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Finite Element Method Representations of Asteroid Gravity Fields

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Abstract. *This work aims to present a new method to model the gravity field of a small body using the Finite Element Method (FEM). Here an asteroid of interest, Bennu is modelled using this new application and the results obtained for the gravity field are compared with the Polyhedron Method to validate the model, making it possible to be used in new and interesting applications.*

Key words: Asteroids; Gravity Field; Finite Element Method; Potential; Disturbances.

1. Introduction

Small solar system bodies, such as asteroids and comets, have been of great interest to multiple communities. Scientifically they hold keys to understand the formation and evolution of the solar system. From a planetary defense perspective, they are perhaps one of the greatest long-term threats to humanity. Finally, they are a very accessible source of resources for future exploration of the solar system. These aspects make them an interesting target for space missions.

Many space missions had and will have small bodies as target, such as NEAR [Verveka et al., 2000] a mission to the asteroid (433) Eros, Hayabusa [Fujiwara et al., 2006] that aimed to collect a sample of (25143) Itokawa, Hayabusa2 [Tsuda et al., 2013] also a mission to collect an asteroid sample but this time with 162173 Ryugu as target, MMX [Usui et al., 2018] to be launched in 2024 to the Martian moons seeking to collect a Phobos sample, Janus [Scheeres et al., 2020] that is going to investigate two binary asteroids, Lucy [Stanbridge et al., 2017] to explore Jupiter Trojans and OSIRIS- Rex [Lauretta et al., 2015] a mission to investigate and collect a sample of the asteroid (101955) Bennu, the asteroid that is going to be used for this paper analysis.

Regarding all these facts, the main idea of this work is to present and validate a new approach to compute and evaluate the gravity field using the Finite Element Method (FEM). FEM definitions were used in previous works study the gravity field with success, the geopotential was computed using the FEM [Junkins, 1976] and it was also used investigate small bodies gravity fields [Park et al, 2010].

The results of simulations to compute the gravity field and the validation comparing the FEM results with the ones obtained using the Polyhedron method will be here presented to validate the method.

2. Methodology

This study uses two methods, the FEM application that is the method to be validated and the Polyhedron Method, that is going to be used as an already validated approach to do the comparison, as this method has been used for years in many applications with success.

The Polyhedron method considers a constant density polyhedron to perform some calculations and obtain the gravity field. This method uses polyhedron with triangular faces, the potential is given by Equation (1) [Werner and Scheeres, 1997].

$$U_P = \frac{1}{2} G \rho_B \left(\sum_{e \in edges} \boldsymbol{r}_e^T \boldsymbol{E}_e \boldsymbol{r}_e \, L_e - \sum_{f \in faces} \boldsymbol{r}_f^T \boldsymbol{F}_f \boldsymbol{r}_f w_f \right) \quad (1)$$

Where

$$\boldsymbol{E}_e = \widehat{\boldsymbol{n}}_A \left(\hat{n}_{12}^A \right)^T + \widehat{\boldsymbol{n}}_B (\hat{n}_{12}^B)^T$$

$$L_e = ln \frac{r_{e1} + r_{e2} + e_{12}}{r_{e1} + r_{e2} - e_{12}}$$

$$\boldsymbol{F}_f = \widehat{\boldsymbol{n}}_f \widehat{\boldsymbol{n}}_f{}^T$$

$$w_f = \frac{\boldsymbol{r}_{e1} \cdot \boldsymbol{r}_{e2} x \boldsymbol{r}_{e3}}{r_{e1} r_{e2} r_{e3} + r_{e1}(\boldsymbol{r}_{e2} \cdot \boldsymbol{r}_{e3}) + r_{e2}(\boldsymbol{r}_{e3} \cdot \boldsymbol{r}_{e1}) + r_{e3}(\boldsymbol{r}_{e1} \cdot \boldsymbol{r}_{e2})}$$

Where $\boldsymbol{r}_{e1,}\boldsymbol{r}_{e1,}$ $\boldsymbol{r}_{e3}$ are vectors from the field point to the vertices of an edge, r_f is a vector from the field point to some fixed point on the face plane, $\widehat{\boldsymbol{n}}_f$, $\widehat{\boldsymbol{n}}_A$ are face normal and $\hat{n}_{12}^A$, $\hat{n}_{12}^B$ edge's normal vectors.

This work combines some FEM tools and Mass Concentration (Mascon) definitions [Werner and Scheeres, 1997] [Venditti and Rocco, 2017]. We start with a polyhedral shape model of the body of interest with a certain number of facets, and from this model, a tetrahedral 3D FEM mesh is created, this mesh also has an internal discretization, considering the interior of the body. From this mesh some information is extracted such as the tetrahedrons volume, coordinates of the nodes and coordinated of the centroids, then Mascon definitions are applied to compute the gravity field of the desired body, Equation (3) to Equation (5).

$$U_e = \frac{\mu_e}{r} \quad (3)$$

$$r = \sqrt{\left(x - x_g\right)^2 + \left(y - y_g\right)^2 + \left(z - z_g\right)^2} \quad (4)$$

$$U_T = \sum_{i=1}^{N} U_e \quad (5)$$

Where μ_e is the gravitational parameter for each element, x, y and z are coordinates of the field point, x_g, y_g and $z_g$are coordinates of each element centroid and N the number of elements.

3. Results and Discussion

To start the analysis, the gravity field over the surface of the asteroid (101955) Bennu was computed using the FEM, we can see these results from Figure 1 to Figure 4. After that the gravity field was generated using the Polyhedron method, and the deviation between both methods are presented from Figure 5 to Figure 8.

Figure 1 shows a 3D view of Bennu and the colormap indicates the gravity field values, and from Figure 2 to Figure 4 we show the xy, xz and yz respectively. Figure 5 to Figure 8 are analogous, however the colormap presents the differences between the Polyhedron and the FEM.

As we can see from Figure 5 to Figure 8, the difference between both methods is very small, reaching the maximum of a magnitude of 10^{-5} on some points of the asteroid, as we can notice from the colormap in the figures

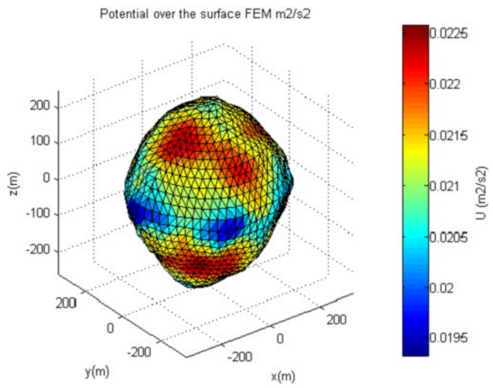

Figure 1. Potential over Bennu surface – FEM.

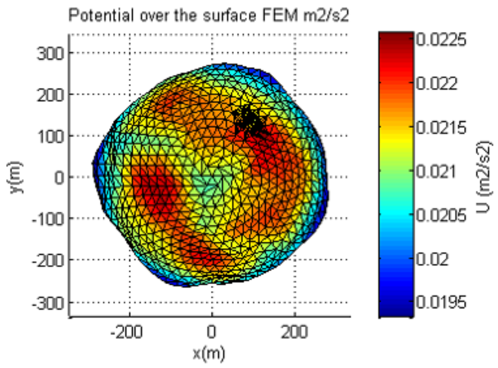

Figure 2. Potential over Bennu surface – FEM (xy view).

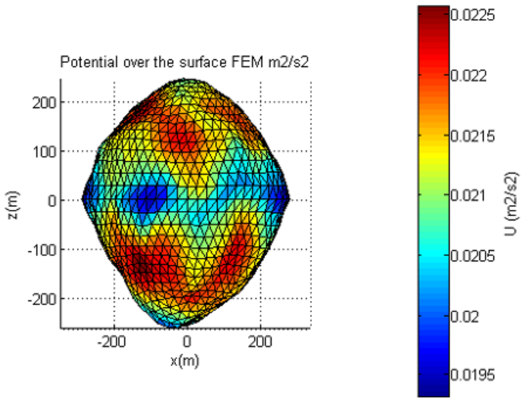

Figure 3. Potential over Bennu surface – FEM (xz view).

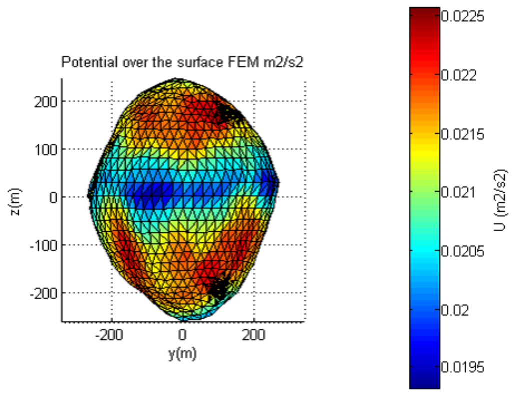

Figure 4. Potential over Bennu surface – FEM (yz view).

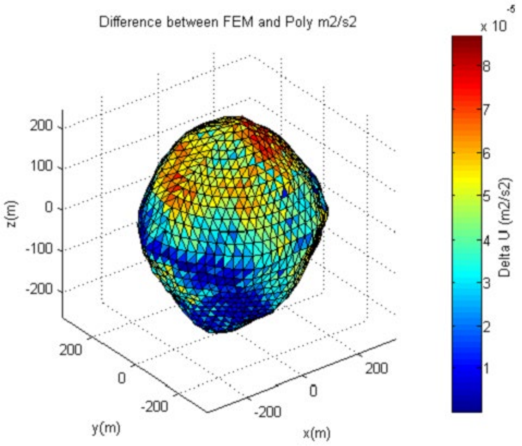

Figure 5. Potential difference over Bennu surface.

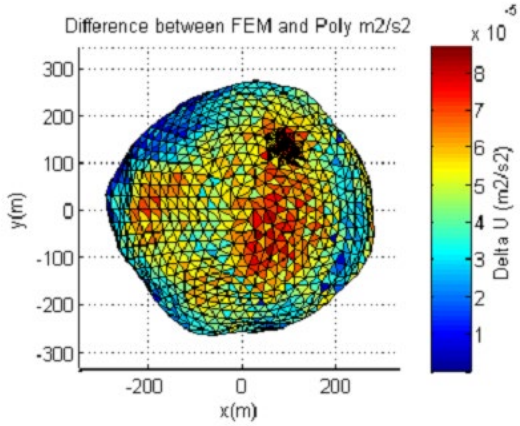

Figure 6. Potential difference over Bennu surface (x-y view).

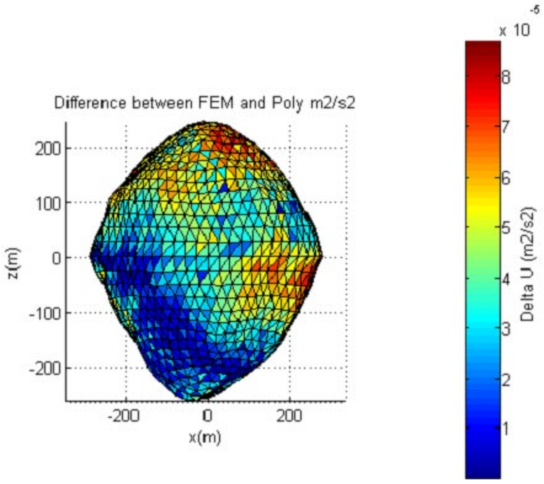

Figure 7. Potential difference over Bennu surface (x-z view).

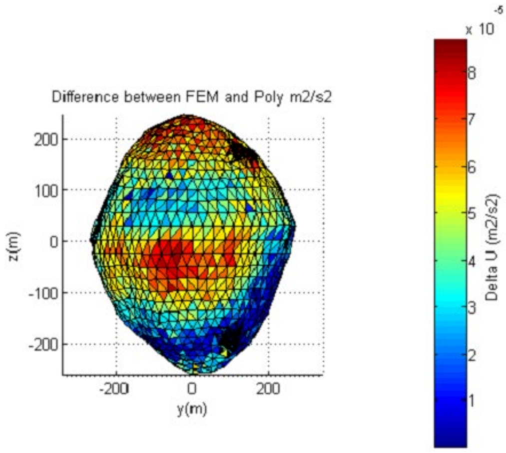

Figure 8. Potential difference over Bennu surface (y-z view).

4. Conclusion

With the results here obtained is possible to validate the FEM application on gravity fields, the results are consistent and enable us to apply the method in other applications, this is important to future works as it's necessary to show a new application to justify the creation of a new method, as we already have many different approaches to compute the gravity field.

The future analysis are going to consider a body with internal different densities and some new studies will be done considering some asteroids formation theories.

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