A NUMERICAL ALGORITHM TO CALCULATE BI-IMPULSIVE THREE-DIMENSIONAL MANEUVERS

Maria Matilde Netto dos Santos Paulo
CDT-ETEP - Av. Barão do Rio Branco 882 - São José dos Campos - SP - 12242-800

Antonio Fernando Bertachini de Almeida Prado
Marcelo Lopes de Oliveira e Souza
Instituto Nacional de Pesquisas Espaciais – São José dos Campos-SP-12227-010
Phone (012) 345-6201 – Fax (012) 3456226 – E-mail: prado@dem.inpe.br

ABSTRACT

In this paper the problem of spacecraft orbit transfer with minimum fuel consumption is considered. The main goal is to develop and implement an algorithm that solves the problem of bi-impulsive three-dimensional orbital transfer. After a search in the literature and analysis of the results available, one selects a method developed by Altman and Pistiner to be the base of the algorithm developed. Their method has the goal of solving the minimum fuel consumption between two fixed positions in space. In the present paper this method is extended to solve the problem of bi-impulsive transfer between two noncoplanar orbits. The spacecraft is supposed to be in Keplerian motion controlled by the thrusts, that are assumed to be impulsive. Results of simulations are presented. The proposed algorithm has two useful characteristics: i) it allows an easy handling of constraints in the region of the orbit that the thrust can be applied, what can be used to avoid burning when the satellite is not visible from the ground stations; ii) it can easily balance between the accuracy of the solution and the time required for solvng the problem, what makes it suitable for on-board implementation in an autonomous satellite.

Key Words: Orbital Transfer, Astrodynamics, Impulsive Maneuvers.

INTRODUCTION

The launching of a geostationary or a heliosynchronous satellite, the orbit corrections, the maintenance of space stations, the interplanetary trips and the interception of celestial bodies are examples of ordinary space missions very popular nowadays due to the great advance of the Space Sciences, and that require orbital maneuvers for their execution. Since it became necessary the use of vehicles equipped with propulsion systems to perform such space missions, it became also necessary the study of the optimal transfer problem of a spacecraft between two given orbits. Some of the papers related to this research are: Hohmann (1925), Hoelker and Silber (1959), Lawden

(1962), Ting (1960), Eckel and Vinh (1984), Jin and Melton (1991), Roth (1967), Prado and Broucke (1994), Eckel (1963), Broucke and Prado (1993, 1996), Rocco (1997). In this paper, we study the tridimensional optimal bi-impulsive transfers extending the formulation of Altman and Pistiner (1964). A more detailed study can be found in Paulo (1998).

DEFINITION OF ORBITAL TRANSFER

An orbital transfer consists of changing the state of a space vehicle. The state is defined as the position, velocity and mass of the vehicle at a given time. Fig. 1 shows an orbital transfer between two points marked by the subscripts "0" and "f".

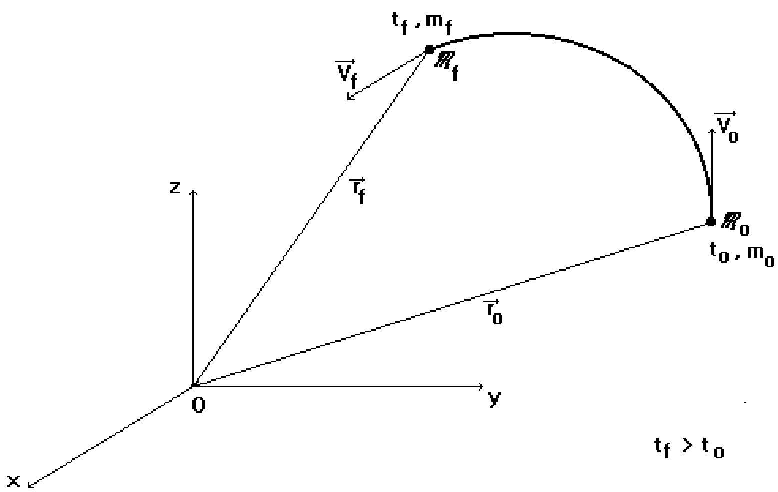

Fig. 1 – Orbital Transfer.

In this paper we study the three-dimensional bi-impulsive transfer, that is shown in Fig. 2 (from Altman and Pistiner, 1964).

The vehicle proceeds in the initial orbit (A) until the transfer point P_1, when a instantaneous change in the velocity makes the vehicle to proceed along P_1P_2, until the final orbit. When arriving in the transfer point P_2, a new impulse places the vehicle in the final orbit (B). The initial and final orbits are defined by their orbital parameters. The transfer points of the orbits A and B (P_1 and P_2) can be defined by the respective true anomalies ϕ_1 and ϕ_2. The plane of the transfer orbit can be determined by three points in space (P_1, P_2, O) or by two vectors with common origin $\vec{r}_1$ and $\vec{r}_2$.

The intersection angles (between the orbital planes) and the central angle (between the transfer points) are determined by vector relationships that are functions of the orbital parameters of the initial and final orbits and of the vectors $\vec{r}_1$ and $\vec{r}_2$.

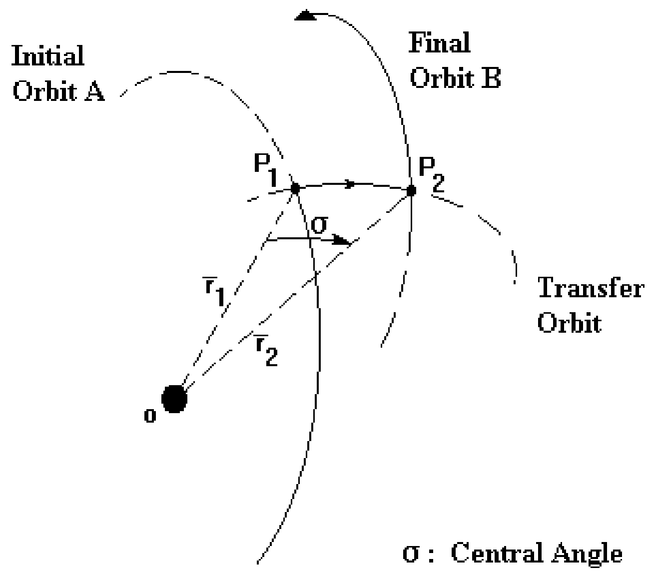

Fig. 2 - Three-Dimensional Transfer

DESCRIPTION OF THE METHOD

To find the orbit and the transfer points of a bi-impulsive transfer with minimum consumption of fuel between two noncoplanar orbits, an algorithm was elaborated and implemented that determines:

- a set of orbital elements (semi-major axis, eccentricity, inclination, argument of the periapsis, longitude of the ascending node) for the transfer orbit;

- the true anomalies of the transfer points, measured in the initial and transfer orbits (first impulse) and in the transfer and final orbits (second impulse);

- the variations of the velocities in the transfer points, and, consequently, the total variation of velocity.

This method solves the problem of the transfer between two given orbits, generalizing the original formulation that solved the problem for two fixed points belonging to the initial and final orbits. The next step is to obtain the Cartesian coordinates of the plane motion starting from the known elements. With this procedure available, two values were guessed for the true anomalies and used, together with Equation 1, to obtain a pair of values for the eccentric anomalies in the initial (u_{1A}, u_{2A}) and in the final orbits (u_{1B}, u_{2B}). With those values we obtained two position vectors for each orbit ($\vec{r}_{1A}, \vec{r}_{2A}, \vec{r}_{1B}, \vec{r}_{2B}$) and the respective velocity vectors in these points ($\vec{v}_{1A}, \vec{v}_{2A}, \vec{v}_{1B}, \vec{v}_{2B}$). The versors ($\vec{s}_A, \vec{s}_B$) of the planes of the orbits (initial and final, respectively) are given by Equation 1.

$$\vec{s}_n = \frac{\vec{r}_{1n} \times \vec{r}_{2n}}{\left|\vec{r}_{1n} \times \vec{r}_{2n}\right|} \tag{1}$$

where n = A (initial orbit) or B (final orbit).

Since this method intends to find the orbit of minimum fuel transfer between two given orbits, these orbits were discretized in terms of the true anomaly (ϕ_1, ϕ_2) of the transfer points. The angle ϕ_1 describes the true anomaly (measured in the initial orbit) of the point of application of the first impulse. The angle ϕ_2 describes the true anomaly (measured in the final orbit) of the point of application of the second impulse. This approach gives the possibility to limit the values of (ϕ_1, ϕ_2) in given intervals. So, it is necessary to give the information of an initial and final values and of an increment for both angles. Thus, the method of Altman and Pistiner (1964) was implemented to obtain the transfer with the smallest total cost ($\Delta V1 + \Delta V2$) between the two fixed points. The number of points of discretization is defined for each case, depending on the accuracy required, and we tested all the possible combinations between these points with the objective of finding the pair of transfer points that generates the transfer with the smallest fuel consumption. The procedure to determine the position and velocity in the transfer points ($\vec{r}_1$, $\vec{r}_2$, $\vec{v}_1$, $\vec{v}_2$), as well as the versor of the transfer orbit plane ($\vec{s}_t$) is the same described previously. So:

$$\vec{s}_t = \frac{\vec{r}_1 \times \vec{r}_2}{\left|\vec{r}_1 \times \vec{r}_2\right|} \tag{2}$$

The central angle σ is the angle that the space vehicle should travel between the instants t_1 and t_2 (during the transfer) and it is defined by the positions $\vec{r}_1$ and $\vec{r}_2$. It is:

$$\cos\sigma = \frac{\vec{r}_1 \cdot \vec{r}_2}{\left|\vec{r}_1\right|\left|\vec{r}_2\right|} \tag{3}$$

The intersection angles ρ (between the planes of the initial and transfer orbits) and λ (between the planes of the transfer and final orbits) are calculated by:

$$\cos\rho = \vec{s}_A \cdot \vec{s}_t \qquad\qquad \cos\lambda = \vec{s}_B \cdot \vec{s}_t \tag{4)-(5}$$

The increments of velocity (ΔV_1, ΔV_2) in the transfer points and, consequently, the ΔV_{total}, as well as the true anomalies (ϕ_{T1}, ϕ_{T2}) of these points, measured in the transfer orbit, are calculated through the following group of equations (Altman and Pistiner, 1964), reminding that the orbital planes are different (see Figs. 5 and 6).

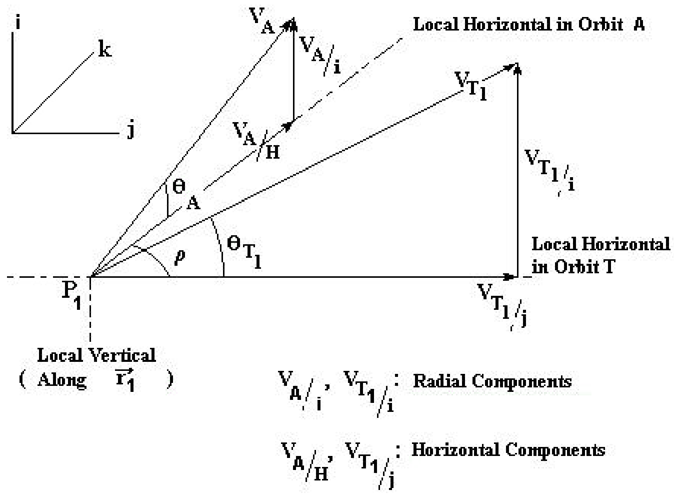

Fig. 5 - Velocity in the initial point of the transfer.

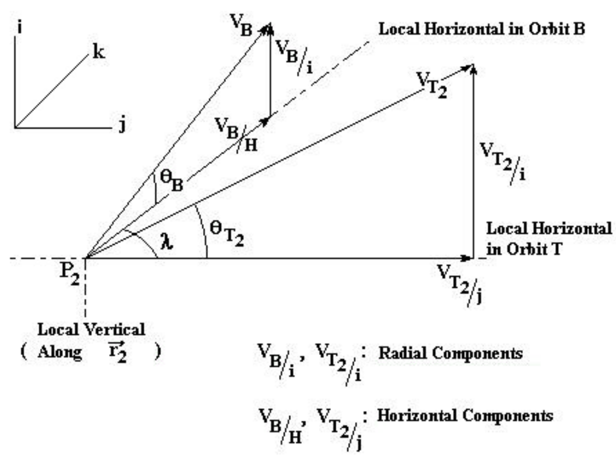

Fig. 6 – Velocity in the final point of the transfer.

The equations are:

radial component of V_A: $v_{A/i} = v_A \,\mathrm{sen}\,\theta_A$ (6)

horizontal component of V_A: $v_{A/H} = v_A \cos\theta_A$ (7)

projection of V_A in the horizontal direction of the orbit T:

$$v_{A/j} = v_{A/H} \cos\rho = v_A \cos\theta_A \cos\rho \tag{8}$$

projection of V_A in the direction perpendicular to the horizontal of the orbit T: $v_{A/k} = v_{A/H} \sin\rho = v_A \sin\theta_A \sin\rho$

(9)

radial component of V_T: $v_{T1/i} = v_{T1} sen\theta_{T1}$ (10)

horizontal component of V_T: $v_{T1/j} = v_{T1} \cos\theta_{T1}$ (11)

projection of V_T in the direction perpendicular to the horizontal of the orbit T:

$$v_{T1/k} = 0 \tag{12}$$

$$\Delta v_1^2 = (v_{T1/i} - v_{A/i})^2 + (v_{T1/j} - v_{A/j})^2 + (v_{T1/K} - v_{A/K})^2 =$$

$$= (v_{T1}\ sen\ \theta_{T1} - v_A\ sen\ \theta_A)^2 + (v_{T1}\ \cos\ \theta_{T1} - v_A\ \cos\ \theta_A\ \cos\ \rho)^2 + (v_A \cos\theta_A\ sen\ \rho)^2 \tag{13}$$

Being C the velocity parameter of the orbit (inversely proportional to the angular momentum h), we have:

$$C_A = \frac{\mu m}{h} = \frac{\mu}{|\vec{r}_{1A}\ x\ \vec{v}_{1A}|} \qquad C_B = \frac{\mu m}{h} = \frac{\mu}{|\vec{r}_{1B}\ x\ \vec{v}_{1B}|} \tag{14)-(15}$$

Consequently:

$$v_A{}^2 = v_{A/i}{}^2 + v_{A/H}{}^2 \tag{16}$$

$$v_{A/i} = \left(v_{C1}{}^2 / C_A - C_A\right)\tan\phi_A \tag{17}$$

$$v_{A/H} = v_{C1}{}^2 / C_A \tag{18}$$

$$v_{T1}{}^2 = v_{T1/i}{}^2 + v_{T1/j}{}^2 \tag{19}$$

$$v_{T1/i} = \left(v_{C1}{}^2 / C_T - C_T\right)\tan\phi_{T1} \tag{20}$$

$$v_{T1/j} = \left(v_{C1}{}^2 / C_T\right) \tag{21}$$

$$v_{cm}{}^2 = \left(C\,v_\phi\right)_m = \mu / r_m \tag{22}$$

where m = 1 ou 2, ϕ_A is in the plane of the initial orbit and ϕ_T is in the plane of the transfer orbit. The equations for the point P_2 are obtained in a similar way. The complete set of the transfer equations is:

$$v_{nm}{}^2 = \left(v_{cm} / C_n\right)^2 + \left(v_{cm} / C_n - C_n\right)^2 \tan^2\phi_{nm} \tag{23–26}$$

$$\text{where}\begin{cases} m = 1 \text{ and} = A \text{ or } T \\ m = 2 \text{ and} = T \text{ or } B \end{cases}$$

$$\Delta v = \left|\Delta v_1\right| + \left|\Delta v_2\right| \tag{27}$$

$$\Delta v_1{}^2 = \frac{\left[a_1\left(C_A - C_T \cos\rho\right)^2 + \left(b_1 C_T{}^2 + c_1 C_T + d_1\right)^2\right]}{C_A{}^2 C_T{}^2 \operatorname{sen}^2 \sigma} + e_1{}^2 \tag{28}$$

$$\Delta v_2{}^2 = \frac{\left[a_2\left(C_B - C_T \cos\lambda\right)^2 + \left(b_2 C_T{}^2 + c_2 C_T + d_2\right)^2\right]}{C_B{}^2 C_T{}^2 \operatorname{sen}^2 \sigma} + e_2{}^2 \tag{29}$$

$$\sigma = (\phi_B - \phi_A) + (\Psi_B - \Psi_A) \tag{30}$$

$$a_1 = v_{C1}{}^4 \operatorname{sen}^2 \sigma \tag{31}$$

$$b_1 = C_A (1 - \cos\sigma) \tag{32}$$

$$c_1 = (C_A{}^2 - v_{C1}{}^2) \tan\phi_1 \operatorname{sen}\sigma \tag{33}$$

$$d_1 = C_A\left(v_{C1}{}^2 \cos\sigma - c_{C2}{}^2\right) \tag{34}$$

$$e_1 = v_{C1}{}^2 \operatorname{sen}\rho / C_A \tag{35}$$

$$a_2 = v_{C2}{}^4 \operatorname{sen}^2 \sigma \tag{36}$$

$$b_2 = C_B(\cos\sigma - 1) \tag{37}$$

$$c_2 = (C_B{}^2 - v_{C2}{}^2) \tan\phi_2 \operatorname{sen}\sigma \tag{38}$$

$$d_2 = C_B(v_{C1}{}^2 - v_{C2}{}^2 \cos\sigma^2) \tag{39}$$

$$e_2 = v_{C2}{}^2 \operatorname{sen}\lambda / C_B \tag{40}$$

$$v_{C1}{}^2 = \mu / r_1 \tag{41}$$

$$v_{C2}{}^2 = \mu / r_2 \tag{42}$$

$$\phi_{T1} = \operatorname{arc\ tan}\left\{\left[\cos\sigma - \frac{C_T{}^2 - v_{C2}{}^2}{C_T{}^2 - v_{C1}{}^2}\right].(\operatorname{sen}\sigma)^{-1}\right\} \tag{43}$$

$$\phi_{T2} = \operatorname{arc\ tan}\left\{\left[\frac{\left(C_T{}^2 - v_{C1}{}^2\right)\operatorname{sen}\sigma}{\left(C_T{}^2 - v_{C2}{}^2\right)} - \cos\sigma\right].(\operatorname{sen}\sigma)^{-1}\right\} \tag{44}$$

We observed that the equations for the calculation of the increments in velocity are expressed as functions of only one independent variable, the parameter C_T. As the optimal trajectory searched is the one that requests the smallest consumption of fuel:

$$\frac{d(\Delta V_{total})}{dC_T} = 0 \tag{45}$$

giving a polinomial equation of eighth degree (Altman and Pistiner, 1964):

$$K_8 C_T{}^8 + K_7 C_T{}^7 + K_6 C_T{}^6 + K_5 C_T{}^5 + K_4 C_T{}^4 + K_3 C_T{}^3 + K_2 C_T{}^2 + K_1 C_T + K_0 = 0 \tag{46}$$

where:

$$K_8 = A\left[\left(G^2 - B^2\right) + 4A\left(D - H\right)\right] \tag{47}$$

$$K_7 = 4A\left[2A\left(J - E\right) + \left(DG - BH\right)\right] + \left[BG\left(G - B\right)\right] \tag{48}$$

$$K_6 = 2A\left[4\left(BJ - EG\right) + \left(GJ - BE\right)\right] + \left[DG^2 - B^2 H\right] \tag{49}$$

$$K_5 = 4A\left[2F\left(G - B\right) + \left(DJ - EH\right)\right] + \left[2BG\left(J - E\right) + B^2 J - EG^2\right] \tag{50}$$

$$K_4 = A\left[\left(J^2 - E^2\right) - 8F\left(D - H\right)\right] + \left[F\left(G^2 - B^2\right)\right] + \left[2\left(DGJ - BEH\right)\right] \tag{51}$$

$$K_3 = -4F\left[2A\left(J - E\right) + \left(DG - BH\right)\right] - \left[2EJ\left(G - B\right)\right] + \left[BJ^2 - E^2 G\right] \tag{52}$$

$$K_2 = 2F\left[-4\left(BJ - EG\right) + \left(GJ - BE\right)\right] + \left[DJ^2 - E^2 H\right] \tag{53}$$

$$K_1 = -4F\left[2F\left(G - B\right) + \left(DJ - EH\right)\right] - \left[EJ\left(J - E\right)\right] \tag{54}$$

$$K_0 = F\left[\left(J^2 - E^2\right) + 4F\left(D - H\right)\right] \tag{55}$$

and

$$A = \frac{b_1{}^2}{C_A{}^2} \tag{56}$$

$$B = \frac{2b_1 c_1}{C_A{}^2} \tag{57}$$

$$D = \frac{\left(a_1 + 2b_1 d_1 + c_1{}^2\right)}{C_A{}^2} \tag{58}$$

$$E = \frac{2\left(C_A a_1 \cos\rho - c_1 d_1\right)}{C_A{}^2} \tag{59}$$

$$F = v_{C1}{}^4 - 2 v_{C1}{}^2 v_{C2}{}^2 \cos\sigma + v_{C2}{}^4 \tag{60}$$

$$G = \frac{2 b_2 c_2}{C_B{}^2} \tag{61}$$

$$H = \frac{\left(a_2 + 2 b_2 d_2 + c_2{}^2\right)}{C_B{}^2} \tag{62}$$

$$J = \frac{2\left(C_B a_2 \cos\lambda - c_2 d_2\right)}{C_B{}^2} \tag{63}$$

To find the parameter C_T, that is one of the roots of Equation 46, we used the method of splitting the interval in two parts. This method requests the previous knowledge of the limits of the interval where to search the root. We know that if an interval (a, b) contains only one root, then the signs of f(a) and f(b) are opposed. After we found C_T, the transfer orbit can be completely identified. The magnitude of the impulses ΔV_1, ΔV_2 can be obtained directly from Equations 28 and 29. The values of ϕ_{T1} and ϕ_{T2} can be obtained through Equations 43 and 44. The semi-major axis (a_T) and the eccentricity (e_T) are obtained by solving a system formed by Equations 64 and 65:

$$r_1 = \frac{a(1-e^2)}{1 + e\cos\phi_{T1}} \qquad r_2 = \frac{a(1-e^2)}{1 + e\cos\phi_{T2}} \tag{64)-(65}$$

Fig. 7 shows that, $\vec{r}_1 = (x_1, y_1, z_1)$ e $\vec{r}_2 = (x_2, y_2, z_2)$, the longitude of the ascending node (Ω), the argument of the periapsis (ω) and the inclination (i) can be calculated by:

$$\Omega = tg^{-1}(\kappa) \tag{66}$$

where:

$$\kappa = \frac{\left(\vec{r}_1 \times \vec{r}_2\right)\hat{i}}{-\left(\vec{r}_1 \times \vec{r}_2\right)\hat{j}} = \frac{(y_1 z_2) - (z_1 y_2)}{(x_1 z_2) - (z_1 x_2)} \tag{67}$$

$$\cos(\omega + \phi_{T1}) = \frac{\vec{r}_1 . \vec{M}}{|\vec{r}_1|\,|\vec{M}|}\ \text{, com } \vec{M} = (1, \kappa, 0) \tag{68}$$

Being $\vec{M}$ a vector in the direction of the intersection line between the planes of the transfer and the initial orbits, where the component in x is assumed unitary, the component in y is given by k and the component in z is null. See Fig. 7.

$$\cos i = \vec{s}_t . \hat{z} \text{ , where } \hat{z} = (0, 0, 1) \tag{69}$$

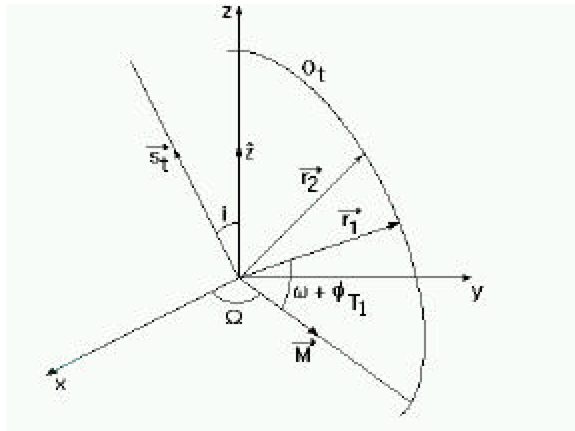

Fig. 7 – Geometry of the transfer.

RESULTS

With the equations developed in this work, the implemented method can be applied for transfers between any conical orbits. The implemented version of the method is not valid only for the cases of bi-impulsive transfers where the impulses are applied in points separated by 180°. These cases include the most well-known problem, the Hohmann transfer (Hohmann, 1925). Despite their importance, they can be easily solved by other methods and they do not need the tools developed in this work.

To compare, we present, in the first case, the magnitude of the total impulse requested by the transfer obtained in the works of Biggs (1978) and Schulz (1997). We observed that this comparison will be limited to the consumption of fuel due to the fact that this problem has more than one solution, that is, different points of application of the impulses can present very close values for the fuel consumption.

Case 1 (Elliptic - Elliptic non-coplanar)

We considered the transfer between two elliptic orbits, of eccentricities 0.02 and 0.016; respectively, the initial and final orbits. The values of the semi-major axis, inclination, argument of the periapsis and longitude of the ascending node are, respectively: 12030.00 km, 0.5°, 182° and 0° for the initial orbit; and 11994.70 km, 0.3°, 175.9° and 8.9° for the final orbit. We obtained, in this example, an elliptic orbit transfer, with eccentricity (e_T) 1.9391 x 10^{-2}, semi-major axis (a_T) equal to 12037.40 km, inclination (i_T) 0.4144°, argument of the periapsis (ω_T) 2.7983° and longitude of the ascending node (Ω_T) equal to 358.5698°. The magnitudes of the necessary increments for the transfer are ΔV_1 = 8.6000 x 10^{-3} km/s for the first impulse and ΔV_2 = 1.7273 x 10^{-2} km/s for the second impulse. Consequently, the total impulse (ΔV_{TOT}) is 2.5873 x 10^{-2} km/s. The true anomalies of the points of application of these impulses are, for the first impulse: ϕ_A = 185.0000° (measured in the initial orbit) and ϕ_{1T} = 5.6320° (measured in the transfer orbit); for the second impulse they are: ϕ_{2T} = 153.4278° (measured in the transfer orbit) and ϕ_B = 330.0000° (measured in the final orbit). The velocity parameter C_T is 5.7555 km/s. In the work of. Biggs (1978), this same transfer requests a total impulse of 0.02210 km/s. In Schulz (1997) we found 0.02222 km/s. We noticed that the result of this study (ΔV_{TOT} = 0.025873 km/s) differs a little from the values obtained in the other two studies. This can be explained by the discretization of the orbits used in this work.

Case 2 (Elliptic - Elliptic non-coplanar)

In this example, we made all the elements to change, except the longitude of the ascending node, that is equal to 0° for both orbits. The semi-major axis varied from 31650 km to 42200 km. The eccentricity of the initial orbit is 0.1 and for the final orbit it is 0.2. The inclination and the argument of the periapsis are both zero for the initial elliptic orbit, and they are 30° and 45°, respectively, for the final orbit. The transfer orbit obtained has semi-major axis of 35773.9244 km, eccentricity 0.1518, inclination 25.4711°, longitude of the ascending node 5.0000°, argument of the periapsis of 91.6139° and parameter velocity of 3.3772 km/s. The total impulse requested by the transfer is 1.9659 km/s, being the magnitude of the first impulse equal to 1,5638 km/s and the second equal to 0.4021 km/s. The true anomalies of the first transfer point, measured in the initial and transfer orbits, are, respectively, 185.0000° and 88.3861°. The true anomalies of the second transfer point, measured in the transfer and final orbits, are, respectively, 238.9566° and 290.0000°.

Case 3 (Elliptic - Elliptic non-coplanar)

The difference between this case and the previous one is the fact that all the elements are varied. The values for the initial elliptic orbit are: semi-major axis: 9567 km, eccentricity: 0.1, inclination: 30°, longitude of the ascending node: 45°, argument of the periapsis: 60°. The Keplerian elements of the final orbit are: semi-major axis: 12756 km, eccentricity: 0.3, inclination: 54°, longitude of the ascending node: 14°, argument of the periapsis: 345°. For this transfer, the first impulse has magnitude 1.9165 km/s and it is given in the point of true anomaly 255.0000°, measured in the initial orbit. In the second point, the increment has magnitude 1.9804 km/s and it is applied with an angle of 160°, counted in the final orbit. The total impulse is 3.8969 km/s. The true anomalies of the transfer points, measured in the transfer orbit are 14.6913° for the first impulse and 151.9871° for the second impulse. The velocity parameter of the transfer orbit is 5.6968 km/s. The inclination angles, longitude of the ascending node and argument of the periapsis of the transfer orbit are, respectively: 39.0084°, 31.9211°, 311.1353°. The semi-major axis and the eccentricity are 13263.9488 km and 0.0272.

Case 4 (Transfer to a Molniya Orbit)

In this example a transfer to a Molniya orbit is made, starting from an initial orbit close to it. The values for the initial elliptic orbit are: semi-major axis: 25000 km, eccentricity: 0.7, inclination: 60°, longitude of the ascending node: 0°, argument of the periapsis: 270°. The Keplerian elements of the final orbit are: semi-major axis: 26600 km, eccentricity: 0.75, inclination: 63.4°, longitude of the ascending node: 0°, argument of the periapsis: 270°. A constraint of allowing the impulses to occur only for true anomalies of the satellite between 90° and 180° both in the initial and final orbit is included, to show this important capability of the method developed. For this transfer, the first impulse has magnitude 0.3188 km/s and it is given in the point of true anomaly 115.0000°, measured in the initial orbit. In the second point, the increment has magnitude 0.0709 km/s and it is applied with an angle of 180°, counted in the final orbit. The total impulse is 0.3897 km/s. The true anomalies of the transfer points, measured in the transfer orbit are 114.8083° for the first impulse and 179.8553° for the second impulse. The velocity parameter of the transfer orbit is 5.6337 km/s. The inclination angles, longitude of the ascending node and argument of the periapsis of the transfer orbit are, respectively: 63.4087°, 1.7703°, 269.3527°. The semi-major axis and the eccentricity are 26904.5252 km and 0.7302.

CONCLUSIONS

We developed, implemented and tested a numerical algorithm that calculates minimum fuel maneuvers between two Keplerian orbits that use a bi-impulsive propulsion system to do the required maneuver. This algorithm can be used for planar and non-planar maneuvers. It is an extension of a method developed by Altman and Pistiner, that was developed to solve the problem of transfers between two fixed points in space. All equations used are derived and explained in some detail. Several tests were made, with four of them shown in detail. They shown the applicability of the method.

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