11º Workshop em Engenharia e Tecnologia Espaciais
18, 19 e 20 de agosto de 2020

Two-Phase Tank Emptying and Burnback Coupled Internal Ballistics Prediction on Hybrid Rocket Motors

Maurício Sá Gontijo [1], Renato de Brito do Nascimento Filho[1]

[1]University of Brasília – Faculdade do Gama, Gama, DF, Brasil

mauricio.sa.gontijo@gmail.com

Abstract. *This work describes the modelling of the behavior of hybrid rocket motors during hot fire tests. To make it possible to accurately model the motor operation a Matlab algorithm, including two-phase flow on feeding system, grain burnback, and internal ballistics, was created and compared with experimental results to validate the theoretical results.*

Keywords: Hybrid Propulsion; Burnback; Nitrous Oxide; Blowdown;

1. Introduction

Hybrid propulsion has been widely studied and developed worldwide. It is highly applicable in space vehicles and satellites. One of the most studied oxidizers on this technology is the dinitrogen monoxide (N_2O), which is a two-phase and self-pressurized fluid. Due to these characteristics, a different approach must be studied to model the tank emptying. Another important analysis is the solid grain burnback that allows the calculation of the motor performance parameters for the whole burn time. For this type of modelling, an algorithm must be developed in order to optimize this kind of system.

2. Methodology

2.1 Grain Burnback

HRMs, likewise SRMs (Solid Rocket Motor), change its internal ballistics characteristics due to the grain burnback. In case of circular port, it is very simple to predict the evolution of the burning area, but on more complex geometries this analysis may get a bit difficult. A common approach to solve this problem is to develop geometrical calculations, but it is needed to develop different series of equations for each kind of geometry. In order to create a general algorithm to burnback any kind of geometry, the level set method [Sethian 1999] using Hamilton-Jacobi equation forwarding on the normal direction for an initial value problem, Equation 1, was applied in this analysis.

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| = 0 \tag{1}$$

The equation 1 provide the ability to analyze the performance of the motor, in terms of port area, for each time step of the burn time.

As shown by [Sethian 1999] it is needed to take in consideration viscosity and entropy solutions on partial differential equations to perform the numerical calculations correctly. In order to make these considerations, hyperbolic conservation laws were used. To solve this numerical scheme, it was used a first order upwind scheme.

In addition to the level set, the minimum distance function (MDF) incorporated on a squared grid was used to recognize the initial port (ϕ=0) and update it to the next time step, similar to [Wilcox et al 2007] but adapted for a 2D geometry. Equation 2 describes the MDF, where z is the horizontal coordinate, y is the vertical coordinate, the "$grid$" subscript is related to grid nodes coordinates and "phi" subscript is related to the actual interface points coordinates.

$$d^2 = \left(z_{grid} - z_{phi}\right)^2 + \left(y_{grid} - y_{phi}\right)^2 \tag{2}$$

Figure 1 displays a grain burnback example for a general star port geometry.

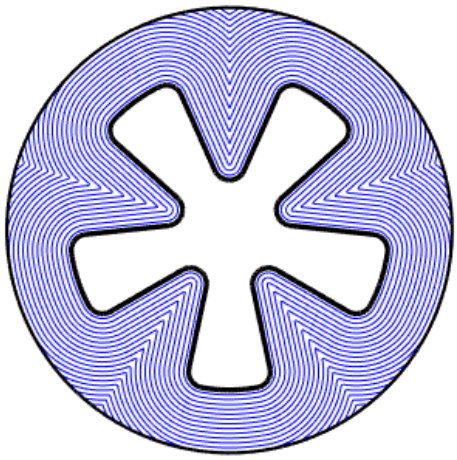

Figure 1. Grain burnback.

To calculate the burning area on each level, ϕ=0, 1, 2, 3…, each square of the grid that is intercepted by the geometry interface is analyzed and the area off the grain is calculated with the Matlab's function *polyfit*. All areas calculated are added up and every cell area fully off the grain are added as well, this will result on the port area.

The external grain circle is the final interface and when the MDF reaches it, the simulation finishes. This provides the right modelling of slivers and burning time.

The grid dimensions are determined by the external diameter of the solid fuel and, similar to any other type of simulation that uses some kind of grid or mesh, the higher

quantity of nodes or elements, the higher is the precision of the model. But, if the grain is too large the grid will be large as well and the user, in most cases, would have to choose between a big simulation time or inaccurate results. To solve this problem, all geometries are scaled down to a unitary radius geometry and at the end of the simulation all parameters are multiplied by a scaling factor, correcting the results. With this approach independently of the grain dimensions the simulation will always be highly accurate, the Figure 2 shows a comparison between a fully geometrical burnback and a level set burnback.

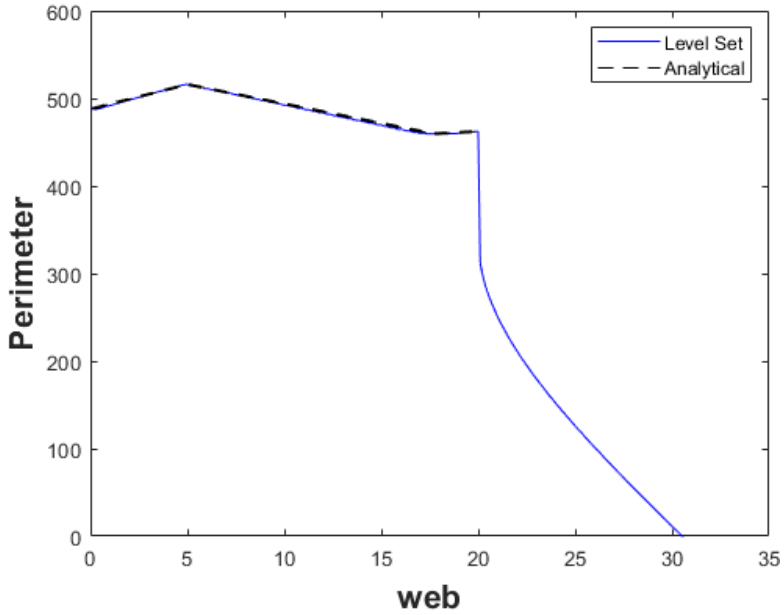

Figure 2. Comparison between Level Set and Analytical analysis.

2.2 Performance Calculations

To determine the performance of the motor the NASA's software CEA (Chemical Equilibrium with Applications) [Gordon & McBride 1994] and [Gordon & McBride 1996] was used. After determining the inputs for the CEA, with the blowndown modelling and the grain burnback, it is executed performing internal ballistics performance calculations. With the CEA's results, some parameters are added to make the model more reliable. The first one is to correct the thrust coefficient (C_T) due to the pressure difference on the nozzle exit. Another coefficient applied on the thrust coefficient is the nozzle efficiency (λ) accounting for losses like drag, slip and others, Equation 3 shows both of these coefficients. Equation 4 describe a correction used for the specific impulse (I_s) calculation similar to the one made at the C_T. The last coefficient is the characteristic velocity (C^*) efficiency (η) that it accounts for the whole combustion process efficiency until the nozzle throat, it is displayed on Equation 4.

$$C_T = \left[C_{T_{CEA}} + \left(\frac{P_e - P_a}{P_c}\right)\left(\frac{A_e}{A_t}\right)\right]\lambda \tag{3}$$

$$I_s = \frac{I_{s\,CEA}}{g_0} + \left(\frac{C_T \cdot \eta C^*}{g_0}\right) \tag{4}$$

Where, P_e is the exit nozzle pressure, P_a is the ambient pressure, P_c is the chamber pressure A_e is the exit area, A_t is the throat area, and the subscript "CEA" is for the CEA's result.

The main coupling between the grain burnback and the performance calculations is the regression rate that determines the step on every ϕ level so the burnback is made and all internal ballistics analysis is made for that level. The regression rate on HRMs depend mostly on the oxidizer mass flux, as shown on Equation 5.

$$\dot{r} = a{G_{ox}}^n x^m \tag{5}$$

Where a, n and m are empirical constants, G_{ox} is the oxidizer mass flux and x is the distance down the port. Figure 3 represents how the regression rate is applied on the geometry and the coupling between both models discussed previously.

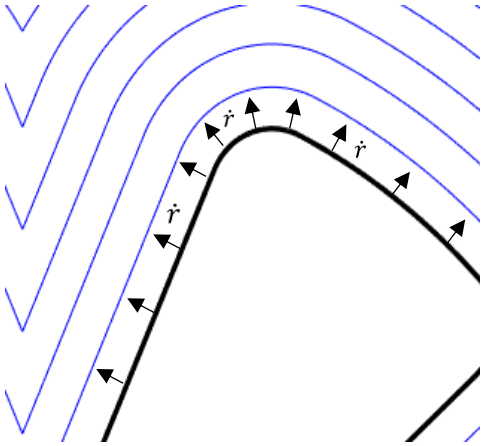

Figure 3. Regression direction on coupling of internal ballistics and burnback.

2.3 Two-Phase Oxidizer Blowdown Modeling

The modeling for two-phase blowdown process requires an engineering model for self-pressurized saturated propellant feed systems. Since N_2O is near the critical point for normal operating temperatures, N_2O cannot be assumed a single-phase fluid, as a matter of fact, at room temperature it exists at both liquid and vapor phase. The assumptions of incompressible liquid and ideal gas cannot be accurately applied to model the mass-flow rate of the propellant because, as stated by [Whitmore & Chandler 2010], the values for saturated-vapor Z factor are approximately 0.53 and for liquid compressibility Z factor are approximately 0.13.

When it comes to self-pressurized systems using saturated fluids, as the emptying process occur, liquid boils into vapor and the fluid quality continuously change. The fluid quality is known as the vapor fraction in the fluid, these changes in vapor fraction lead to variations in the internal tank pressure and effective fluid density, two key factors to determine the injector mass-flow rate and therefore the operation conditions of the rocket motor. Therefore, the correct modeling of the fluid properties across the injector outlet are determinant to predict the overall performance of the propulsion system.

The models used in this work are the Nonhomogeneous Nonequilibrium [Dyer et al 2007] to predict N_2O mass flow rate and an adiabatic expansion model [Whitmore & Chandler 2010] where the entropy of the oxidant tank at any instant during the flow, added to the entropy of the propellant portion that was discharged, equals the initial entropy in the oxidant tank. Together with this model, the properties of N2O are calculated at each instant with the aid of thermodynamic property tables for two-phase fluids.

2.3.1 Saturation States

In order to determine the saturation properties of the N_2O it was used the database from the National Institute of Standards and Technology (NIST), that is available online [NIST]. With the charts exported from the NIST database, it was possible to obtain the properties of the fluid, for both liquid and vapor phases, as function of the fluid temperature or pressure. These properties are density, enthalpy and entropy for both phases, and together with the quality of the fluid, the effective properties of the fluid can be determined continuously during the evacuation process.

For upstream the injector, the saturation properties are taken as function of the temperature in the tank, after the injector the properties are taken as function of the pressure downstream. With this, the trend over time of the saturation properties can be determined for the entire evacuation process, and so the mass-flow rate and overall conditions of the propellant tank.

2.3.2 Two-Phase Injector Model

An important step in any tank model is the characterization of the outlet flow, which comes with the use of the correct model that describes the outlet flow regime. In a propulsion system, the flow regime after the injector orifice depends on the injector configuration and the chamber pressure, as shown below. For this work, the two-phase injector model proposed by [Dyer et al 2007] will be used:

$$\dot{m}_{out} = \left[\left(1 - \frac{1}{1+k}\right)\dot{m}_{SINC} + \frac{1}{1+k}\dot{m}_{HEM}\right] \tag{6}$$

$$\dot{m}_{SINC} = C_d A_c \sqrt{2\rho_1 (P_1 - P_2)} \tag{7}$$

$$\dot{m}_{HEM} = C_d A_c \rho_2 \sqrt{2(h_1 - h_2)} \tag{8}$$

$$k = \sqrt{\frac{P_1 - P_2}{P_{vapor} - P_2}} \tag{9}$$

The Equation 6, the Nonhomogeneous Nonequilibrium model, shown above is the correction made by Solomon from the [Dyer et al 2007] equation. In Equation 8, h_2 is found assuming that the fluid expands isentropically across the injector and in Equation 9 P_{vapor} is the saturated vapor pressure of N_2O. In the equations above C_d is the discharge coefficient, A_c in the injection area, ρ is the effective fluid density, h is the effective specific enthalpy, P is the pressure and the subscripts 1 and 2 relates to the values taken upstream and downstream the injector, respectively.

The Nonhomogeneous Nonequilibrium model ($\dot{m}_{out}$) is a Two-Phase Injector Model that is a combination of the Bernoulli derived Single Phase Incompressible ($\dot{m}_{SINC}$) mass flow rate model and the Homogeneous Equilibrium Model (HEM) mass flow rate model. Each of these two models, SINC and HEM, describes a possible flow regime through the injector.

The model proposed a modified form of the cavitation number that is proportional to the ratio of bubble growth time τ_b to the liquid residence time τ_r, to account for the vaporization that can happen inside the injector. That ratio is the weighting parameter k in Equation 6.

$$\tau_b = 1 \Big/ \frac{d_r}{d_t} = \sqrt{\frac{3}{2}\frac{\rho_L}{P_{vapor} - P_2}} \tag{10}$$

$$\tau_r = \sqrt{\frac{\rho_L}{2(P_1 - P_2)}} \tag{11}$$

The k parameter is used to builds weighting coefficients that describes the principle of the Nonhomogeneous Nonequilibrium model. For a bubble growth time higher than a liquid residence time, the SINC model predicts the mass flow rate better, since there is insufficient time for heat and mass transfer between the phases. As for a bubble growth time smaller them a liquid residence time, the HEM model better predicts the mass flow rate, since now that is sufficient time for interphase heat and mass transfer.

2.3.3 Adiabatic Two-Phase Entropy Model

Here is described the engineering model implemented on this work, as proposed by [Whitmore & Chandler 2010], for the N_2O tank evacuation. The model assumes that only

liquid phase is leaving the tank and is built upon the isentropic assumption for the expansion process, where the entropy of the oxidant tank at any instant during the flow, added to the entropy of the propellant portion that was discharged, equals the initial entropy in the oxidant tank. Therefore, for the entire empty process, the flow is isentropic, but for the propellant tank control volume, the flow is non-isentropic. Later the data presented show that the isentropic assumption allows the accurate prediction of critical parameters for the empty of the tank, as tank pressure, temperature and exit mass flow.

The initial conditions of the tank are calculated given the initial temperature and pressure in the tank, T_0 and P_0, and using these and interpolating the NIST charts calculates the initial densities of the saturated liquid and vapor phases, ρ_L and ρ_V. With the tank volume and initial propellant mass as constants, the initial fluid quality [Çengel & Michael 2010] can be calculated as:

$$X = \frac{(\rho_V.\rho_L)V_{tank} - \rho_V.m_0}{m_0(\rho_L - \rho_V)} \tag{12}$$

Also, from interpolation of the NIST charts, the specific entropy for both saturated liquid and vapor phases, s_L and s_V, can be found and used together with the value of X to calculate the initial specific entropy s_0 with a simple thermodynamic correlation.

The initial total entropy S_0 in the tank can be calculated multiplying the value of initial specific entropy by the initial mass in the tank. As said before, as the tank evacuates, the total entropy in the tank decreases with the mass evacuated, but at any time during the emptying, the sum of the entropy in the tank and the entropy evacuated from the tank equates the initial value.

With the initial conditions defined, all parameters that is needed to initialize the time history calculation for the propellant properties are known. Using the Equations from 6 to 9, and with Equation 7 assuming that the vapor pressure in the tank is sufficient to ensure that the outlet fluid is liquid and only flashes to vapor after entering the injector port, the mass flow rate is calculated. The mass in the tank now equals the difference between the initial mass and the liquid mass expelled from the tank and the new total entropy in the tank is the total entropy from the previous step minus the total entropy expelled with the outlet mass flow.

In the equation below, i is the discrete time index and Δt is the difference between the time instants of the indexes i and $i+1$ and m_{i+1} is the mass that remains in the tank after every mass flow calculated in the previous step.

$$m_{i+1} = m_i - (\dot{m}_{out} \,.\, \Delta t) \tag{13}$$

The new fluid stated is finally defined by calculating the current step effective specific entropy s_{i+1} as the ratio between the effective total entropy and the total mass in the

tank, and the effective density ρ_{i+1} as the ratio between the total mass of propellant and the tank volume.

With those values, it is possible to calculate the new temperature, pressure and fluid quality in the tank using the NIST charts. To find the next step tank temperature the previous step temperature is decremented by a constant chosen value and new values for X are calculated using the values of s_{i+1} and ρ_{i+1} and assessing whether the error between the two values of X is less than 1%. If the error is higher than 1%, we continue to decrease the temperature for the next step.

By the time when the value of X equates to 1, all the liquid in the tank is evacuated and only saturated vapor exits the tank. For this case, the NHNE model does not apply to predict the mass flow and only data for saturated vapor is available and the there are two possible equations to calculate the mass flow rate of propellant, depending on whether the flow is chocked or not. The choked condition is determined by calculating the ratio between downstream and upstream pressures.

For a choked condition, the chocked-flow compressible version of the discharge coefficient equation is used.

$$\dot{m}_{V,out} = C_d A_c \sqrt{\gamma P_1 \rho_V \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} \tag{14}$$

The tank pressure can drop significantly at the end of the emptying and in this case, the outlet flow will no longer remain choked. For this case, the subsonic version of the discharge coefficient equation is used.

$$\dot{m}_{V,out} = C_d A_c \sqrt{\frac{2\gamma}{\gamma-1} P_1 \rho_V \left[\left(\frac{P_2}{P_1}\right)^{2/\gamma} - \left(\frac{P_2}{P_1}\right)^{(\gamma+1)/\gamma}\right]} \tag{15}$$

Following these steps, the time history of the properties of N_2O can be tracked during all the emptying process, in special the tank pressure, temperature and outlet mass flow. With the trend over time of those critical parameters, the overall performance of the HRM can be calculated using the equations described in section 2.2.

3. Results and Discussion

Coupling the models described above in order to predict performance it is possible to obtain important tank and motor parameters over burning time. Figure 4 shows the curves for tank pressure, temperature, and the chamber pressure from the two-phase oxidizer blowdown modeling described in section 2.3, the performance calculations described in section 2.2 and the burnback model described in section 2.1. The key parameter to join the models is the

calculation of the oxidizer mass flow rate from the tank emptying model, which makes it possible to calculate the oxidizer mass flux G_{ox}, used in Equation 5.

From the results shown in figure 4 it is possible to see how the slope in the tank pressure curve implies in a slope in the chamber pressure curve, a characteristic in HRM's that uses self-pressurized saturated propellant feed systems. It is also possible to observe the exact instant that all the liquid phase in the tank was expelled and only vapor remains, around 3.6 s. These results provide a valuable tool for the design of HRM feed systems and evaluating general system performance. Only the pressure curves and tank temperature were presented here, but from this result it is possible to obtain other parameters of the motor, like thrust and specific impulse.

The curves can be further compared with experimental measurements of temperature and pressure, from fire tests, in order to validate the models and improve the analysis of the HRM system.

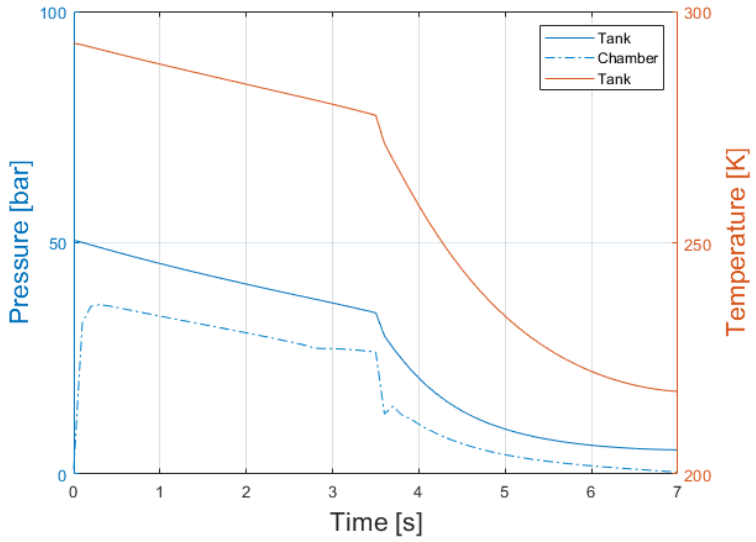

Figure 4. Pressure and temperature traces.

4. Conclusion

The models presented are a reliable way of calculating key parameters of critical sub-systems of hybrid propulsion systems. With the coupling of the three algorithms it is possible to obtain the performance and other thermodynamics parameters within the whole motor operation. These results are obtained in a low computational cost software and guarantying precise data to be used in several analysis and optimizations.

References

Çengel Y., Michael B., "Thermodynamics, An Engineering Approach," 7th Edition, 2010.

Dyer, J., Doran, E., Dunn, Z., and Lohner, K., "Modeling Feed System Flow Physics for Self-Pressuring Propellants," 43rd AIAA/ASME/SAE/ASEE Joint Propulsion Conference&Exhibit, AIAA Paper 2007-5702, 2007.

Gordon S., McBride B. J., "Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications", NASA Reference Publication 1311, Part I: Analysis, 1994.

Gordon S., McBride B. J., "Computer Program for Calculation of Complex Chemical Equilibrium Compositions and Applications", NASA Reference Publication 1311, Part II: Users Manual and Program Description, 1996.

NIST Chemistry WebBook (web-site), <https://webbook.nist.gov/chemistry/> (Accessed in 01/06).

Sethian J. A., "Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials", Cambridge University Press; Edition: 2, 1999.

Whitmore S. A., Chandler S. N., "Engineering Model for Self-Pressurizing Saturated-N_2O-Propellant Feed Systems," Journal of Propulsion and Power, Vol. 26, No. 4, 2010.

Willcox M. A., Brewster M. Q., Tang K. C., Stewart S. D., "Solid Propellant Grain Design and Burnback Simulation Using a Minimum Distance Function", Journal of Propulsion and Power Vol. 23, No. 2, March–April 2007.