Nr 74 Prace Naukowe Instytutu Konstrukcji i Eksploatacji Maszyn Politechniki Wrocławskiej Nr 74

Konferencje Nr 20 1993

POTENTIAL FLOW CALCULATION IN CENTRIFUGAL IMPELLERS OF TURBOMACHINES BY MEANS OF THE PANEL METHOD

WALDIR DE OLIVEIRA
EFEI - Escola Federal de Engenharia, Itajubá, MG, Brazil
NELSON MANZANARES FILHO
EFEI - Escola Federal de Engenharia, Itajubá, MG, Brazil
JERZY TADEUSZ SIELAWA
INPE - Cachoeira Paulista, SP /EFEI, Itajubá, MG, Brazil

The potential flow through centrifugal impellers of turbomachines is analysed with the purpose of determining the theoretical characteristics of the flow. The method of singularities is used and the numerical solution is obtained by means of the panel method. Some theoretical results are compared with the experimental ones obtained by means of tests with centrigugal fans.

INTRODUCTION

In the design of a turbomachine, a reliable method for predicting the flow condition in its interior is essential to secure a good performance. As it is well known, the flow inside any turbomachine is extremely complex due to the geometry and the interdependence between the rotating and stationary components. Therefore, a satisfactory theory that takes into account the combined effect of all components on a real flow through the turbomachine is not yet available. For this reason, some simplifying assumptions are necessary. For example, it is common to analyse each component of a turbomachine separately. In the case of the impeller, besides of geometric complexities and viscous flow effects, there is the additional influence of the rotation. Certainly, the simplest flow model to be considered is the potential one.

The purpose of this paper is to determine the characteristics of the potential incompressible steady flow inside centrifugal impellers, such as, pressure distribution on the blade surface, pressure and flow coefficients .

The method of singularities is used following the classical formulation proposed by Martensen based on a vortex distribution on the blade contour. The centrifugal impellers can have blades of arbitrary shape, variable width and finite thickness. Some theoretical results for blades with finite thickness are compared with those for infinitely thin blades obtained by means of another panel technique described by Manzanares (1982).

Theoretical results for some global performance parameters of centrifugal impellers are compared with experimental results obtained in tests with centrifugal fans.

THEORETICAL ANALYSIS

Figures 1 and 2 show, respectively, a radial cascade and a rectilinear cascade of variable width. The theoretical analysis is based on the assumptions that the flow is potential, incompressible, quasi two-dimensional and steady state.

The equation of continuity for the absolute flow is

$$\frac{\partial c_\sigma}{\partial \sigma} + \frac{1}{r}\frac{\partial c_\theta}{\partial \theta} + \left(\frac{1}{r}\frac{dr}{d\sigma} + \frac{1}{b}\frac{db}{d\sigma}\right)c_\sigma = 0$$

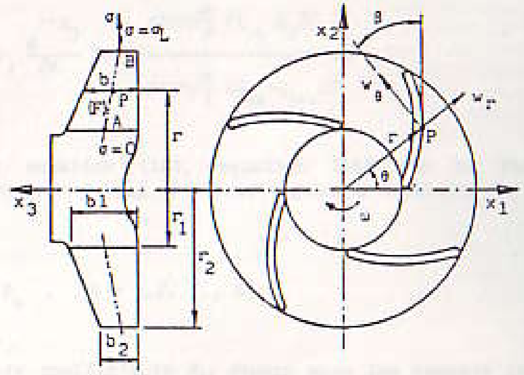

Figure 1. Physical plane (radial cascade)

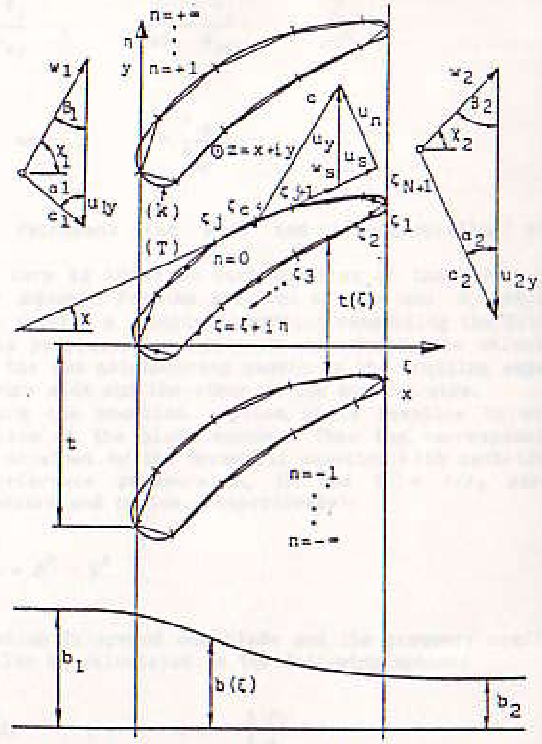

Figure 2. Mapped plane (rectilinear cascade)

and the equation expressing the irrotationality of the absolute flow is

$$\frac{\partial c_\theta}{\partial \sigma} - \frac{1}{r}\frac{\partial c_\sigma}{\partial \theta} + \frac{1}{r}\frac{dr}{d\sigma} c_\theta = 0 \qquad (2)$$

The axisymmetric stream surface (σ,θ) of the channel can be mapped on the plane (x,y) by the equations

$$x = \frac{Nt}{2\pi}\int_0^\sigma \frac{d\sigma}{r} \qquad \text{and} \qquad y = \frac{Nt}{2\pi}\theta \qquad (3)$$

The velocity (c) in the mapped plane is related to the velocity (c_R) in the physical plane by

$$c = \frac{2\pi r}{Nt} c_R . \qquad (4)$$

By using equation (4), equations (1) and (2) can be written for the mapped plane as

$$\frac{\partial c_x}{\partial x} + \frac{\partial c_y}{\partial y} = -\frac{1}{b}\frac{db}{dx} c_x \qquad (5)$$

and

$$\frac{\partial c_y}{\partial x} - \frac{\partial c_x}{\partial y} = 0 \qquad (6)$$

The potential function ϕ of the flow field in the mapped plane satisfies the following Poisson type equation

$$\nabla^2\phi = -\frac{1}{b}\frac{db}{dx} c_x = B(x)c_x \quad , \qquad (7)$$

The boundary conditions are

$$w_n = 0 \quad \text{on the blades,} \qquad (8)$$

$$\left.\frac{\partial\phi}{\partial x}\right|_{x=-\infty} = c_{1x} \qquad \text{and} \qquad \left.\frac{\partial\phi}{\partial y}\right|_{x=-\infty} = c_{1y} \qquad (9)$$

$$\left.\frac{\partial\phi}{\partial x}\right|_{x=+\infty} = c_{2x} \qquad \text{and} \qquad \left.\frac{\partial\phi}{\partial y}\right|_{x=+\infty} = c'_{2y} \qquad (10)$$

It is possible to develop an integral formulation satisfying equation (7) and the boundary conditions (equations (8), (9) and (10)) by applying the Green integral theorem, as given by Nyiri (1970):

$$c_s(\zeta) - \frac{1}{t}\oint_{(k)}\lambda_I(\zeta,\zeta')c_s(\zeta')ds' = [c\infty_x + c_{Bx}(\zeta)]\cos\chi +$$
$$+ [c\infty_y + c_{By}(\zeta)]\sin\chi + \frac{1}{t}\oint_{(k)}\lambda_{II}(\zeta,\zeta')u_n(\zeta')ds' \qquad (11)$$

Equation (11) is the Fredholm integral equation of the second kind. The kernel λ_I of this equation is limited when the point ζ' approaches the point ζ. The kernels $\lambda_I(\zeta,\zeta')$ and $\lambda_{II}(\zeta,\zeta')$ are given by Nyiri (1970) and

The conjugate complex velocity induced by a vortex distribution on the panel "j" of the cascade at the control point ζ_{ck} is given by Giesing (1964):

$$\bar{v}_j(\zeta_{ck}) = i\gamma_j \frac{e^{-i\chi_j}}{2\pi} \log\left(\frac{\sinh[\frac{\pi}{t}(\zeta_{ck}-\zeta_j)]}{\sinh[\frac{\pi}{t}(\zeta_{ck}-\zeta_{j+1})]}\right) \qquad (18)$$

Considering equation (18), equation (14) can be represented in nondimensional form by a system of linear algebraic equations for the unknowns W_{sj}:

$$\sum_{j=1}^{M} A_{kj} W_{sj} = B_k \quad , \quad k = 1,2,\ldots, M \qquad (19)$$

The influence coefficients A_{kj} depend upon the cascade geometry only. The values B_k depend also upon aerodynamic parameters. In the final formulation, it is convenient to consider the following definitions:

$$W_{sj} = \frac{w_{sj}}{u_{2y}} = \frac{\gamma_j}{u_{2y}} \quad , \qquad u_{yj} = \frac{u_j}{u_{2y}} \qquad (20.a,b)$$

$$\Phi = \frac{c_{2x}}{u_{2y}} \qquad \text{and} \qquad \Omega = \frac{c_{1y}}{u_{2y}} \quad , \qquad (20.c,d)$$

where ϕ and Ω represent the flow and pre-circulation coefficients, respectively.

A constant term is added to each equation of the system (19) and is treated as another unknown. For the solution of this new system of equations it is necessary to specify a unicity constraint resembling the Kutta-Joukowsky condition. For this purpouse, in this work one equates the velocities at the control points of the two neighbouring panels at the trailing edge, one panel being on the pressure side and the other on the suction side.

After solving the equation system it is possible to calculate the velocity distribution on the blade contour. Then the corresponding pressure distribution p is obtained by the Bernoulli equation with centrifugal effect, by adopting a reference pressure p_o (P and R = r/r_2 represent the nondimensional pressure and radius, respectively):

$$P = \frac{2(p-p_o)}{\omega^2 r_2^2} = R^2 - W^2 \qquad (21)$$

The circulation Γe around one blade and the pressure coefficient ψ of the impeller can also be calculated in the following manner:

$$\Gamma e = \oint_{(k)} c_s ds \quad , \qquad \psi = \frac{2\,\Gamma e}{t\,u_{2y}} \qquad (22a,b)$$

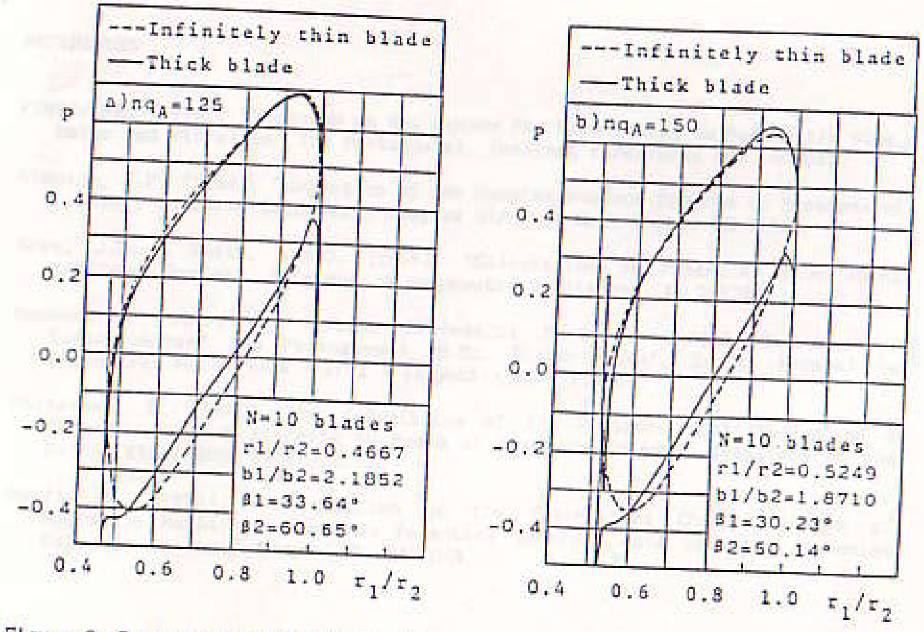

Figure 3. Pressure distribution on the circular arc blade with Ω=0 and $\phi = \phi_{sc}$

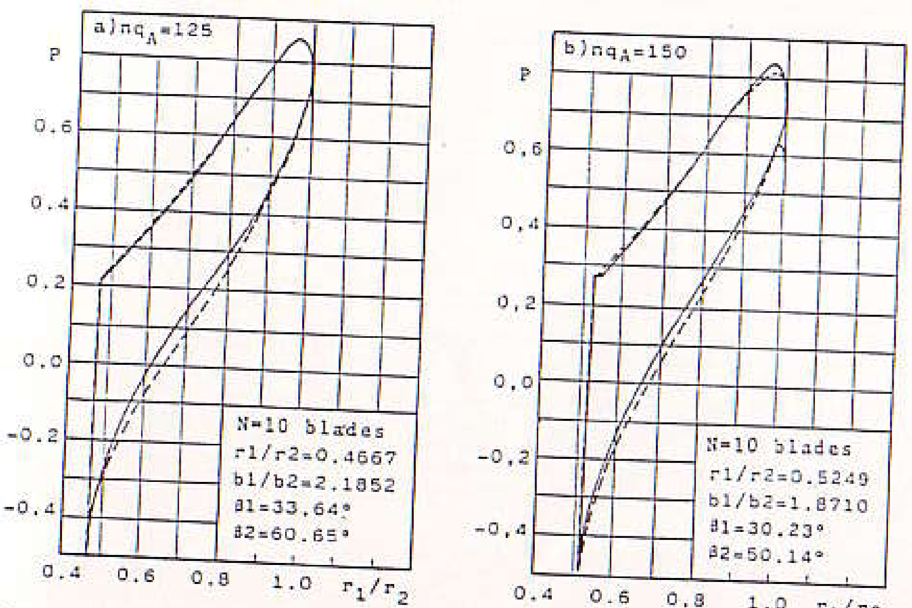

Figure 4. Pressure distribution on the circular arc blade with Ω=0 and $\phi = \phi_{opt}$

RESULTS

Some theoretical results were obtained for two cases of centrifugal impellers with circular arc blades and width varying linearly with radius. The cases represent impellers of fans actually constructed and tested, as described in the FINEP/EFEI Contract Report (1978). The Addison design specific speed is n_{qA}=125 for one case and n_{qA}=150 for the other one. The blade thicknes is 1.3% of external radius for n_{qA}=125 and 1.5% for n_{qA}=150. The blades have rounded leading edges; the trailing edges follow the external circunference of the impeller. Results were obtained for theoretical smooth entry conditions (indicated by an sc index) and for flow coefficients corresponding to the best efficiencies of the tested fans (indicated by an opt index).

In figures 3 and 4, one shows the pressure distributions calculated by the present method (300 panels) in comparasion with those obtained by the panel method developed by Manzanares (1982) in wich the blades are considered infinitly thin (40 panels). One observes a reasonable agreement between the two methods except near the leading and trailing edges. This would be expected since the blades are slightly thick. No experimental results are available for the pressure distributions.

The global results for the flow (ϕ) and pressure (ψ) coefficients are shown in Table 1, in comparison with experimental results. Again, both calculation methods agree reasonably. One observes that the theoretical pressure coefficients for nominal flow conditions, $\psi(\phi_{opt})$, agrees well with the experimental values of ψ_{opt}/η_{max}. One also notes that the theoretical flow coefficient for the smooth entry condition, ϕ_{sc}, is substantially greater than the experimental nominal flow coefficient, ϕ_{opt}. This indicates that the maximum efficiency operation occurs at a positive incidence angle at the leading edge.

Table 1. Experimental and theoretical results

	Experimental Results				Theoretical Results					
	thick blade				thick blade			infinitely thin blade		
n_{qA}	ϕ_{opt}	ψ_{opt}	η_{max}	ψ_{opt}/η_{max}	ϕ_{sc}	ψ_{sc}	$\psi(\phi_{opt})$	ϕ_{sc}	ψ_{sc}	$\psi(\phi_{opt})$
125	0.292	1.005	0.743	1.352	0.501	1.120	1.292	0.491	1.121	1.324
150	0.271	0.890	0.762	1.168	0.449	0.974	1.200	0.442	0.968	1.226

CONCLUSIONS

The panel method utilizing a vortex distribution along the contour of the thick blade is a versatile tool for analysing the potential flow in centrifugal impellers of turbomachines with blades of arbitrary shape. The present method allows a designer to explore many preliminary blade designs in a short period of time. Hence, the present method is considered promising for evolving a rational procedure for the design of low specific speed impellers.

In order to evaluate the potentialities of the method it is necessary to perform systematical experimental analysis of the flow in isolated centrifugal impellers without influence of the volute.

REFERENCES

FINEP/EFEI (1978), "Research on New Shapes for Fans Aiming to Reduce its Size, Noise and Vibration" (in Portuguese), Contract FINEP/EFEI num. 000082.

Giesing, J.P. (1964), "Extension of the Douglas Neumann Program to Problems of Lifting, Infinite Cascades", Douglas Aircraft Co., Report LB-31653.

Hess, J.L. & Smith, A.M.O. (1966), "Calculation of Potential Flow About Arbitrary Bodies", Progress in Aeronautical Sciences, 8: 1-138.

Manzanares Filho, N. (1982), "Potential Flow in Radial Cascades of Turbomachines" (in Portuguese), M.Sc. Dissertation", Escola Federal de Engenharia de Itajubá - EFEI - Itajubá - MG - Brazil.

Martensen, E. (1971), The Calculation of the Pressure Distribution on a Cascade of Thick Airfoils by Means of Fredholm Integral Equations of the Second Kind, NASA TT F-702.

Nyiri, A. (1970), Determination of the Theoretical Characteristics of Hydraulic Machines, based on Potential Theory, "Acta Technica Academiae Scientiarium Hungaricae", 69: 243-273.

Prace Naukowe Instytutu Konstrukcji i Eksploatacji Maszyn

Nr 74 Politechniki Wrocławskiej Nr 74

Konferencje Nr 20 1993

THREE-DIMENSIONAL VORTEX MODEL OF ANNULAR AIRFOIL WAKE

Tomasz Tabaczek, Jan Kulczyk

Technical University of Wroclaw, Department of Mechanical Engineering

ul. I. Lukasiewicza 7/9, 50-371 Wroclaw, Poland

The model based on the concept of vorticity is applied to simulate three-dimensional wake evolution downstream of the trailing edge of annular airfoil. The flow is assumed to be potential outside the bounded wake region of non-zero velocity rotation. In order to solve the problem numerically annular airfoil at incidence is substituted by the lifting line of prescribed sinusoidal distribution of circulation and vorticity in wake is concentrated in number of vortex filaments of finite core radius. The vortex sheet evolution equation is applied to trace the wake development. The results of computations agree well with the results obtained by the other authors for two-dimensional cylindrical vortex sheet.

1. INTRODUCTION

In wide range of vortex models reported in the literature in application to lifting flows one can recognize, taking approach to wake treatment as a criterion, two categories: models with assumed simplified shape of the wake (a) and those in which the wake is subject to the vortex sheet evolution equation (b). In the first category the wake shape is assumed to be known apriori and, in many cases, to coincide with stream sheet of an undisturbed flow. If empirical observations reveal strong deformation of a wake and formation of highly rolled-up regions then the model is simplified to several vortex filaments imitating regions of high rotation. In the cases where the shape simplification occurs too crude, some empirical or semiempirical coefficients are introduced to algorithms to modify the shape of the wake modelling vortex filaments or surface. It is the case in ship screw propeller calculations where the slipstream contraction and pitch change are often taken into account in this manner. Those models (a) provide with sufficiently good results as far as the magnitudes related to an object in the flow are concerned (e.g. lift force or pressure distribution).

In those cases where the wake's influence on the magnitudes of interest, e.g. on velocities in some distance downstream of an object, is so strong that any simplification of wake geometry leads to excessive errors the models of category (b) are in use. The sample ideas of approach to modelling 3-D wake geometry are those by Koronowicz [1] in application to elliptically loaded wing, Katz [2] - to delta wing with leading edge separation, or