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CONSTRUCTING CHAOTIC TRAJECTORIES FOR PATROL MOBILE ROBOTS

Luiz S. Martins-Filho[1], *Elbert E.N. Macau*[2], *Ronilson Rocha*[3]

[1]Departamento de Computação, Universidade Federal de Ouro Preto, Ouro Preto/MG, Brazil, luizm@iceb.ufop.br
[2]Lab. Associado de Computção e Mat. Aplicada, Inst. Nac. de Pesquisas Espaciais, S. José dos Campos/SP, Brazil, elbert@lac.inpe.br
[3]Departamento de Controle e Automação, Universidade Federal de Ouro Preto, Ouro Preto/MG, Brazil, rocha@em.ufop.br

Abstract: This article presents a path planning strategy for mobile robots based on dynamical features of chaotic systems. This peculiar methodology of constructing trajectories envisages missions for terrain exploration, with specific purpose of search or surveillance. The proposed way to achieve fast scanning of the robot workspace consist of a scheme of imparting a chaotic motion behavior to the mobile robot using a path-planner based on the conservative Standard map. As a consequence, for a external observer, the robot trajectories resemble highly opportunistic and unpredictable, with characteristics that quickly scans the surveillance space. The kinematic modelling and the closed-loop control of the robot are described. Presentation of results and analysis of numerical simulations close the paper.

Keywords: Mobile robots, Patrol, Mobile robot motion-planning, Motion control, Nonlinear systems, Chaos

1. INTRODUCTION

The chaos theory became an important study subject in the last twenty years. Many works have investigated deeply into this phenomenon after pioneer Edward Lorenz studies on numerical simulation on weather prediction in 1961 [**?**]. More recently, the studies concerning chaos surpassed the focus on its properties and features, and have investigated the possible application of this peculiar dynamical behavior. For instance, remarkable chaos utilization are taking place in secure data transmission and medical diagnostics. In the fist case, chaos provides a key for encryption of restricted information. In the medical applications, chaos has been used for lung and cardiologic diseases through lung sounds and blood pressure signals analysis.

The main goal of the present work is to apply chaotic systems in obtaining special trajectories for mobile robot performing search and patrol missions. The mobile robotics keeps been the subject of intensive researches for different useful applications. These robots have been used for floor-cleaning tasks, parts transportation in industry plants, scientific missions where environment represents important risks to humans (e.g. vulcan and planetary explorations), and so on [1].

The scientific interface between mobile robotics and chaos theory appears in some works like [6–8]. For instance, the integration between the robot motion system and a chaotic system, the Arnold dynamical system, is used to impart a chaotic behavior to the robot in [9]. An extension of this motion control strategy, applying diverse chaotic systems on integration with the robot kinematic model, can be seen in [10]. In [11], an open-loop control approach is proposed to produce unpredictable trajectories, using state variables of the Lorenz chaotic system to command the robot wheels velocities. Conversely, this article proposes a path-planning strategy on a closed-loop locomotion control scheme to produce trajectories by a sequence of objective points path following, i.e. the design and the execution of trajectories that will cause the robot to reach a sequence of partial targets' locations. The partial targets are defined in real-time by an auxiliary module based on conservative chaotic system. In a conservative chaotic dynamics, the chaotic region occupy continuous regions of the state space. This behavior can be contrasted with the one presented by dissipative chaotic systems in which the chaotic trajectories live on a chaotic attractor whose dimension is in general less than the dimension of the space. As so, it is always possible to make a one-to-one association between the continuous region of the state space associated with the physical region that we wish the scans and the chaotic region of the conservative chaotic system that we use to generate the sequence of objective points. The advantage of this proposed approach is an interesting combination of a deterministic planning of unpredictable trajectories with a quite simple control strategy that reinforces the characteristics of an apparently unplanned robot movement in a surveillance mission context.

2. MOBILE ROBOTS

The mobile robot considered in this work is a typical differential motion robot with two degrees-of-freedom, composed by two active, parallel and independent wheels, a third passive wheel with exclusive equilibrium functions (a sort of free steered standard wheel), and proximity sensors capable of obstacles detection. The active wheels are independently controlled on velocity and sense of turning. The sensors provide short-range distances to obstacles. For instance, these sensors can be sonar or infrared devices commonly used in mobile robots, with adequate accuracy. Additionally, the robot is supposed to be equipped with specific sensors for

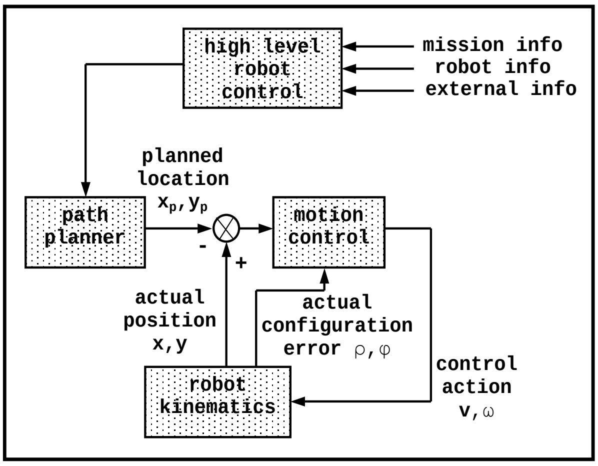

Figure 1 – Scheme of the overall robot control including a high level control module, the proposed path planner, the motion control, and the interaction with the physical robot.

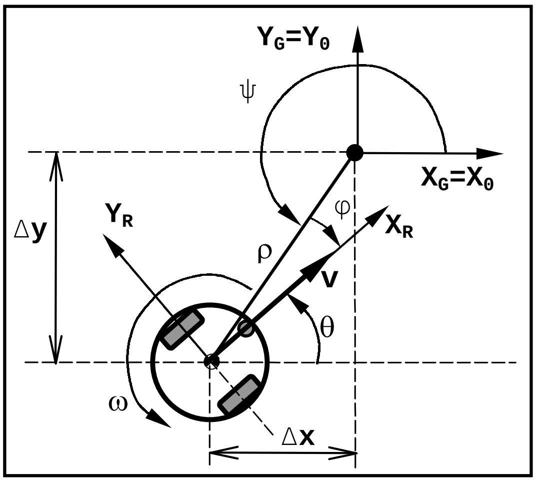

Figure 2 – Geometry of the kinematic control showing the error variables (ρ, φ), robot linear and angular velocities (v, ω), the robot frame ($X_R Y_R$), and the desired location frame ($X_G Y_G \equiv X_0 Y_0$).

detection and recognition of searched objects. This robot model represents an interesting compromise between control simplicity and degrees of freedom that allows the robot to accomplish mobility requirements [13], and it is largely adopted in several researches on mobile robotics like in [9], [10], [14], [15].

The robot motion is obtained by driving the active wheels. The resultant motion is described in terms of linear velocity $v(t)$ and direction $\theta(t)$, describing an instantaneous linear motion of the medium point of the wheel axis and a rotational motion (rotational velocity $\omega(t)$) of the robot body over this same point.

The robot motion control can be done providing the wheels velocities, $\omega_l(t)$ and $\omega_r(t)$, or equivalently the body linear and angular velocities, $v(t)$ and $\omega(t)$, called input or control variables. The mathematical model of this kinematic problem considers these two input variables and three state variables: the robot position and orientation ($x(t)$, $y(t)$, $\theta(t)$) [13]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

This class of systems, nonholonomic one, is not obvious to control, however it has been deeply studied by various research groups. For instance, interesting and adequate solutions are presented and discussed in [14, 15].

2.1. Motion control

The motion control strategy adopted in this work involves a real-state feedback controller, proposed in [15], which is an appropriate approach to produce a desired trajectory described by a sequence of coordinate (x_p, y_p). It means that the path-planning task is given by a specialized robot module, independent of the motion control module, that sets intermediate positions lying on the requested path. A general scheme of this proposed architecture can be seen in 1.

The adopted control law considers the geometric situation shown in Fig. 2. In this figure, the robot is placed at an arbitrary configuration (position x, y and orientation θ), and a desired position (the target x_p, y_p: the origin of frame $X_G Y_G$) is defined by the robot path-planner. In the robot reference frame ($X_R Y_R$), the configuration error vector is defined as $e = [\rho \;\; \varphi]^T$, where ρ and φ localize the target position, and define a coordinate change:

$$\begin{aligned} \rho &= \sqrt{\Delta x^2 + \Delta y^2} \\ \varphi &= 180 + \theta - \psi \end{aligned} \quad (2)$$

The robot kinematic model is described by Eq. 1, where $\dot{x}(t)$ and $\dot{y}(t)$ are the linear velocity components on absolute reference frame (fixed on the workspace). We define the angle φ between the X_R axis of body reference frame and the vector connecting the robot center and the desired position. The other configuration variables, ρ and ψ, describe respectively the distance between present and desired positions, and the angle between the direction to the target and the axis X_0.

The description of the motion in the new coordinates becomes

$$\begin{bmatrix} \dot{\rho} \\ \dot{\varphi} \end{bmatrix} \begin{bmatrix} -\cos\varphi & 0 \\ \frac{1}{\rho}\;\sin\varphi & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (3)$$

Concerning these polar coordinates system descriptions, it's necessary to remark that the coordinate transformation is not defined at $x = y = 0$, i.e. when the robot achieves the goal location. The control law, in terms of error feedback (ρ, φ) to determine the value of system inputs v, ω, is given by:

$$\begin{aligned} v &= k_1 \;\rho\; \cos\varphi \\ \omega &= -k_1 \;\sin\varphi\; \cos\phi - k_2\;\varphi \end{aligned} \quad (4)$$

It can be shown that this control law stabilizes the systems, i.e. leads its state variables to the origin. The details of this prove, based on Lyapunov functions, can be found in [15].

Evidently, if an obstacle is found on this trajectory, a specific navigation competence, the obstacle avoidance, must be used to drive the robot about the obstacle. Is this work, the obstacle avoidance problem is not treated, nevertheless a simple solution can be implemented.

A second kinematic control is based on a very simple and discontinuous control scheme. In this scheme, the robot executes two phases control action. The first one consists of an exclusive rotation motion, with constant angular velocity about its own center, to point the robot straightforward to the next target location. Completed the robot-pointing phase, the robot can execute a straight trajectory with constant velocity toward the desired position.

2.2. Patrol missions

The utilization of autonomous and remotely operated robots in search or patrol missions can deal with hazardous tasks involving detection of dangerous materials or intruders, avoiding risks to humans. An example of this kind of mission is the patrol of a known terrain for perception and identification of any unauthorized intruder.

In this kind of robotic application, the physical robot and its systems architecture and software have been deeply studied for military and civil operations. Some surveillance mobile robots are already commercially available (e.g. [2]). Many examples of published works involving vigilance robots can be found in [3] and [4]. The core of these works is about the target perception and identification, the robot localization, the terrain map updating, and the optimization of communication with the operation center or other robots. Another important issue is the planning of patrolling path [5]. Actually, the robot motion strategy is a crucial factor for the success of any robot surveillance mission. High unpredictability of robot trajectories and fast scanning of the workspace are strongly suitable. In this work we exploit the dynamical behavior of a conservative chaotic system to accomplish these goals. As a spin-off of this approach, previous mapping is no longer necessary.

3. CHAOTIC TRAJECTORIES

In the context of deterministic systems, sensitive dependence on initial conditions is the main well known characteristic of the chaotic behavior. It means that arbitrarily close initial conditions imply trajectories that exceed a fixed, finite distance after some time. It is this property of a chaotic system that makes long term prediction of a chaotic trajectory based on finite time measurements practically impossible because of the limited accuracy associated to measurements sensors. However, another intrinsic characteristic of a chaotic evolution is the transitivity. A deterministic system is transitive on an invariant set if for any two open subsets U and V of this invariant set, there exist trajectories originated from U that passes through V after some time. This property means that we always can use a chaotic trajectory as a transportation path between regions belonging to the chaotic invariant set. It implies the "mixing property" founded in chaotic systems that ensures that the system cannot be broken down into subsystems that do not interact with each other.

As a consequence of these two properties, for the perspective of an external a chaotic trajectory presents a complicated behavior that does not exhibit any recurrent pattern. Long term prediction from time series is practically unfeasible and trajectory keeps changing its behavior continuously with a behavior that looks like random for the external observer. The transitivity property gives an extra ingredient to the chaotic behavior: a chaotic trajectory fills all its invariant set. As a consequence, a chaotic trajectory is reported by an external observer as an erratic trajectory that quickly move among different regions of a certain invariant set. As so, the trajectory is evolving about a region and suddenly after it starts to evolve in another region away from the previous one. It is precisely this behavior that we exploit in this work to run the movement of a robot to make its behavior perfect to be used as an opportunistic surveillance tool.

The space to be scanned for the robot can be viewed as a kind of a continuous subset with an integer dimension. As so, to accomplish our goal, the chaotic trajectory must fill this integer dimension continuous subset. This requirement provides an extra ingredient that delimits the class of chaotic system to be used to fulfill our purpose. In a dissipative chaotic system, the chaotic invariant set is an attractor. In general, this attractor has a not integer dimension, i.e., its geometric picture on the phase space is a fractal. However, in our problem, the robot must cover in its phase space an integer dimension region. As so, we consider more appropriated to chose a class of chaotic system: a Hamiltonian chaotic system. On of the basic properties of Hamiltonian systems is that they preserve volumes in the phase space. As a consequence of this property, Hamiltonian systems do not have attractors. Because of that, in a a chaotic Hamiltonian system the chaotic regions cover integer dimension regions of the phase space. Note that this fact individualize our approach in the scenario of previous works that uses a chaotic system to run the robot dynamics. Because those previous approaches use dissipative chaotic systems, they require subterfuges to make the robot wander opportunistically through the surveillance area. In our approach this is not necessary at all. For the knowledge of the authors, this is the first time that this concept is applied in the area of mobile robots.

In this work we decided to use an area preserving map that is considered as a paradigm for chaotic Hamiltonian systems: the Standard map. It is a two dimensional map which results from a periodic impulsive kicking of a rotor. This map was firstly proposed by Brian Taylor and then independently obtained by Boris Chirikov to describe the dynamics of magnetic field lines on the kicked rotor [12]. It's also called Taylor-Chirikov map and constitutes a family of area-preserving maps. The dynamics effect of this system is expressed mathematically through the equations of the map, given by

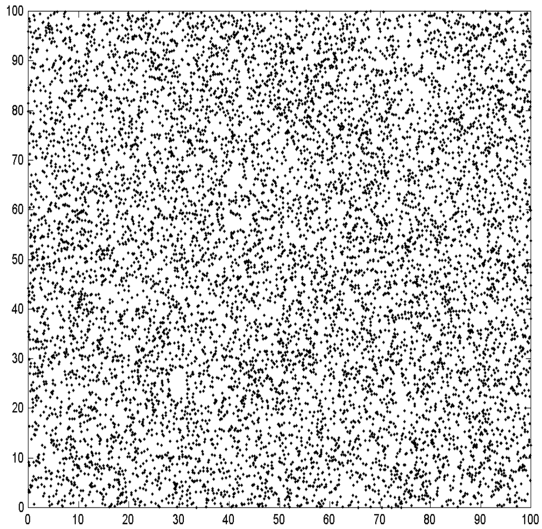

Figure 3 – Terrain covering by subgoals planned points using Standard maps after 10,000 planning iterations (considered map gain value is $K = 7$).

$$\begin{aligned} x_{n+1} &= x_n + K \sin y_n \\ y_{n+1} &= y_n + x_{n+1} \end{aligned} \tag{5}$$

Here x is a periodic configuration variable (angular position) and y is the momentum variable, or angular speed, both computed $\mathrm{mod}(2\pi)$. The map has a single parameter K that represents the strength of the nonlinear kick. Standard map is so interesting mainly because it can show chaos under very basic conditions.

To show the characteristics of this map, we simulated the Standard map and verified if its covering properties satisfy the mission request for fast terrain scanning. We define a square terrain with dimensions 100×100 in a normalized measurement unit. The scan simulation begins with an arbitrary initial position. The result of partial goals' locations planned after 10,000 iterations, considering a Standard map with gain values $K = 7$, is shown in Fig. 3. We can see that it can cover completely the considered terrain (in fact, the necessary condition for this complete scan is $K > 6$).

It is quite evident that a faster complete area covering could be obtained using a systemic scan without passing two or more times at one terrain cell, but this classic strategy is absolutely predictable.

A similar sequence of planned locations, or objective points to be reached by the robot path, could be generated using random number generator with uniform distribution covering the terrain space (see Figure 4). However, the planning nature is quite different of the conservative Standard map, as will be discussed later in this article.

4. NUMERICAL SIMULATIONS

To validate the proposed approach, we numerically simulated the robot motion applying the closed-loop control law discussed in the previous section to track a sequence of objective points planned provided by the Standard map, and we examine (i) the trajectory unpredictability, (ii) the terrain covering by the robot motion, and (iii) the general characteristics of obtained trajectories.

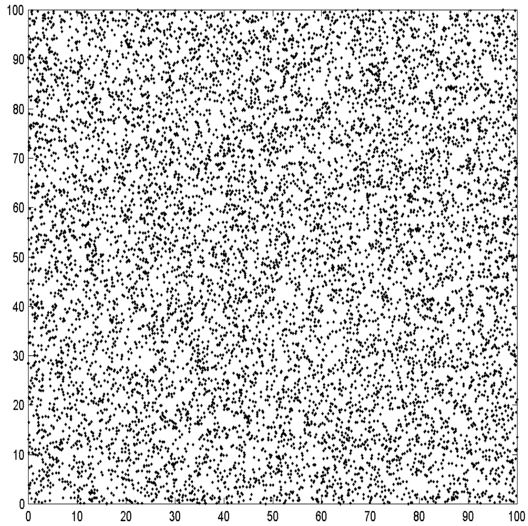

Figure 4 – Terrain covering by subgoals planned points using random numbers generator after 10,000 planning iterations.

The adopted control law provides smooth trajectories between two subsequent objective points, reinforcing the apparently erratic nature of the movement, that constitutes an interesting feature for surveillance missions. The trajectory results of the application of this control law are shown in Fig. 5. For any other terrain shape, the planning process can fit the interest area inside a square Standard map, ignoring the points planned outside the terrain but ensuring the desired fast workspace scan.

Before analyze the simulation results, a necessary comment should be placed here: the robot can percept a target (a searched object on a search mission, or an intruder on a patrol mission) inside the sensor range region, but the range of this perception field depends on the properties of the device used to perceive external objects. Therefore, the perception field trajectory has a width centered on the robot body trajectory. In this work, we do not take in account this extra area covered by the sensors. Consequently, we can say that the terrain scan analysis presented here is based on a worst case in which the perception field collapse to a single point.

Considering the basic requirements of surveillance missions, and the main ideas of our approach, we can configure three different scenarios concerning the construction of the mobile robot trajectories combining motion control laws and the determination of the regions to be visited by the robot. In the first one, the approach proposed in this article, we adopt a continuous control law that can provide smooth trajectories minimizing unnecessary maneuvers, and consequent control switches, and we apply a planning strategy based on Hamiltonian chaotic systems to establish a sequence of objective locations. In the second scenario, adopting the same control law of the first scenario, a different strategy is applied to de-

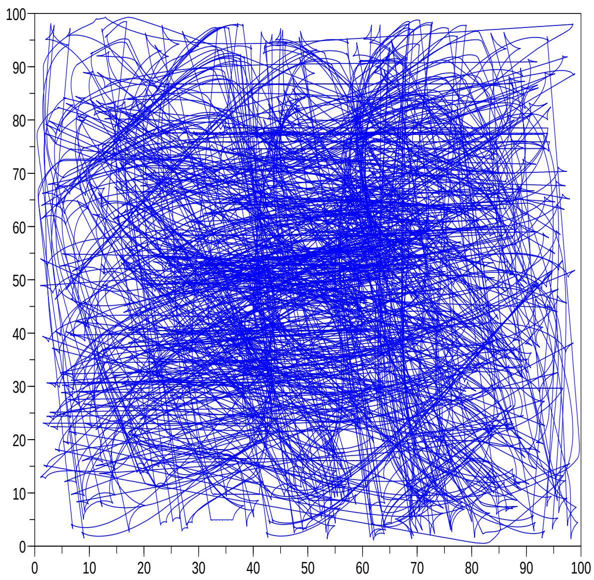

Figure 5 – The mobile robot trajectory evolution using the adopted continuous control law.

termine the sequence of objective points: a random sequence of locations uniformly distributed in the patrolled space that provides a similar result in terms of terrain covering, as it was showed in the Section 3. The last case, applying the same random strategy to define the objective points that the robot must pass through, a discontinuous control law based on a initial rotational maneuver around its own center to orientate the robot head towards the next desired position, followed by a straight trajectory directly to the objective.

Analyzing the advantages and disadvantages of the two aspects composing the three scenarios, i.e., control laws and visitation planning, the differences can be remarked and a conclusion about our proposed approach can be established. Examining the options of control laws, continuous or discontinuous, it is easily confirmed that the continuous one offers advantages: the smoother trajectories save control switches and maneuvers; moreover, they contribute to perform unpredictable trajectories for external observers. While discontinuous control produces piece wise predictable straight trajectories. The choice between chaotic and random visitation planning can result, in a first analysis, in very similar results in terms of terrain scan and surveillance space covering. However, there is a fundamental advantage of chaotic trajectories over random walk: the deterministic of the sequence of objective points path.

Considering the chaotic approach, the robot navigation system maintains complete control and knowledge of the planning process for the reason that it's a deterministic system and its dynamical behavior is precisely defined and deeply studied. In terms of navigation competencies, this path-planning determinism represents an important advantage over navigation based on a sort of random walk trajectory. For instance, this determinism can facilitates the frequent robot localization procedure, which is a crucial function because the knowledge of the robot position with appropriate precision is very necessary information for the robot itself and also for the mission operation center. This importance is associated to the executed trajectory supervision, the terrain scan information, the precise localization of an eventual intruder or target, and so on.

5. CONCLUSION

The presented strategy to deal with terrain exploration missions for mobile robots achieve adequately the main requirements of fast scanning of the workspace. Imparting chaotic motion behavior to the robot motion through the Standard map-based path-planner, this approach ensures high unpredictability of robot trajectories, resembling a non-planned motion from external observers' point of view. Validation tests, based on numerical simulations of closed-loop motion control to follow the sequence of objective points on the robot trajectory, confirm that the chaotic planning procedure can obtain interesting results. The advantageous property of the proposed chaotic motion planning over unplanned or randomly planned motion resides on the deterministic nature of chaotic behavior, that can be useful for important functions of the robot motion control like localization and terrain mapping.

This research shows that the application of dynamical behaviors of nonlinear systems on solutions for mobile robots control problems can represent an interesting interdisciplinary interface for researchers of both scientific domains, opening perspectives of future works including experimental realizations.

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