Optimum Detector to Non-Orthogonal PAM Signals with Spectral Overlapping

Antonio Macilio Pereira de Lucena, João César Moura Mota and Charles Casimiro Cavalcante

Abstract— This work presents the optimum detector for detection of two non-orthogonal *n*-PAM signals with spectral overlapping through an AWGN band-limited channel based on joint maximum-likelihood criterion. The performance of signal detection in terms of symbol error rate for the new detector is evaluated using Monte Carlo simulations. The obtained results from this strategy represent a benchmark to this sort of communication system and comparative simulations are carried out in order to evaluate the performance faced to existing detection techniques for the discussed system showing the obtained lower bound.

Index Terms— PAM, multi-user communication, spectral overlapping, maximum-likelihood, detection.

I. Introduction

THERE are few references in literature about digital communication systems that utilizes non-orthogonal signal intentionally to transmit information. The $\{m\text{-QAM}\}^2$ modulation system [1] transmits simultaneously two non-orthogonal *m*-QAM signals with spectral overlapping but requires a very large channel bandwidth to prevent signal distortion [2].

In [3] and [4] a communication system which transmits at the same time two non-orthogonal band-limited *n*-PAM signals with partial spectral superposition over AWGN channels has been presented. The structure of symbol detection system proposed in [3] is based on source separation concepts. In [4], the symbol detector uses the maximum-likelihood criteria and is implemented by an extension of Viterbi algorithm applied separately to each one of demodulated symbols.

This work presents a symbol detector based on maximum-likelihood criteria jointly applied to the two demodulated signals. The solution turns out to be the optimum detector for the system we are handling. The new system performance in terms of symbol error rate is determined and compared with the results shown in [3-4].

The rest of the paper is organized as follows. In Section II, the system model is described, expression to signals and to noise are presented as well as the cross correlation between the noise part of demodulated signals. In Section III, the foundations of joint optimization using the maximum-likelihood criteria are discussed. Monte Carlo simulation results for symbol error rate are presented in Section IV. Finally in Section V, our conclusions and perspectives are stated.

II. System Description

The block diagram of system transmitter is given in the Fig.1. The signals $x_1(k)$ and $x_2(k)$ are the transmitted symbols at instant k from two independent sources. Symbol duration is T, and f_1 and f_2 are the carrier frequencies of each *n*-PAM signal. It is supposed $f_2 > f_1$ and $\Delta f = f_2 - f_1 < 1/T$ in order to have spectral overlapping between the transmitted signals. The pulse shaping filter is represented by $g(t)$ and we suppose the pulse spectrum is a square root raised cosine with roll-off factor equals zero, i.e., $g(t)=\text{sinc}(t/T)/T^{1/2}$.

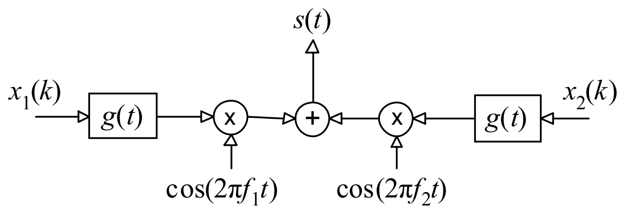

Fig. 1- Transmitter block diagram.

The receiver block diagram is shown in Fig. 2. We assume perfect synchronization at receiver end. The function of low-pass filters (LPF) at mixer outputs is to eliminate the signal around $2f_1$, $2f_2$, and f_1+f_2. The filters $g(t)$ are matched filters identical to the transmitter ones. The channel is considered AWGN with bandwidth $B=1/T+\Delta f$. Thus, the noise $n(t)$ at the receiver input is supposed to be white gaussian with power spectral density $N_0/2$ and zero mean.

The demodulated and sampled signals $d_1(m)$ and $d_2(m)$, as indicated in Fig. 2, are the inputs of a symbol detection system that deliver the estimate of transmitted symbols $x_1(m)$ and $x_2(m)$.

The signal $r(t)$ at receiver input is given by:

$$r(t) = [\sum_{k=0}^{\infty} x_1(k) g(t-kT)] \cos 2\pi f_1 t + [\sum_{k=0}^{\infty} x_2(k) g(t-kT)] \cos 2\pi f_2 t + n(t). \qquad (1)$$

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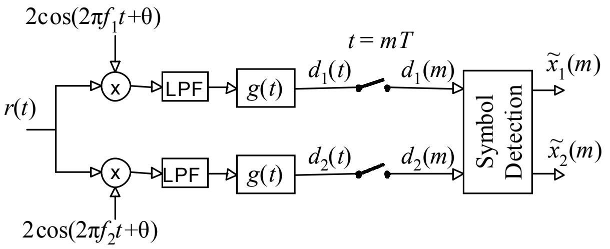

Fig. 2- Receiver structure.

The signals $d_1(m)$ and $d_2(m)$ are the demodulated signals sampled at time t=mT and given by [3-4]:

$$d_1(m) = x_1(m-L) + (1-T\Delta f)\sum_{j=0}^{2L} \mathrm{sinc}[(1-T\Delta f)(j-L)]\cos[\pi T\Delta f(2m+L-j)]x_2(m-j) + n_1(m-L), \tag{2}$$

$$d_2(m) = x_2(m-L) + (1-T\Delta f)\sum_{j=0}^{2L} \mathrm{sinc}[(1-T\Delta f)(j-L)]\cos[\pi T\Delta f(2m+L-j)]x_1(m-j) + n_2(m-L). \tag{3}$$

where $2L$+1 are the total number of significant terms of sequence in k sinc[(1-$T\Delta f$)(m-k)] for each instant m, $n_1(m\text{-}L)$ and $n_2(m\text{-}L)$ are individually white gaussian noise with variance $\boldsymbol{N}_0$ and zero mean, and the cross-correlation between them is given by [2]:

$$E[n_1(l)n_2(k)] = N_0 \int_{-\infty}^{\infty} \cos(2\pi\Delta f Tx)\mathrm{sinc}(l-x)\mathrm{sinc}(k-x)dx. \tag{4}$$

Notice by Eq. (4) that the random process $n_1(m)$ and $n_2(m)$ are not jointly stationary.

The time discrete model representing the system since transmission up to demodulation is shown at Fig. 3 [3-4].

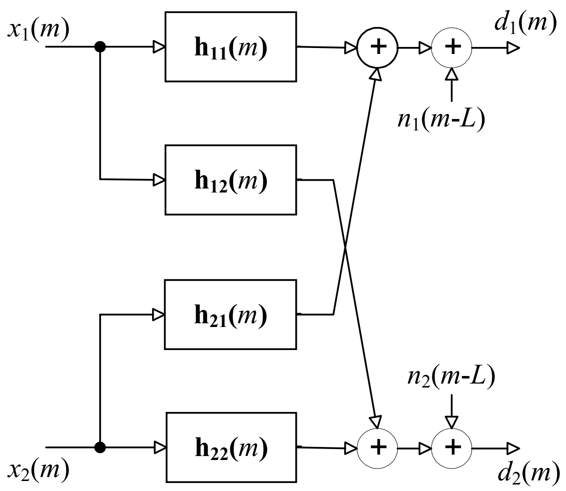

Fig. 3 – System Model.

The filters $\mathbf{h}_{11}(m)$, $\mathbf{h}_{21}(m)$, $\mathbf{h}_{12}(m)$, and $\mathbf{h}_{22}(m)$ can be described in vectorial representation as following:

$$\mathbf{h}_{11}(m) = \mathbf{h}_{22}(m) = [\ 0\ \ 0\ \ ...\ \ 1\ \ ...\ 0\ \ 0\ ] \tag{5}$$

$$\mathbf{h}_{12}(m) = \mathbf{h}_{21}(m) = [\ h_0(m)\ \ ...\ \ h_i(m)\ \ ...\ h_{2L}(m)\ ] \tag{6}$$

where each vector component $h_i(m)$, for i = 0,...,$2L$, is given by:

$$h_i(m) = (1\text{-}T\Delta f)\ \mathrm{sinc}[(1\text{-}T\Delta f)(i\text{-}L)]\cos\pi T\Delta f(2m+L\text{-}i). \tag{7}$$

We would like to remark that each demodulated signal, $d_1(m)$ and $d_2(m)$, contains the desired symbol plus gaussian noise and $2L$+1 interference terms originated from another n-PAM signal. In fact, the interfering signal is equivalent to the output of a linear time variant FIR filter which has the second user symbol as its input.

III. Optimal Symbol Detection

If the symbol detection is performed directly from demodulated signals $d_1(m)$ or $d_2(m)$, utilizing a simple decision circuit, e.g. a hard limiter, the symbol error rate should be very high due to the $2L$+1 interference terms [3-4]. The Fig. 4 shows the symbol error rate (SER) for binary PAM with frequency superposition ($T\Delta f$ =1/3) and L = 1, 3, 5, compared to conventional PAM.

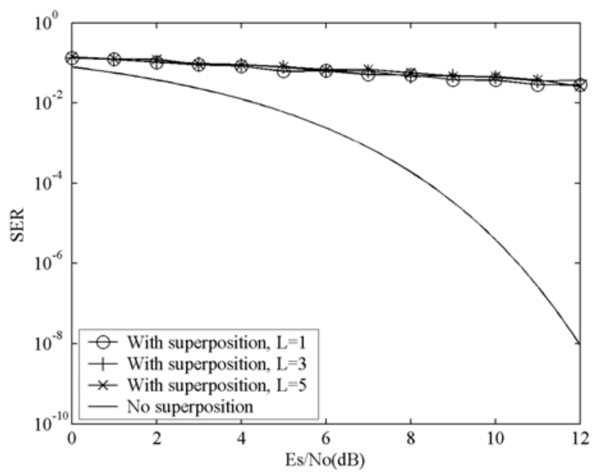

Fig. 4 - SER for overlapped 2-PAM system with superposition ($T\Delta f$ =1/3) compared to conventional 2-PAM.

In [4], the proposed detection techniques, based on maximum likelihood criteria individually applied to each sequence of demodulated symbol, although efficient, is not optimum because only half of available data sequences was used in the optimization process.

In this work we present the maximum likelihood sequence estimator (MLSE) when both sequences of demodulated signals $\{d_1(m)\}$ and $\{d_2(m)\}$ are jointly considered. Such strategy is globally optimum and minimizes the joint error probability of detection [5-6].

Let suppose that N symbol sequences for each demodulated signal $d_1(m)$ and $d_2(m)$, represented by vectors $\mathbf{d_1} = [\ d_1(1), d_1(2), \ ... \ , d_1(N)\ ]^T$ and $\mathbf{d_2} = [\ d_2(1), d_2(2), \ ... \ , d_2(N)\ ]^T$, are observed by detection system. Since the processes $n_1(m)$ and $n_2(m)$ are Gaussian, the conditional joint probability density function of $\mathbf{d_1}$ and $\mathbf{d_2}$, supposing that $\mathbf{x_1} = [\ x_1(1\text{-}L), x_1(2\text{-}L), \ ... \ , x_1(N)\ ]^T$ and $\mathbf{x_2} = [\ x_2(1\text{-}2L), x_2(2\text{-}2L), \ ... \ , x_2(N)\ ]^T$ are known, can be written as [5]

$$p(\mathbf{d}_1, \mathbf{d}_2 \mid \mathbf{x}_1, \mathbf{x}_2) = \frac{1}{(2\pi)^N (\det \mathbf{C})^{1/2}} \exp[-(\mathbf{d}-\boldsymbol{\mu})^{\mathrm{T}} \mathbf{C}^{-1} (\mathbf{d}-\boldsymbol{\mu})/2] \tag{8}$$

where $\mathbf{d}=[d_1(1), d_1(2), \ ... \ ,d_1(N), d_2(1), d_2(2), \ ... \ ,d_2(N)\ ]^{\mathrm{T}}$, $\boldsymbol{\mu}= \{\mu_1(1), \mu_1(2), \ ... \ , \mu_1(\mathrm{N}), \mu_2(1), \mu_2(2), \ ... \ , \mu_2(\mathrm{N})\ ]^{\mathrm{T}}$, being $\mu_1(m)$, $\mu_2(m)$ the mean value of $d_1(m)$ and $d_2(m)$, and $\mathbf{C}$ denotes the covariance matrix of vector $\mathbf{d}$. It is easy to verify that the covariance matrix $\mathbf{C}$ is $2N$x$2N$ square matrix with the following form:

$$\mathbf{C} = \begin{bmatrix} N_0\mathbf{I} & \mathbf{R} \\ \mathbf{R}^T & N_0\mathbf{I} \end{bmatrix},$$

where $\mathbf{I}$ denotes the identity matrix with dimension N and the elements r_{ij} of matrix $\mathbf{R}$ (NxN) are given by

$$r_{ij} = E[n_1(i)n_2(j)], \tag{9}$$

which can be valued by Eq. (4). The components $\mu_1(m)$, $\mu_2(m)$ of mean vector are given by

$$\begin{aligned} \mu_1(m) &= x_1(m-L) + \sum_{j=0}^{2L} h_j(m) x_2(m-j), \\ \mu_2(m) &= x_2(m-L) + \sum_{j=0}^{2L} h_j(m) x_1(m-j), \end{aligned} \tag{10}$$

where the coefficients $h_j(m)$ are given by Eq. (7). It is interesting to remark that although the coefficients $h_j(m)$ are time variant they are known at receiver when there is a perfect carrier and symbol synchronization.

The MLSE detector assigns the sequences $\{x_1\}$ and $\{x_2\}$ as the transmitted ones which maximize the probability density function defined by Eq. (8), or equivalently minimize the metric M expressed by

$$M = (\mathbf{d}-\boldsymbol{\mu})^{\mathrm{T}} \mathbf{C}^{-1} (\mathbf{d}-\boldsymbol{\mu}). \tag{11}$$

In order to compute the metric in Eq. (11), the MLSE detector should have, for binary PAM, 2^{2N} sequences to make the decision. Obviously the complexity of such detector is prohibitive in practice. But it is very important to know the performance of MLSE detector for the present case of two non-orthogonal n-PAM signal with frequency overlapping because such performance represents a bench mark to the system. Next section presents some Monte Carlo simulation results of symbol error rate for the MLSE detector.

IV. Numerical Results

We have simulated a system with binary PAM signals, ΔfT=1/3, and L=1. We adopt L=1 because the SER, like shown in Fig. 4, is almost the same for all values of L. In this case, the filters for the equivalent model described in Fig. 3 are the following:

$$\mathbf{h_{11}}(m) = \mathbf{h_{22}}(m) = [0\ 1\ 0]\ ,$$

$$\mathbf{h_{12}}(m) = \mathbf{h_{21}}(m) = [h_0(m)\ \ h_1(m)\ \ h_2(m)],$$

where

$$h_0(m)=0{,}2757\cos[\pi(2m+1)/3],$$

$$h_1(m)=0{,}6667\cos[\pi(2m)/3],$$

$$h_2(m)=0{,}2757\cos[\pi(2m\text{-}1)/3].$$

The block diagram of equivalent channel to demodulated signal $d_1(m)$ is shown in Fig. 5. The channel structure to the signal $d_2(m)$ is completely analogous to this one.

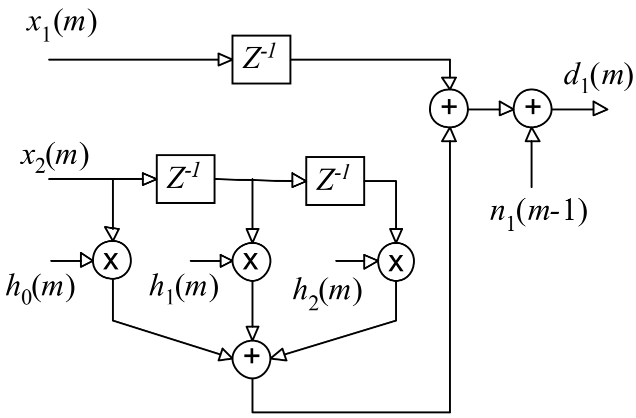

Fig. 5- Equivalent channel to signal $d_1(m)$.

It worth noting that the filter coefficients, $h_0(m)$, $h_1(m)$, and $h_3(m)$, although time variant, are periodic with period $P = 3$, resulting in only three possible filters:

$$\mathbf{h}_{12}(0) = \mathbf{h}_{21}(0) = [\ 0{,}1379 \quad 0{,}6667 \quad 0{,}1379],$$
$$\mathbf{h}_{12}(1) = \mathbf{h}_{21}(1) = [-0{,}2757 \quad -0{,}3333 \quad 0{,}1379],$$
$$\mathbf{h}_{12}(2) = \mathbf{h}_{21}(2) = [\ 0{,}1379 \quad -0{,}3334 \quad -0{,}2757].$$

In order to reduce the complexity and the computation time, the detector implemented in the simulation uses symbol sequences with length N equal to 5. It is expected that a better performance should be reached if it was used sequences with larger length. Fig. 6 shows the symbol error rate (SER) at different values of symbol energy by noise density (E_s/N_0) for the MLSE detector scheme compared with the PAM without frequency superposition ($\Delta fT = 1$) and with the situation when the detection is performed directly from demodulated signal without interference cancellation (hard limiter).

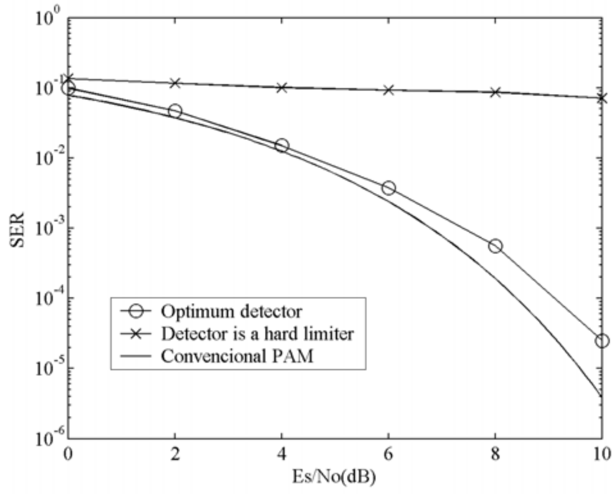

Fig. 6 - SER for overlapped 2-PAM system, with $T\Delta f = 1/3$, when detection is done by optimum detector, hard limiter and compared to conventional 2-PAM.

The results shown in Fig. 6 clearly indicate the system with optimum detector has a very good performance. The loss of performance compared to the PAM without frequency overlapping is less than 1.0 dB for SER=10^{-5}.

Results comparing the performance of MLSE detector with the techniques proposed in [3] and [4] are shown in Fig. 7.

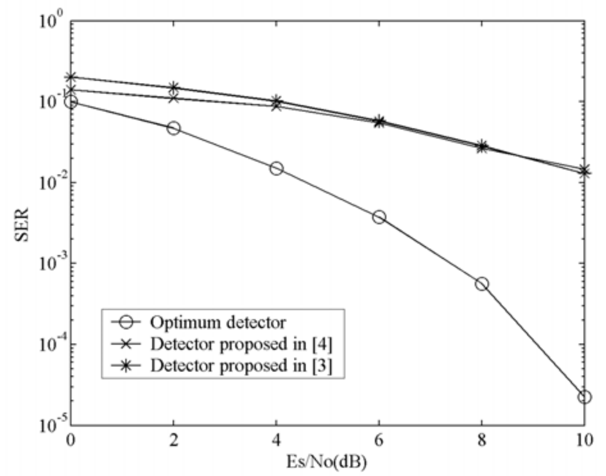

Fig. 7- Performance comparison of the optimum (MLSE) detector with alternative detection strategies in overlapped ($T\Delta f = 1/3$) 2-PAM system.

The very good performance of optimum detector, and the large difference of symbol error rate between the MLSE detector and the proposed techniques described in [3-4] suggest that may exist other suboptimum detection strategies with performance closer to the optimum.

The presented results also suggest the possibility of two signals sharing the same frequency band without high degradation in terms of performance.

V. Conclusion and Perspectives

The detector based on joint ML criterion presents a very good performance and almost completely eliminates the effect of interference terms for the present communication system which employ two non-orthogonal *n*-PAM signal with frequency overlapping. Probably, the high efficiency of optimum detector for this case is explained by the well-conditioned nature of interference.

The large difference of symbol error rate between the MLSE detector and the proposed techniques described in literature [3-4] suggest that it must exist other suboptimum detection strategies with performance closer to the optimum. Such possibility is interesting because, due to the high complexity, the use of MLSE detector is prohibitive in practice.

Investigation of new suboptimum detectors with low complexity and application to other modulation schemes are being considered in our research.

References

[1] E. C. Giraudo, F. R. Baldini and R. R. Scarabucci, , "*On the {m-QAM}^{2} Modulation*". IEEE Communications Letters, Vol. 5, n^o. 10, pp. 426-428, O*ct*. 2001.

[2] Lucena, A. M. P., "*Study on Communication System with Non-Orthogonal Signals Overlapped in Frequency*", *Technical Report*, Dep. of Teleinformatics Eng., Federal University of Ceará-UFC, Brazil, February *2006.*

[3] A. M. P. de Lucena, J. C. M. Mota, C. C. Cavalcante, "*Detection of Non-Orthogonal PAM Signal with Spectral Overlapping*", Proceedings of The Sixth IEEE International Workshop on Signal Processing Advances in Wireless Communications, New York, June 2005.

[4] A. M. P. de Lucena, Charles Casimiro Cavalcante and João Cesar M. Mota, "*Detection of Non-Orthogonal PAM Signal with Spectral Superposition Using ML Criterion*", Proceedings of the XXII Brazilian Telecommunication Symposium (SBrT2005), September 4th-8th, Campinas-SP, Brazil, 2005.

[5] J. G. Proakis, "*Digital Communications*" , 3rd ed. New York, McGrawHill, 1995.

[6] Verdu, S.; "Minimum Probability of Error for Asynchronous Gaussian Multiple-Access Channels", IEEE Trans. On Information Theory, Vol. IT-32, No. 1, January 1986.