Multiscale analysis of Eta forcasts: Preliminary analysis

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Motivation of this work

Are the short and long range Eta model runs seeing the same time scales?

Wavelet analysis

- Tool to understand the multiscale aspects of functions or signals.
- Synthesis and synergy of:
  - **robust mathematic results**
  - **efficient computational algorithms**
  - under the interest of a broad community
- The use of wavelet techniques has exponentially grown, since late 80's

[Jaffard,Meyer, Ryan (2001), Meneveau(91), Chen(83), Morlet(83)] .

Atmospheric applications

- The more popular characteristic of the wavelet techniques are the introduction of the time-scale decomposition.
- Musical structure => events localized in time.
- A piece of music can be understood as a set of musical notes characterized by four parameters:
  - frequency, time of occurrence, duration and intensity

[Domingues(2005), Daubechies(92), Lau&Weng(95), Farge(92)] .

Continuous wavelet transform (CWT)

CWT of a time series f is defined by

$$\mathfrak{W}_f^{\psi}(a,b) = \int_{-\infty}^{\infty} f(u)\bar{\psi}_{a,b}(u)\,du \qquad a > 0,$$

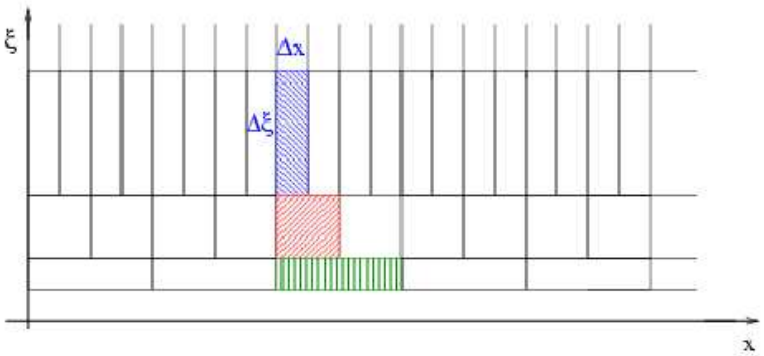
where

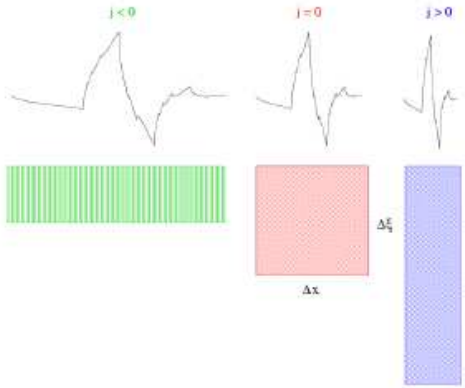
$$\psi_{a,b}(u) = \frac{1}{\sqrt{a}}\,\psi\left(\frac{u-b}{a}\right)$$

represents a chosen wavelet function family, named mother-wavelet.

CWT

- Can be used in the analysis of non-stationary signals to obtain:
  - Information on the pseudo-frequency or scale variations
  - The detection of structures localization in time and/or in space.





CWT

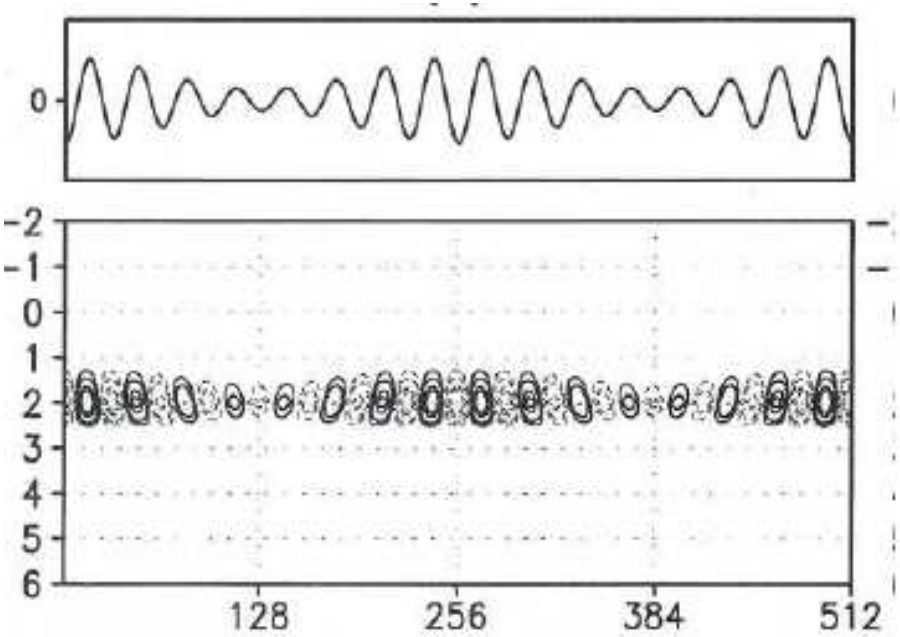
- CWT - when scale and localization parameters assume continuous values.

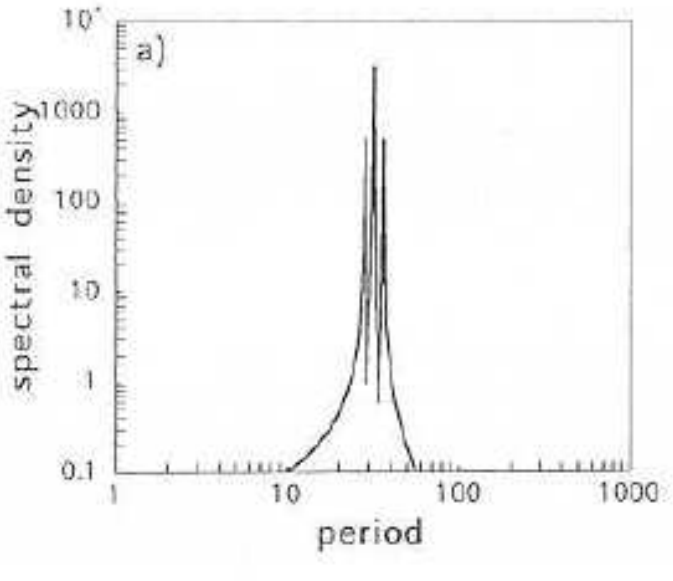
A wavelet function must satisfy the following conditions.

1) The integral of the wavelet function, usually denoted by ψ, must be zero. This assures that the wavelet function has a wave shape and it is known as the admissibility condition.
2) The wavelet function must have unitary energy. This assures that the wavelet function has compact support or has a fast amplitude decay (in a physical vocabulary *e-folding time*), warranting a physical domain localization.

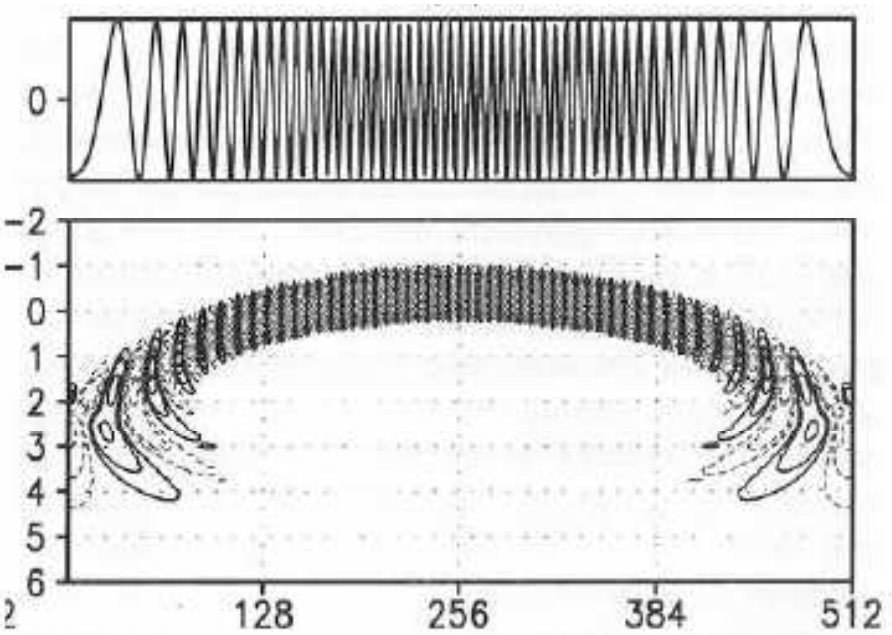
Examples: CWT

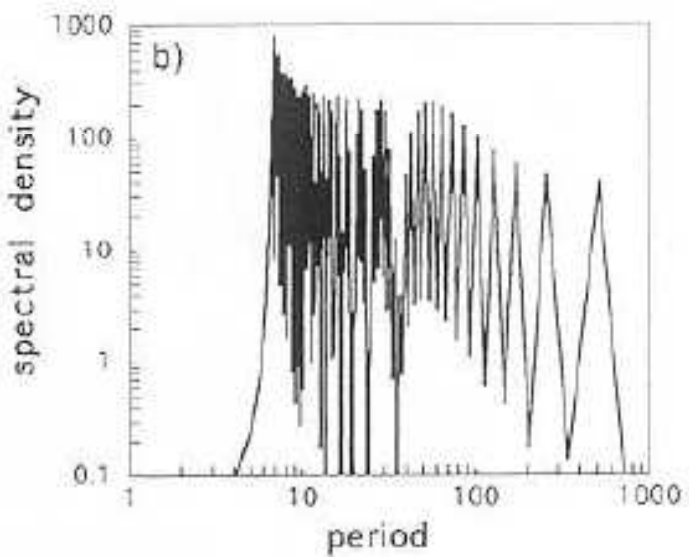
Amplitude modulation



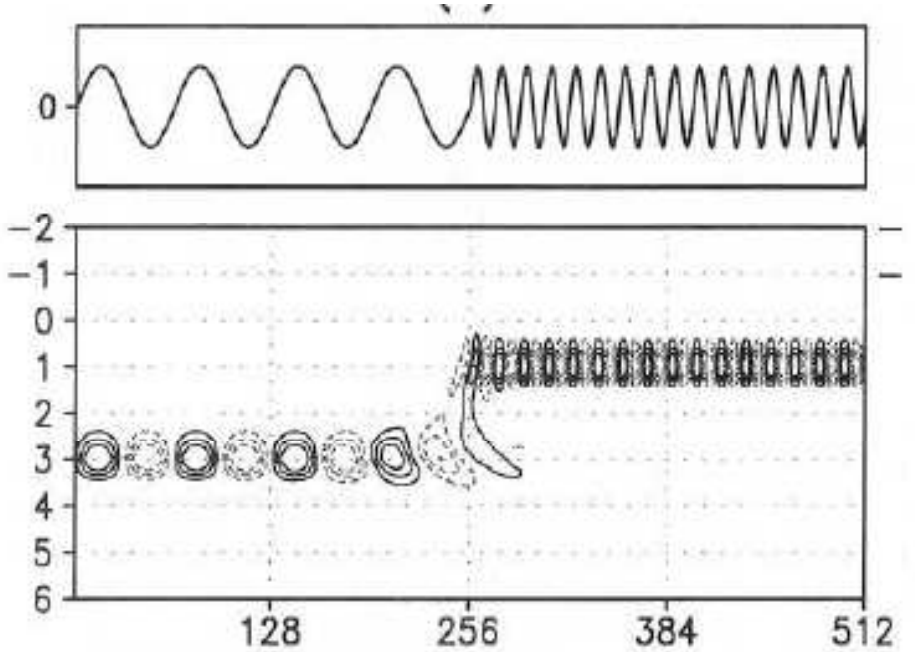


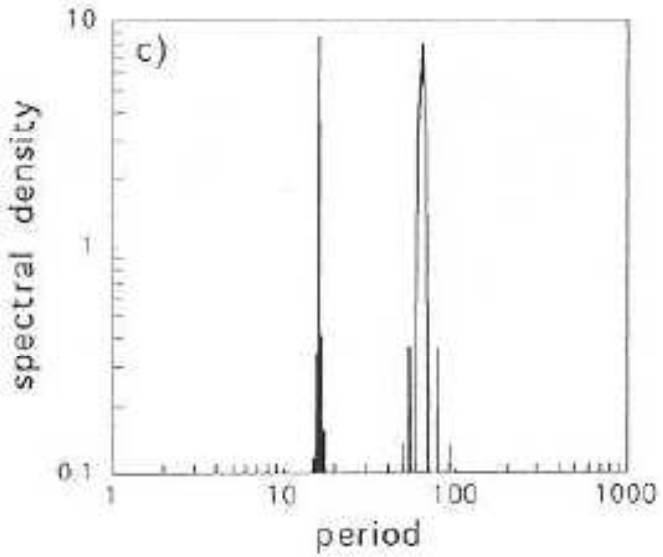
Frequency modulation





Abrupt changes in time





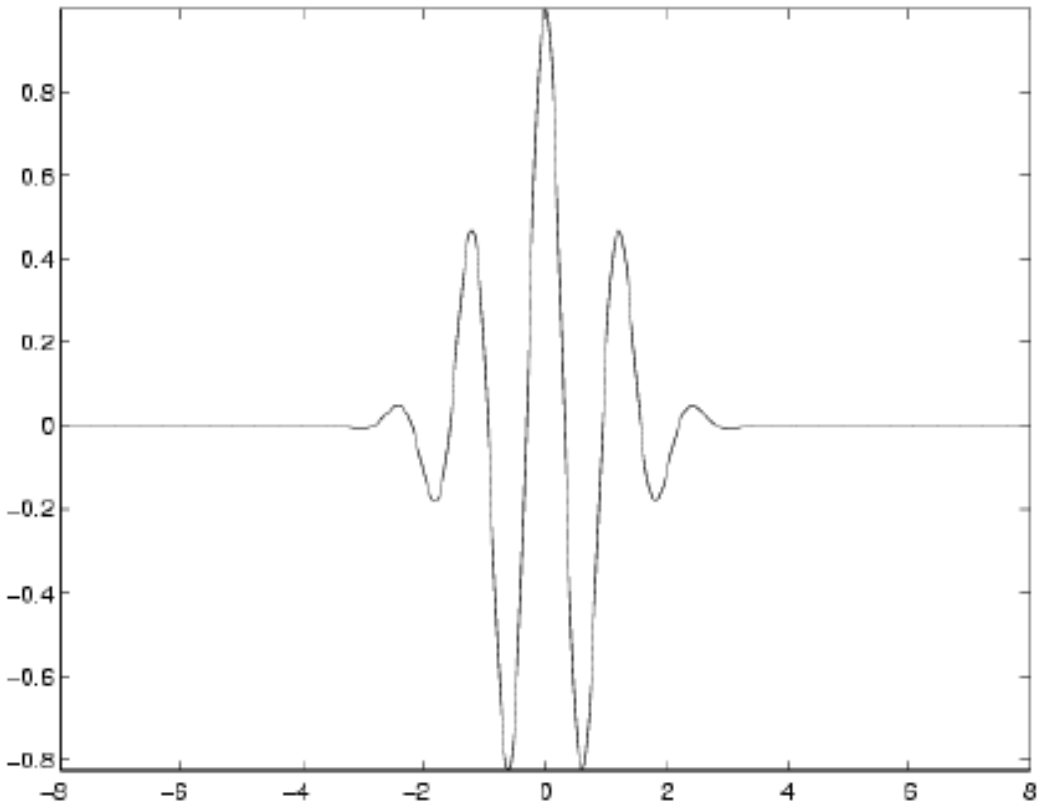
Morlet wavelet

It is formed by a plane wave modulated by a gaussian function and it is given by

$$\psi(x) = \pi^{-\frac{1}{4}} \left(e^{i\xi x} - e^{-\frac{\xi^2}{2}} \right) e^{-\frac{x^2}{2}},$$

where ξ is a non dimensional value.

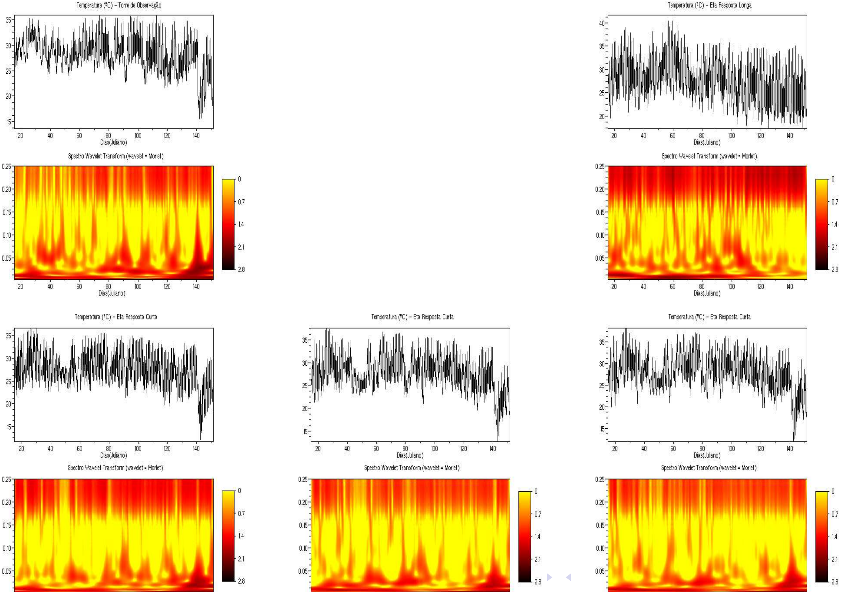
Morlet wavelet - real part



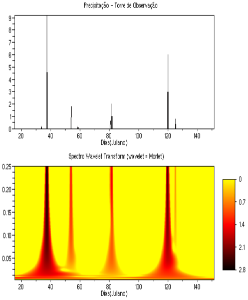
Methodology:

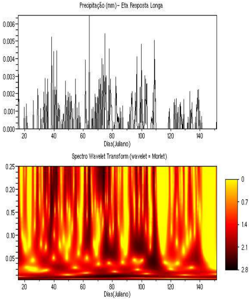
- short and long range Eta model runs
- an observation station data sets
- during part of a summer/fall season
- analysis of variance wavelet:scalogram
- using the continuous **wavelet** transform with Morlet mother-wavelet, family 6.

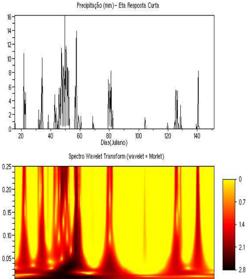
Air Temperature (2 meters)

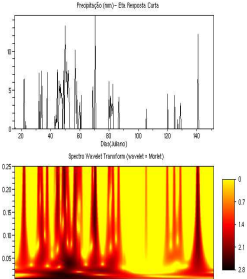


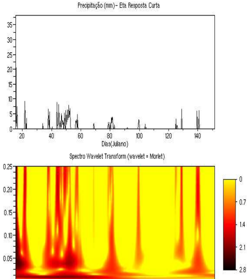
Precipitation (mm/day)



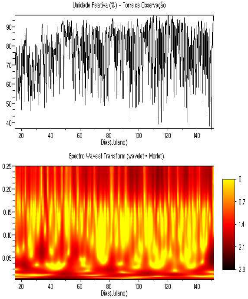


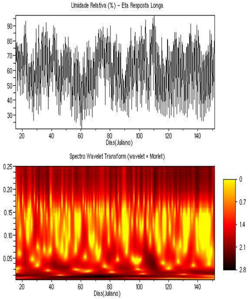


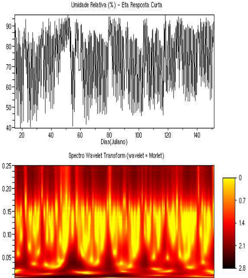


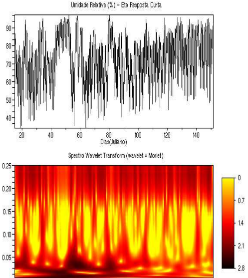


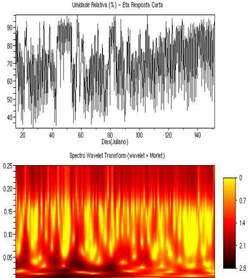
Relative Humidity (%)



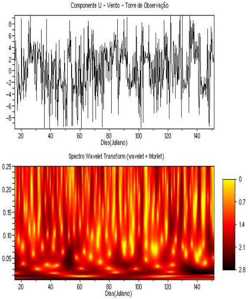


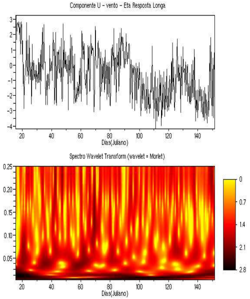


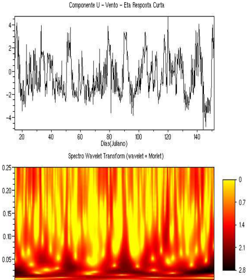


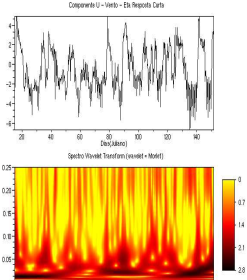


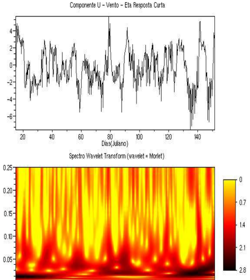
Zonal wind (m/s)



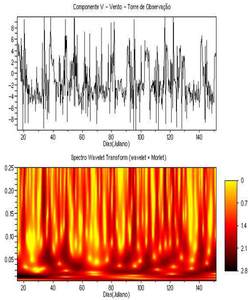


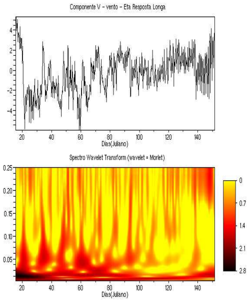


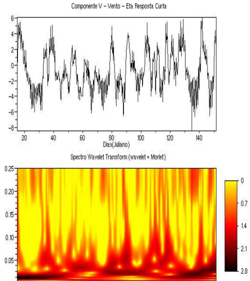


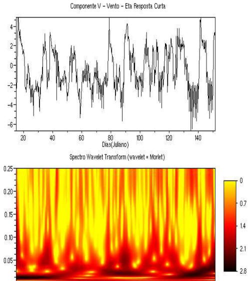


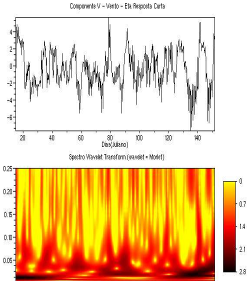
Meridional wind (m/s)







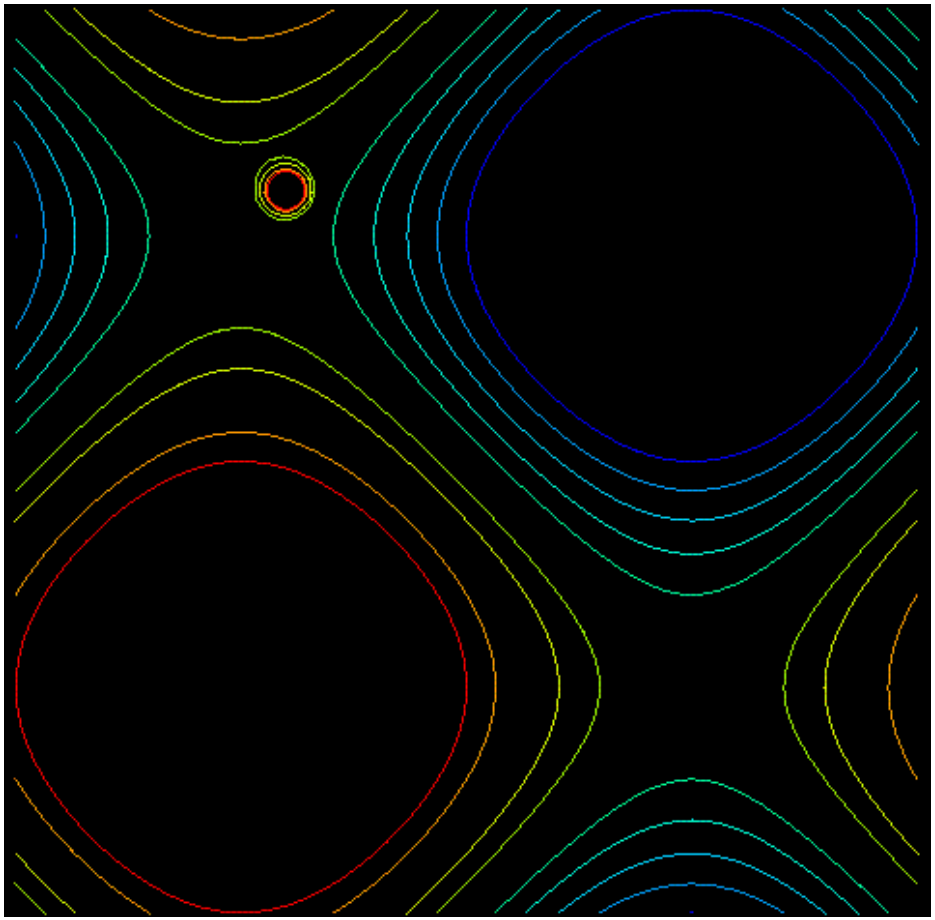


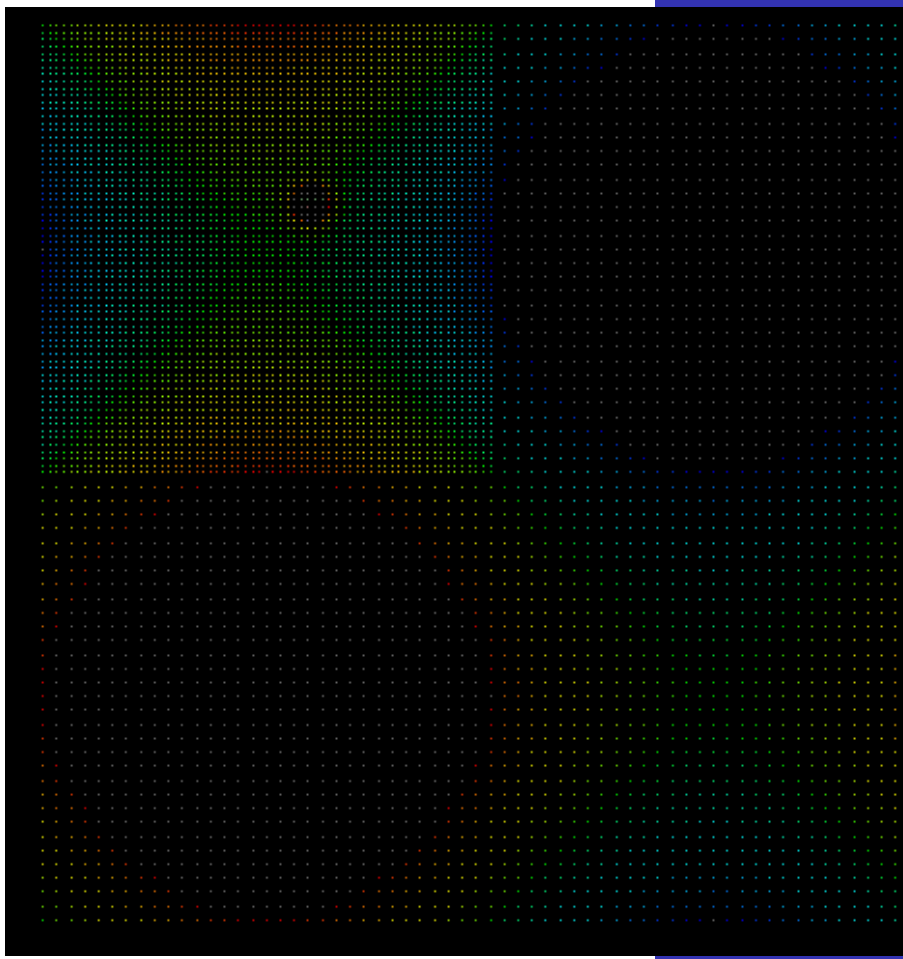


Next steps !

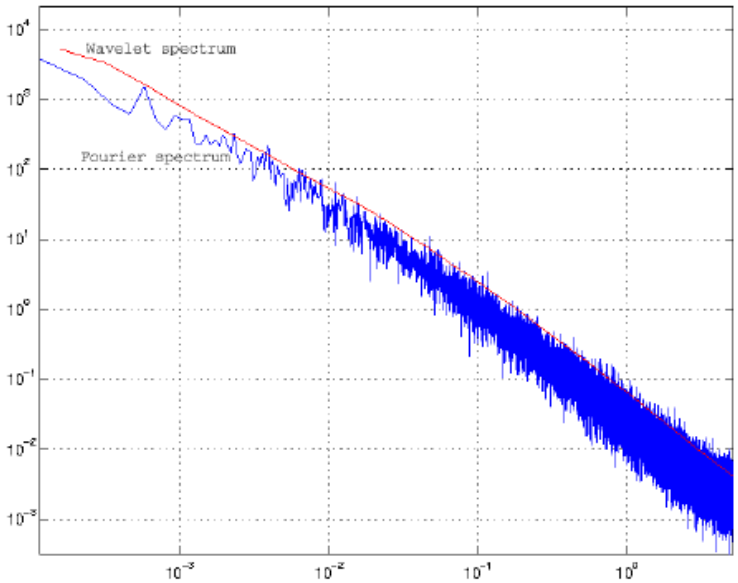
- To use more features of this wavelet, as the **phase** and the global wavelet aspects.
- To identify why could be the causes of these differences;
- To study if this behaviour is representative in space:
  - Using a two or three dimensional transform - time-space multiscale analysis.

Other examples: automatic mesh refinement





Other examples: turbulence analysis



Obrigada! Thanks!

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C. H. Chen, editor.

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I Daubechies.

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