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Experimental evidence of phase coherence of magnetic field fluctuations in the solar wind using GEOTAIL satellite

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Large amplitude magnetohydrodynamic (MHD) waves are commonly found in the solar wind. Non-linear interactions between the MHD waves are likely to produce finite correlation among the wave phases. For discussions of various transport processes of energetic particles, it is fundamentally important to determine whether the wave phases are randomly distributed (as assumed in the quasi-linear theory) or they have a finite coherence. Using a method based on a surrogate data technique, we analysed GEOTAIL magnetic field data to evaluate the phase coherence among the MHD waves in the Earth's foreshock region. The correlation among the wave phases does exist, including that non-linear interactions between the waves are in progress.

Keywords: MHD turbulence,non-linear interactions, phase coherence

1. Introduction

The solar wind, a hot supersonic and super-Alfénic plasma flow streaming from the sun, provides an ideal environment for the study of nonlinear plasma waves. A wealth of nonlinear phenomena can be found in the solar wind, especially around the planetary bow shocks and interplanetary shocks. The magnetic and electric fields show turbulent fluctuations. A particular interest is the low-frequency large amplitude magnetohydrodynamic (MHD) waves. Such MHD waves are considered to play crucial roles in heating of plasma and acceleration of energetic particles.

Transport of the energetic particles by MHD turbulent field has been analysed mainly in terms of the quasi-linear theory (Sagdeev & Galeev 1969). Two major assumptions are imposed to the theory. The first is that the amplitudes of the wave perturbations are sufficiently small, so that truncation of terms at the second power of the wave amplitude is possible. The second assumption is the so-called *random phase approximation*, which is supposed to destroy any effect of wave-wave coherence due to phase mixing.

However, the MHD waves in space do not necessarily satisfy these assumptions. We note that the temporal variations can be often regarded as spatial fluctuations on the solar wind flow since the solar wind speed is much higher than that of satellite, i.e., the Taylor hypothesis. Thus we often observe spatial fluctuations rather than time-developping oscillation although it depends on the eddy turn over time. Such MHD waves often have amplitude comparable or larger than the ambient magnetic field, and their wavefoms are not likely to be stochastic. For instance, the so-called *shocklets*, commonly found in the upstream region of the Earth's bow shock (Hoppe et al. 1981), that of planetary bow shocks (Fairfield & Behannon 1976; Hoppe & Russell 1981), and near comets (Tsurutani 1991), have the wave magnetic field amplitude comparable or even a few times larger than the average of local magnetic field, and nonlinearly developed waveform. They are often accompanied by monochromatic whistler wave trains, which presumably are reminiscent of soliton-trains. The shocklets are shown to be consequences of nonlinear evolution of obliquely propagating almost monochromatic waves by numerical simulation studies (Hada et al. 1987; Omidi & Winske 1990).

Another typical example suggesting that the two central assumptions made in the quasi-linear theories may be violated is the *SLAMS* (short large amplitude magnetic structures), often detected upstream of quasi-parallel shock waves (Schwartz & Burgess 1991; Schwartz et al. 1992). They are short duration ultra-low-frequency waves ($\sim$ 10 s), characterized by a well-defined single magnetic structure. Similar structures are reproduced in numerical simulations (Akimoto et al. 1991; Scholer 1993). It has suggested possibilities of non-classical diffusion of charged particles by coherent large amplitude MHD waves in terms of numerical simulations (Kirk et al. 1996; Kuramitsu & Hada 2000). From the viewpoints of those observations and numerical simulations, we thus expect that in the MHD turbulence there may exist a state in which the quasi-linear theory is no longer valid.

From this point of view, we introduce a method to estimate the phase coherence among MHD waves quantitatively in §2. By applying this method, we evaluate the phase coherence among MHD waves using magnetic field time series data observed by the GEOTAIL satellite during the period 8 October 18:00 UT to 9 October 06:00 UT 1995 in §3. Finally, we summarize the results in §4.

2. Phase Coherence Index

When we attempts to obtain a phase information from data, the Fourier transform has traditionally been the starting point for this purpose. This transformation of a time series data $x(t)$ is defined as

$$\hat{X}(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt, \tag{2.1}$$

where ω indicates the angular frequency, which brings us the information of amplitude $|\hat{X}(\omega)|$ and phase distribution $\phi(\omega) = tan^{-1}(\Im(\hat{X}(\omega))/\Re(\hat{X}(\omega)))$. An example is given in figure 1. In space plasma physics research, the amplitude (power spectrum) has been discussed in a large number of literature over many years, e.g., the classifications of geomagnetic pulsations (Saito 1969), power-law type spectrum of magnetic field turbulence in the solar wind (Goldstein & Roberts 1999) and in geomagnetic activities (Tsurutani et al. 1990). The phase distribution $\phi(\omega)$, on the other hand, has not been paid much attention in space plasma applications. A possible reason for this may be that the phase distribution in Fourier space looks almost completely random as seen in figure 1(c).

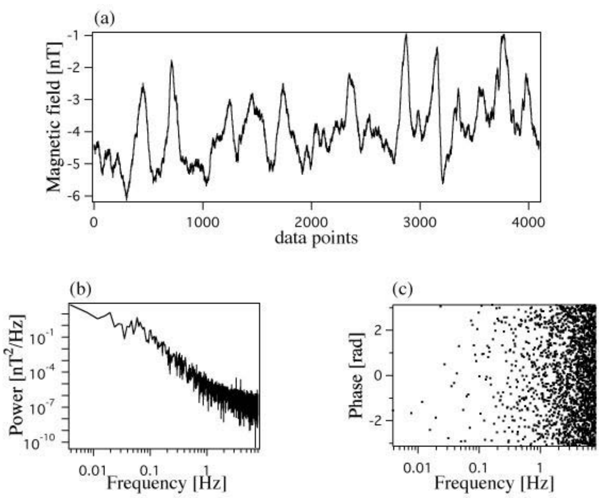

Figure 1. Fourier transform of a time series data: (a) time series data (magnetic field), (b) the power spectrum, (c) the phase distribution.

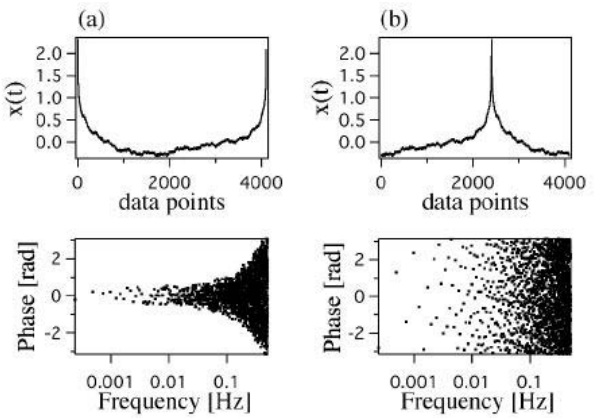

Figure 2. The coordinate origin dependence of the phase distribution: (a) the peak position of the wave $x_0 = 0$, (b) $x_0 = 2404$. These values are taken arbitrarily.

Furthermore, distribution of the wave phase depends on the choice of the coordinate origin, which is arbitrary. In figure 2 we show two solitary waveforms in upper panels and their phases in Fourier space in lower panels. These waves are exactly the same except that they are differently shifted in the horizontal direction. The distribution of phases for figure 2(a) is coherent at small ω. On the other hand, when the shift is 2404 sampling periods, which is an arbitrary number, then the phase distribution appears to be almost completely random as shown in figure 2(b) (note the periodic boundary conditions). This may be the reason why the information of phase has been overlooked in the past studies. In order to avoid the influence of the choice of the coordinate origin, we need to depart from the estimation in Fourier space. To this end, we pay attention to the waveform in real space.

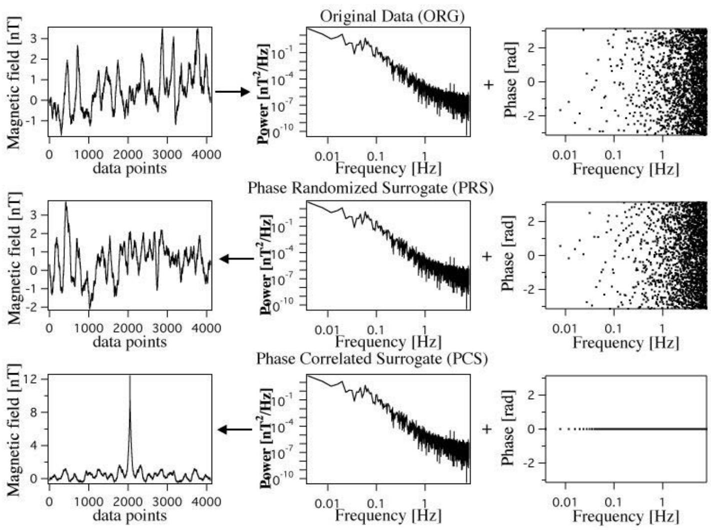

Figure 3. Making process of the surrogate data. With the information of the original power spectrum (ORG, top row) unchanged, the phase randomized surrogate (PRS, middle row) and the phase correlated surrogate (PCS, bottom row) are generated by shuffling and setting to zero the wave phases, respectively.

Hada et al. (2003) and Koga & Hada (2003) introduced a method to evaluate the degree of phase coherence among Fourier modes quantitatively. Here we explain the method in detail. Suppose we have a sequence of data, $x(t)$, for instance a measurement of magnetic field by spacecraft in the solar wind. From the original data (ORG), we can make two surrogate data (see figure 3). First we decompose the original data into the power spectrum and the phases by Fourier transform. We then randomly shuffle the phases, but keep the power spectrum unchanged, and from these two information in Fourier space, we perform the inverse Fourier transform to create *Phase Randomized Surrogate* (PRS). In a similar way we can make *Phase Correlated Surrogate* (PCS), in which the phases are all made equal. The three data, ORG, PRS and PCS, share exactly the same power spectrum, while their phase distributions are all different. We note that the calculation of PRS is performed using the average over 100 realizations of the phase shuffling.

The distribution of phases of the ORG data looks almost as random as that of the PRS in figure 3, due to the arbitrary choice of the coordinate origin. However, we can characterize the differences in the phase distribution by the differences in the waveforms in real space, instead of the Fourier space: when the phases are correlated, the *path length* of the curve tends to be shorter than the case where the phases are random. The fractal nature of the curve which has a fine structure can be extracted by measuring the path length of the curve with a ruler (or unit norm) τ. Thus, the difference of the curves can be most naturally captured by the definition of the 1st order structure function $L(\tau) = \sum_t |x(t+\tau) - x(t)|$ (Higuchi 1988) where τ is a measure characterizing the coarse-graining of the curve. Thus, we can evaluate the degree of phase coherence as the difference of geometrical characteristic of each data, without being influenced by the coordinate origin. Figure 4 shows the path

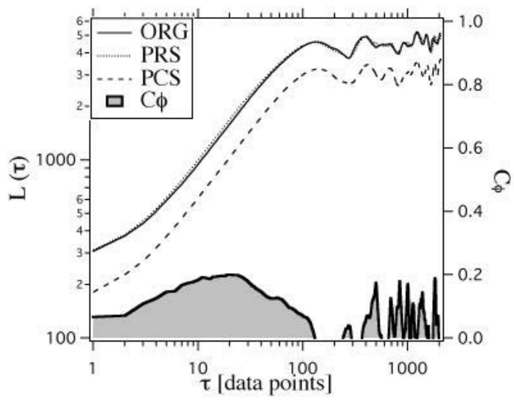

Figure 4. $L(\tau)$ for the ORG, PRS, and PCS data and C_ϕ plotted versus τ. The dotted, dashed and solid line show the path length of PRS, PCS, and ORG, respectively.

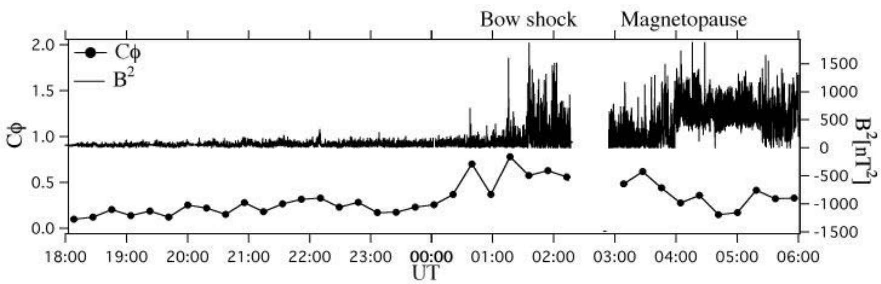

Figure 5. Evolution of C_ϕ and magnetic energy as the GEOTAIL satellite travels from far upstream toward the Earth's bow shock, inside of the magnetosheath, and into the magnetosphere.

length of each data (ORG, PRS and PCS). In general, the path length of PRS is longer than that of PCS, and they construct extremes of the value. To evaluate the degree of phase coherence, we therefore define the *phase coherence index*,

$$C_\phi(\tau) = \frac{L_{PRS}(\tau) - L_{ORG}(\tau)}{L_{PRS}(\tau) - L_{PCS}(\tau)}. \tag{2.2}$$

If the original data has random phase, C_ϕ should be ~ 0, while $C_\phi = 1$ if the data is completely phase correlated. The phase coherence index C_ϕ is shown in figure 4 (right axis).

3. Applications

In this section, we evaluate the phase coherence among MHD waves using the GEOTAIL 16 Hz magnetic energy data observed from 8 October 18:00 UT to 9 October 06:00 UT 1995. During this period, the GEOTAIL was approaching the Earth's bow shock from far upstream, passing through the shock, and entering into the magnetosphere. We separate the entire period into 37 data sets, with

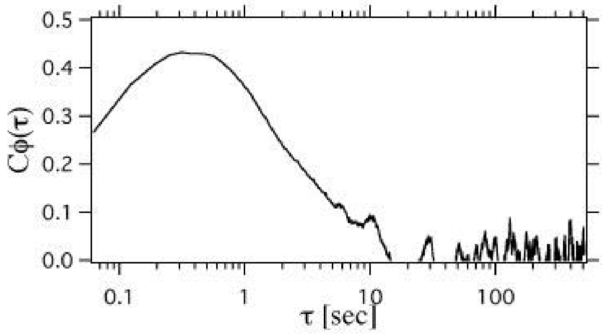

Figure 6. Typical profile of C_ϕ in the upstream region of the Earth's bow shock.

16384 sample each (=about 17 min. duration). We examine evolution of C_ϕ for the long sequence of the GEOTAIL data. We let $\tau = 0.0625$ sec throughout the analysis. The result is shown in figure 5 where the blank in the figure indicates data gap. As the GEOTAIL approaches the Earth's bow shock, the magnetic energy (turbulence level) gradually increases, becomes large within the magnetosheath, and decreases again as the GEOTAIL enters into the magnetosphere. The evolution of C_ϕ approximately follows the evolution of the magnetic field turbulence level. This is a natural consequence if the phase coherence is generated via non-linear interactions between the finite amplitude MHD waves.

The typical profile of C_ϕ in the upstream region of the Earth's bow shock is shown as a function of τ in figure 6. The value of C_ϕ increases from $\tau \sim 10$ s to $\tau \sim 1$ s. The range where the C_ϕ increases corresponds, approximately, to a frequency range of $\sim 0.1\Omega_i$ to $\sim \Omega_i$ where Ω_i denotes the local ion-cyclotron frequency evaluated in the upstream region (~ 1 sec). In the upstream region, low-frequency MHD waves ($\sim 0.1\Omega_i$) are mainly excited due to ion-beam instabilities (Fairfield 1969; Gary et al. 1984). Once they become finite-amplitude perturbations, they can evolve non-linearly due to inhomogeneous spatial distribution of plasma. Therefore, this result indicates that such non-linear evolution of MHD waves and its high-C_ϕ region are related to each other. The decrease of the PC index under 1 s ($\sim \Omega_i$) implies that non-linear interactions among MHD waves become weak due to energy dissipation processes such as Landau damping.

4. Summary

In this study the phase correlation among MHD waves observed by the GEOTAIL satellite near the Earth's bow shock are evaluated. We find that there exists finite phase coherence ($C_\phi > 0$) among the wave phases in the upstream and the downstream region of the Earth's bow shock. Typically, C_ϕ ranges between 0.1 and 0.4, but sometimes it can be as large as ~ 0.6 near the shock. Furthermore, we find that the phase coherence is mainly generated in the characteristic frequency band $0.1\Omega_i \le \omega \le \Omega_i$ in the upstream region.

We finally would like to make a remark that the assumptions used in the quasi-linear theory, *i.e.*, small amplitude and random phase approximation, are not valid for the actual MHD turbulence near the Earth's foreshock region. Since non-linear interactions among MHD waves bring coherent and intermittent field therein, we expect that the non-classical particle transport transcending the quasi-linear theory can be realized under the circumstance as reported by Kirk et al. (1996) and Kuramitsu & Hada (2000). Furthermore, the attempt to examine phase coherence (or synchronization) has become main stream in science, it is important for the understanding of non-linear dynamics in turbulence and chaos (He & Chian 2003; Nariyuki & Hada 2005).

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