{e}_d^T \begin{bmatrix} (1+\varepsilon) AP_e H_e A^T + \\ + \left(\gamma_e + \frac{\gamma_e}{\varepsilon}\right) D_1 D_1^T - P_e \end{bmatrix} \mathbf{e}_d = -\mathbf{e}_d^T Q\, \mathbf{e}_d \qquad (20)$$

Hence:

$$\mathbf{e}_d^T \begin{bmatrix} (1+\varepsilon) AP_e H_e A^T + \\ + \left(\gamma_e + \frac{\gamma_e}{\varepsilon}\right) D_1 D_1^T - P_e + Q \end{bmatrix} \mathbf{e}_d = 0 \qquad (21)$$

The term between brackets correspond to modified Riccati equation presented at theorem.